

# Abstract Book

## Emerging issues in nonlinear elliptic equations: singularities, singular perturbations and non local problems

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## Talk Abstracts

# Anisotropic semipositone quasilinear problems

Oscar Agudelo

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## Abstract

We present the existence of positive solutions for a degenerate (or singular) quasilinear equation in a ball or in the entire space  $\mathbb{R}^N$ . As differential operator we consider a weighted  $p$ -Laplacian with a continuous coefficient satisfying a mild integrability condition towards the boundary of the ball or at infinity in  $\mathbb{R}^N$ . Our main result applies when the nonlinearities  $f(u)$  have  $(p - 1)$ -sublinear growth at infinity and singular semipositone structure at the origin.

This is a joint work with Pavel Draábek from the University of West Bohemia in Pilsen.

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# Periodic solutions of the point vortex Hamiltonian system

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**Abstract:** The dynamics of  $N$  point vortices  $z_1, \dots, z_N$  in a bounded domain  $\Omega \subset \mathbb{R}^2$  is described by the the Hamiltonian system

$$\Gamma_k \dot{z}_k = \nabla_{z_k}^\perp H(z_1, \dots, z_N)$$

with Hamiltonian

$$H(z_1, \dots, z_N) = -\frac{1}{2\pi} \sum_{\substack{j,k=1 \\ j \neq k}}^N \Gamma_j \Gamma_k \log |z_j - z_k| - \sum_{j,k=1}^N \Gamma_j \Gamma_k g(z_j, z_k).$$

Here  $g : \Omega \times \Omega \rightarrow \mathbb{R}$  is the regular part of the Green's function of the Dirichlet Laplacian in  $\Omega$ , and  $\Gamma_k \neq 0$ ,  $k = 1, \dots, N$ , are real parameters. The system arises as a limit of the Euler equation in vorticity form when the vorticity concentrates at  $N$  points in the domain.

We prove the existence of periodic solutions of the system that are localized near critical points of the Robin function  $h(x) = g(x, x)$ . After a blow-up these solutions look like a periodic solution of the point vortex Hamiltonian system in the plane. Using a degree theory for  $S^1$ -equivariant potential operators we also show that our solutions lie on a global continuum of periodic solutions.

This is joint work with Qianhui Dai and Björn Gebhard.

# Blow up phenomena for Liouville systems

Luca Battaglia

We consider the following general system of two Liouville-type PDEs:

$$\begin{cases} -\Delta u_1 = 2\lambda_1 e^{u_1} - a\lambda_2 e^{u_2} & \text{in } \Omega \\ -\Delta u_2 = 2\lambda_2 e^{u_2} - b\lambda_1 e^{u_1} & \text{in } \Omega \\ u_1 = u_2 = 0 & \text{on } \partial\Omega \end{cases},$$

where  $\Omega \subset \mathbb{R}^2$  is a smooth bounded domain and  $a, b, \lambda_1, \lambda_2 > 0$  are positive parameters.

Using a fixed-point argument, we construct some families of solutions which blow up as  $\lambda_1, \lambda_2 \rightarrow 0$ . In particular, such solutions blow up with a *tower of bubble* profile, namely they look like a superposition of rescaled entire solutions of singular Liouville equations (bubbles).

# FUJITA'S BLOWUP PROOF REVISITED

PIOTR BILER

ABSTRACT. We discuss sufficient conditions for blowup of nonnegative solutions of the Cauchy problem for the semilinear heat equation  $u_t = \Delta u + u^p$  in  $\mathbb{R}^d$  and radially symmetric solutions of chemotaxis systems with fractional diffusion  $u_t = -(-\Delta)^{\alpha/2}u + \nabla \cdot (u\nabla v)$ ,  $\Delta v + u = 0$ ,  $\alpha \in (0, 2]$ . The proof of blowup follows the original H. Fujita argument analyzing the moment  $\int G(x, t)u(x, t)dx$  where  $G$  solves the backward heat equation  $G_t + \Delta G = 0$  in  $\mathbb{R}^d \times (0, T)$  with the final condition  $G(\cdot, T) = \delta_0$ . A sufficient condition for the blowup is then interpreted in terms of a critical Morrey space norm, either  $M^{d(p-1)/2}(\mathbb{R}^d)$  or  $M^{d/\alpha}(\mathbb{R}^d)$ , of the initial condition  $u_0$ . On the other hand, if this norm of  $u_0$  is small, the solution is global in time.

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# ON MULTIPLICITY OF EIGENVALUES AND SYMMETRY OF EIGENFUNCTIONS OF THE $p$ -LAPLACIAN

VLADIMIR BOBKOV

Let  $B$  be a ball in  $\mathbb{R}^N$ ,  $N \geq 2$ . Consider the eigenvalue problem

$$(0.1) \quad \begin{cases} -\Delta_p u = \lambda |u|^{p-2} u & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

where  $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ ,  $p > 1$ . It is well-known that a sequence of variational eigenvalues of (0.1) can be obtained by means of the following minimax variational principle:

$$\lambda_k(p; B) := \inf_{\mathcal{A} \in \Gamma_k(p)} \max_{u \in \mathcal{A}} \frac{\int_B |\nabla u|^p dx}{\int_B |u|^p dx}, \quad k \in \mathbb{N},$$

where  $\Gamma_k(p)$  is a family of symmetric and compact subsets of  $W_0^{1,p}(B)$  with Krasnosel'skii genus greater than or equal to  $k$ .

In the linear case  $p = 2$ , it is well-known that  $\lambda_2(2; B) = \dots = \lambda_{N+1}(2; B) = \lambda_\ominus(2; B)$ , where  $\lambda_\ominus(2; B)$  is the eigenvalue of the Laplacian which has an associated eigenfunction whose nodal set is an equatorial section of  $B$ . We are interested in the generalization of this fact to the nonlinear case. One of our main results in this direction is formulated as follows.

**Theorem 1.** *The following chain of inequalities is satisfied:*

$$\lambda_2(p; B) \leq \dots \leq \lambda_{N+1}(p; B) \leq \lambda_\ominus(p; B).$$

If  $\lambda_2(p; B) = \lambda_\ominus(p; B)$ , as it holds true for  $p = 2$ , this result implies that the variational (algebraic) multiplicity of the second eigenvalue is at least  $N$ . Moreover, in the case  $N = 2$ , we can deduce from [1] that any *third* eigenfunction of the  $p$ -Laplacian on a disc is nonradial.

This is a joint work with BENJAMIN AUDOUX and ENEA PARINI (Aix Marseille Univ, CNRS, Marseille, France), see [2].

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# On a class of asymptotically linear systems

Maya Chhetri

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We will consider a class of asymptotically linear elliptic system with zero Dirichlet boundary conditons. We will discuss the existence of unbounded connected components of the solution set and discuss the nodal properties of solutions on these components. As a consequence, we infer the existence and multiplicity of solutions in a neighborhood of the bifurcation parameter containing the simple eigenvalue of the associated eigenvalue problem.

EXISTENCE AND MULTIPLICITY RESULTS  
FOR SOME QUASILINEAR ELLIPTIC  
PROBLEMS

Jorge Cossio\*  
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In this lecture we study the existence of solutions for the quasilinear elliptic boundary value problem

$$\begin{cases} \Delta_p u + f(u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where  $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$  is the  $p$ -Laplace operator,  $p > 1$ ,  $\Omega \subset \mathbb{R}^N$  ( $N \geq 2$ ) is a bounded and smooth domain, and  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a nonlinear function such that  $f(0) = 0$ .

First we present some results concerning the existence of multiple *radial* solutions for problem (1), when the nonlinearity is either  $p$ -asymptotically linear at infinity or  $p$ -asymptotically superlinear at the origin. The main tools that we use are bifurcation theory and the shooting method.

Additionally, we prove the existence of multiple solutions for problem (1), when the  $p$ -derivative at zero and the  $p$ -derivative at infinity are greater than the first eigenvalue of the  $p$ -Laplace operator. We extend to quasilinear equations a result due to J. Cossio, S. Herrón, and C. Vélez [2] for the semilinear case. Our proof uses bifurcation from infinity and bifurcation from zero to prove the existence of unbounded branches of positive solutions (resp. of negative solutions). We show the existence of multiple solutions and we provide qualitative properties of these solutions.

\* Joint work with S. Herrón and C. Vélez (Universidad Nacional de Colombia)

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## 2D QUASI-GEOSTROPHIC EQUATION; SUB-CRITICAL AND CRITICAL CASES

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Emerging issues in nonlinear elliptic equations: singularities,  
singular perturbations and non local problems,  
Bdlewo, June 2017

We are using a version of the 'vanishing viscosity technique' to study the Dirichlet problem for *critical 2D Quasi-Geostrophic equation*:

$$(1) \quad \begin{aligned} \theta_t + u \cdot \nabla \theta + \kappa(-\Delta)^\alpha \theta &= f, \quad x \in \Omega \subset \mathbb{R}^2, t > 0, \\ \theta(0, x) &= \theta_0(x), \end{aligned}$$

where  $\theta$  represents the potential temperature,  $\kappa > 0$  is a diffusivity coefficient,  $\alpha \in [\frac{1}{2}, 1]$  a fractional exponent, and  $u = (u_1, u_2)$  is the *velocity field* determined by  $\theta$  through the relation:

$$(2) \quad u = \mathcal{R}^\perp \theta \quad \text{with } \mathcal{R} = \nabla(-\Delta)^{-\frac{1}{2}}.$$

Solving first the *sub-critical approximating equations* (1) with  $\alpha \in (\frac{1}{2}, 1]$ , we let then  $\alpha \rightarrow \frac{1}{2}$  to obtain a solution corresponding to the critical value of exponent  $\alpha = \frac{1}{2}$ . We discuss next in some details properties of solutions of the critical problem; in particular their uniqueness, regularity and long time behavior. The lecture is based on the recent publication [1], joint with Chunyou Sun, and uses the technique presented in [2].

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# Periodic orbits near heteroclinics.

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## Abstract

Let  $W : \mathbb{R}^m \rightarrow \mathbb{R}$ ,  $m \geq 1$  be a nonnegative potential with exactly two distinct zeros  $a_{\pm} \in \mathbb{R}^m$ . In the scalar case  $m = 1$  phase plane analysis shows that the Newton equation

$$(0.1) \quad u'' = W_u(u)$$

possesses a heteroclinic solution  $u^\infty$  that connects  $a_-$  to  $a_+$  and a family of periodic solutions  $u^T$  that converge in compacts to  $u^\infty$  as  $T \rightarrow +\infty$ .

We prove that, under the assumption that  $W$  is invariant under the reflection  $\gamma$  that exchange  $a_-$  to  $a_+$ , the same is true in the vector case  $m > 1$ .

We also extend this result to an infinite dimensional setting. We assume that  $W$  is invariant under a reflection  $\sigma$  that fixes  $a_{\pm}$ ,  $\sigma a_{\pm} = a_{\pm}$ , and that there exist exactly two distinct heteroclinic solutions of (0.1)  $\bar{u}_-$  and  $\bar{u}_+$  that satisfy

$$\bar{u}_- = \sigma \bar{u}_+.$$

Under a non degeneracy assumption on  $\bar{u}_{\pm}$  we show that, for  $L > L_0$ , the PDE system

$$\Delta u = W_u(u),$$

has a solution  $u^L : \mathbb{R}^2 \rightarrow \mathbb{R}^m$  which is  $L$ -periodic in  $x$ ,  $u^L(x + L, y) = u^L(x, y)$ , and such that the restriction of  $u^L$  to  $(-\frac{L}{4}, \frac{L}{4}) \times \mathbb{R}$ , converges along a subsequence to a heteroclinic connection  $u^\infty : \mathbb{R}^2 \rightarrow \mathbb{R}^m$  between  $\bar{u}_-$  and  $\bar{u}_+$ :

$$\begin{aligned} \lim_{L \rightarrow +\infty} u^L(x, \cdot) &= u^\infty(x, \cdot), \\ \lim_{x \rightarrow \pm\infty} u^\infty(x, \cdot) &= \bar{u}_{\pm}. \end{aligned}$$

**Asymptotic behavior of nodal radial solutions for  
Moser-Trudinger problems in the ball**

Let  $B$  be the unit ball in  $\mathbb{R}^2$ . In 1990-1992 Adimurthi and Yadava computed the *border line* nonlinearity for the existence-nonexistence of *nodal* solutions in Moser-Trudinger problems in  $B$ . More precisely they proved the following results,

**Theorem (Adimurthi and Yadava)**

*Let us consider the problem*

$$\begin{cases} -\Delta u = \lambda u e^{u^2+|u|^\beta} & \text{in } B \\ u = 0 & \text{on } \partial B. \end{cases}$$

*Then we have that,*

*i) if  $1 < \beta < 2$  there exists a radial solution with  $k$  interior zeros for any integer  $k \geq 1$  and for any  $\lambda \in (0, \lambda_1)$ ,*

*ii) if  $0 \leq \beta \leq 1$  there exists  $\lambda = \lambda_{AY} > 0$  such that for any  $0 < \lambda < \lambda_{AY}$  there exist no solution.*

We will study the asymptotic behavior of the radial solution  $u_\varepsilon$  with  $k$  nodal zeros of the problem

$$\begin{cases} -\Delta u = \lambda u e^{u^2+|u|^{1+\varepsilon}} & \text{in } B \\ u = 0 & \text{on } \partial B. \end{cases}$$

with  $0 < \lambda < \lambda_{AY}$  and  $\varepsilon \rightarrow 0$ . This is a joint result with Daisuke Naimen (MIT, Hokkaido, Japan).

# THE FREDHOLM ALTERNATIVE FOR THE $p$ -LAPLACIAN IN EXTERIOR DOMAINS. PART I

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ABSTRACT. In this talk we study the behavior of the energy functional associated with the following resonant problem

$$\begin{cases} -\Delta_p u = \lambda_1 K(x)|u|^{p-2}u + h & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \end{cases} \quad (1)$$

where  $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$  is the  $p$ -Laplacian with  $p > 1$ ,  $B_1^c$  is the complement of the closed unit ball  $B_1$  in  $\mathbb{R}^N$  ( $N \geq 2$ ), the weight  $K$  and the function  $h$  are chosen appropriately,  $\lambda_1$  is the first eigenvalue of  $-\Delta_p$  in  $B_1^c$  relative to the weight  $K$ . Similar to known results for resonant problems on a bounded domain or on the entire space  $\mathbb{R}^N$  we show that the energy functional associated with (1) has “a saddle point geometry” when  $1 < p < 2 \leq N$ . On the other hand, we prove an improved Poincaré inequality and show that the energy functional has a “global minimizer geometry” when  $2 \leq p < N$ . The behavior of the energy functional will be used to obtain the existence of solutions to problem (1). The striking difference between our case and the entire space case is also discussed.

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This is a joint work with Pavel Drábek and Abhishek Sarkar (Department of Mathematics and NTIS, University of West Bohemia, Czech Republic).

# Emden - Fowler type equations. An approach to regularity via strongly nonlinear multiplicative inequalities

Agnieszka Kałamajska

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## Abstract

We deal with Emden–Fowler -type equations like:  $f''(x) = g(x)f^{-\theta}(x)$ , where  $\theta \in \mathbf{R}$ ,  $x \in (a, b)$ ,  $g$  belongs to  $L^p((a, b))$ , and their  $n$ -dimensional counterparts:  $\Delta f(x) = g(x)f^{-\theta}(x)$ , where  $x \in \Omega \subset \mathbf{R}^n$ . We obtain the *a priori* estimates for the solutions, information about their asymptotic behavior near boundary points and some existence results. As a tool we use certain nonlinear Poincaré inequalities, and some variants of strongly nonlinear multiplicative inequalities:

$$\int_{\{x \in (a,b): f(x) > 0\}} |f'(x)|^p (f(x))^{\theta p} dx \leq C \int_{\{x \in (a,b): f(x) > 0\}} \sqrt{|f(x)f''(x)|}^p (f(x))^{\theta p} dx,$$

where  $f$  is nonnegative.

Results are based on several works obtained together with Jan Peszek, Katarzyna Mazowiecka and Tomasz Choczewski.

# Singular elliptic problems on unbounded domains

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## Abstract

We consider the problem

$$\begin{cases} -\Delta u = \lambda K(x)f(u) & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \\ u(x) \rightarrow 0 & \text{as } |x| \rightarrow \infty, \end{cases}$$

where  $B_1^c = \{x \in \mathbb{R}^n \mid |x| > 1\}$ ,  $\lambda$  is a positive parameter,  $K$  belongs to a class of functions which satisfy certain decay assumptions and  $f$  belongs to a class of functions which are asymptotically linear and may be singular at the origin, namely  $\lim_{s \rightarrow 0^+} f(s) = -\infty$ . We discuss the existence and non existence of a positive solutions to such problems for certain values of parameter  $\lambda$ . We also obtain similar existence results when the domain is  $\mathbb{R}^n$ .

# BIFURCATION FROM INFINITY FOR AN ELLIPTIC PROBLEM IN $\mathbb{R}^N$

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The talk is based on [2] and [1]. We consider the asymptotically linear Schrödinger equation

$$-\Delta u + V(x)u = \lambda u + f(x, u), \quad x \in \mathbb{R}^n$$

as well as an abstract problem of the form

$$Lu = \lambda u + N(u),$$

where  $L$  is a linear (unbounded) operator in a Hilbert space. We show that if  $\lambda_0$  is an isolated eigenvalue for the linearization at infinity, then under some additional conditions there exists a sequence  $(u_n, \lambda_n)$  of solutions such that  $\|u_n\| \rightarrow \infty$  and  $\lambda_n \rightarrow \lambda_0$ . If the potential  $V \in L^\infty(\mathbb{R}^N)$ , then we use degree theory if the multiplicity of  $\lambda_0$  is odd and Morse theory (or more specifically, Gromoll-Meyer theory) if it is not. In the case of a possible potential-well, i.e., when  $V = V_0 + V_\infty$ , where  $V_\infty \in L^\infty$  and is strictly positive and  $V_0 \in L^p(\mathbb{R}^N)$  with  $p > N$ , then our approach is based on the version of the Conley index due to Rybakowski and the existence relies on a variant of the Landesman-Lazer conditions.

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# Nonradial solutions of nonlinear scalar field equations

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We look for nonradial solutions of the following nonlinear scalar field equation

$$\begin{cases} -\Delta u = g(u), & u \in H^1(\mathbb{R}^N), \quad N \geq 3, \end{cases}$$

with a nonlinearity  $g$  satisfying the general assumptions due to Berestycki and Lions. We present how to build a critical point theory on the Pohozaev manifold associated with the problem, which is only a topological manifold. In particular, we find nonradial solutions under some additional restrictions imposed on the dimension  $N$  or the nonlinearity  $g$ .

**Infinite time blow-up for critical heat equation in  
 $\mathbb{R}^3$  with fast decay initial condition**

MONICA MUSSO

**THE DIRICHLET PROBLEM FOR SOME SINGULAR  
ELLIPTIC EQUATIONS INVOLVING THE 1-LAPLACE  
OPERATOR**

FRANCESCO PETITTA

ABSTRACT. We will discuss singular elliptic problems whose simplest model is

$$\begin{cases} -\Delta_1 u = fu^{-\gamma} & x \in \Omega \\ u = 0 & \partial\Omega, \end{cases}$$

where  $\Delta_1 u = \operatorname{div} \frac{Du}{|Du|}$  is the usual 1-laplace operator,  $0 \leq f \in L^N(\Omega)$ , and  $\gamma > 0$ . For a fairly general class of such problems we prove existence and uniqueness of solutions. We also discuss *BV*-regularity of such solutions and how it can be related to the regularity of  $\partial\Omega$ .

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## Some multi-species mean field equations arising in 2D turbulence

We compare some mean field equations with exponential nonlinearity describing turbulent Euler flows in equilibrium according to Onsager's theory. We consider the multi-species case where the variable vorticities are subject either to a deterministic distribution or to a stochastic distribution. We show that such models lead to different critical temperatures (corresponding to optimal Moser-Trudinger inequalities). Although the stochastic model is in general more similar to the single vorticity case (standard mean field equation) than the deterministic model, we will show some situations where such a situation is reversed.

# 3D vortex approximation construction and applications to the study of global minimizers for the Ginzburg-Landau functional below and near the first critical field

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The Ginzburg-Landau model is a phenomenological description of superconductivity. An essential feature of type-II superconductors is the presence of vortices (similar to those in fluid mechanics, but quantized), which appear above a certain value of the applied magnetic field called the *first critical field*. We are interested in the regime of small  $\varepsilon$ , where  $\varepsilon > 0$  is the inverse of the *Ginzburg-Landau parameter* (a material constant). In this regime, the vortices are at main order codimension 2 topological singularities.

This talk will provide a polyhedral approximation of the vorticity in 3D, which allows one to obtain a Jacobian (or vorticity) estimate and a lower bound for the Ginzburg-Landau free energy. These estimates are optimal, analogous to the 2D ones, and work at the  $\varepsilon$ -level. We then apply these results, together with arguments from the calculus of variations and geometric measure theory, to describe the behavior of global minimizers for the 3D Ginzburg-Landau functional below and near the first critical field.

**Title: A heat equation with exponential nonlinearity and with singular data in  $R^2$**

*Abstract:*

We consider a semilinear heat equation with exponential nonlinearities and singular data in  $R^2$ .

In  $R^n$ ,  $n \geq 3$ , critical growth related to singular initial data is polynomial and has been studied by several authors:

Existence and non-existence results for singular initial data in suitable  $L^p$ -spaces were obtained by F. Weissler and H. Brezis - T. Cazenave; furthermore, non-uniqueness results for certain singular initial data were given by W.-M. Ni - P. Sacks and E. Terraneo.

In  $n = 2$  critical growth is given by nonlinearities of exponential type (cf. N. Trudinger - J. Moser). We prove that similar phenomena, namely existence, non-existence and non-uniqueness, occur for suitable exponential nonlinearities and singular initial data in certain Orlicz spaces.

This is joint work with N. Ioku (Ehime University) and E. Terraneo (University of Milan).

# ADAMS' INEQUALITY WITH THE EXACT GROWTH CONDITION

FEDERICA SANI (Università degli Studi di Milano)

Adams' inequality is the complete generalization of the Trudinger-Moser inequality to the case of Sobolev spaces involving higher order derivatives. In this talk we discuss the optimal growth rate of the exponential-type function in Adams' inequality when the problem is considered in the whole space  $\mathbb{R}^n$ . This is a joint work with Nader Masmoudi.

# THE FREDHOLM ALTERNATIVE FOR THE $p$ -LAPLACIAN IN EXTERIOR DOMAINS. PART II

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ABSTRACT. In this talk we investigate the existence and multiplicity of solutions of the following problem

$$\begin{cases} -\Delta_p u = \lambda K(x)|u|^{p-2}u + h & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \end{cases} \quad (1)$$

where  $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$  is the  $p$ -Laplacian with  $p > 1$ ,  $B_1^c$  is the complement of the closed unit ball  $B_1$  in  $\mathbb{R}^N$  ( $N \geq 2$ ),  $\lambda > 0$  is a parameter, the weight  $K$  and the function  $h$  satisfy certain conditions.

By using the saddle point geometry of the energy functional and the improved Poincaré inequality obtained in Part I, we obtain multiplicity of solutions for (1) when  $1 < p < 2$  and  $2 < p < N$ , respectively employing techniques of Calculus of Variations. This work can be seen as a complement to the Fredholm alternative for the  $p$ -Laplacian in an exterior domain for the resonant case.

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This is a joint work with Pavel Drábek and Ky Ho (Department of Mathematics and NTIS, University of West Bohemia, Czech Republic).

# Antimaximum principle in exterior domains

Sarath Sasi

## Abstract

We consider the antimaximum principle for the  $p$ -Laplacian in the exterior domain:

$$\begin{cases} -\Delta_p u = \lambda K(x)|u|^{p-2}u + h(x) & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \end{cases}$$

where  $\Delta_p$  is the  $p$ -Laplace operator with  $p > 1$ ,  $\lambda$  is the spectral parameter and  $B_1^c$  is the exterior of the closed unit ball in  $\mathbb{R}^N$  with  $N \geq 1$ . The function  $h$  is assumed to be nonnegative and nonzero, however the weight function  $K$  is allowed to change its sign. For  $K$  in a certain weighted Lebesgue space, we prove that the antimaximum principle holds locally. A global antimaximum principle is obtained for  $h$  with compact support.

*Joint work with Anoop T. V., Pavel Drábek and Lakshmi Sankar*

# Uniqueness results for classes of semipositone problems

Ratnasingham Shivaji



## Abstract

We consider steady state reaction diffusion equations on the exterior of a ball, namely, boundary value problems of the form:

$$\begin{cases} -\Delta_p u = \lambda K(|x|)f(u) & \text{in } \Omega_E, \\ u = 0 & \text{on } |x| = r_0, \\ u \rightarrow 0 & \text{when } |x| \rightarrow \infty, \end{cases}$$

where  $\Delta_p z := \operatorname{div}(|\nabla z|^{p-2} \nabla z)$ ,  $1 < p < n$ ,  $\lambda > 0$  and  $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0 > 0\}$ . Here the weight function  $K \in C^1([r_0, \infty), (0, \infty))$  satisfies  $\lim_{r \rightarrow \infty} K(r) = 0$ , and the reaction term  $f \in C^1([0, \infty))$  is strictly increasing and satisfies  $f(0) < 0$ ,  $\lim_{s \rightarrow \infty} f(s) = \infty$ ,  $\lim_{s \rightarrow \infty} \frac{f(s)}{s^{p-1}} = 0$  and  $\frac{f(s)}{s^q}$  is nonincreasing on  $[a, \infty)$  for some  $a > 0$  and  $q \in (0, p - 1)$ . We establish uniqueness results for positive radial solutions for  $\lambda \gg 1$ .

# Existence and compactness for a singular mean field problem with sign changing potentials

Rafael López Soriano\*

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**Abstract.** In this talk, we study the existence of solutions of the following mean field problem

$$-\Delta_g u = \lambda \left( \frac{K e^u}{\int_{\Sigma} K e^u} - \frac{1}{|\Sigma|} \right) - 4\pi \sum_{i=1}^m \left( \alpha_i \delta_{p_i} - \frac{1}{|\Sigma|} \right) \text{ in } \Sigma, \quad (1)$$

where  $\Sigma$  is a compact surface without boundary, equipped with a certain Riemannian metric  $g$ . Here  $\Delta_g$  is the Laplace-Beltrami operator,  $\lambda$  is a positive parameter,  $|\Sigma|$  denotes the area of  $\Sigma$  and  $K$  is a given function defined in  $\Sigma$ . Mean field problems of Liouville-type arise in differential geometry and physics

Following the ideas of [3], the solvability of this and other related problems has been settled under the assumption of  $K$  positive. We will focus on the case in which  $K$  is allowed to change sign. Under extra hypotheses on  $K$ , existence results are obtained by means of variational methods. In addition, we present a compactness criterion for solutions, see [1, 2].

## Referencias

- [1] F. De Marchis, R. López-Soriano. *Existence and Non Existence Results for the singular Nirenberg problem*. Calc. Var. Partial Differential Equations, **55** (2016), no. 2, Art. 36, 35 pp.
- [2] F. De Marchis, R. López-Soriano, D. Ruiz. *Compactness, existence and multiplicity for the singular mean field problem with sign-changing potentials*. preprint (2016), arXiv:1612.02080.
- [3] Z. Djadli, A. Malchiodi. *Existence of conformal metrics with constant Q-curvature*. Ann. of Math. **168** (2008), no. 3, 813–858.

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# Diffusive Hamilton-Jacobi equations and their singularities

PHILIPPE SOUPLET

ABSTRACT. I will consider diffusive Hamilton-Jacobi equations, of the form  $u_t - \Delta u = |\nabla u|^p$  (or, more generally, with nonlinear diffusion).

Such equations arise in the viscous regularization of the Hamilton-Jacobi equations from control theory, as well as in KPZ type models for interface growth in ballistic deposition processes. They possess both global smooth and (gradient) blowup solutions, and display a variety of interesting behaviors.

We will in particular discuss the phenomenon of gradient blow-up (GBU) on the boundary and consider such issues as: localization of singularities, single-point GBU, Bernstein estimates, time rate of GBU, spatial GBU profiles, continuation in the viscosity sense after GBU and loss of boundary conditions.

The Nehari-Pankov manifold revisited  
Andrzej Szulkin (Stockholm)

*Abstract:* We study the Schrödinger equations

$$-\Delta u + V(x)u = f(x, u) \quad \text{in } \mathbb{R}^N$$

and

$$-\Delta u - \lambda u = f(x, u) \quad \text{in a bounded domain } \Omega \subset \mathbb{R}^N.$$

We assume that  $f$  is superlinear but of subcritical growth and  $u \mapsto f(x, u)/|u|$  is non-decreasing. In  $\mathbb{R}^N$  we also assume that  $V$  and  $f$  are periodic in  $x_1, \dots, x_N$ . We show that these equations have a ground state solution and also briefly discuss existence of infinitely many solutions if  $f$  is odd in  $u$ . Our results generalize older ones where  $u \mapsto f(x, u)/|u|$  was assumed to be strictly increasing. This seemingly small change forces us to go beyond methods of smooth analysis.

This is joint work with Francisco Odair de Paiva and Wojciech Kryszewski.

## Poster Abstracts

# Nonlinear (fractional) Schrödinger equations with sign-changing nonlinearities

**Bartosz Bieganowski**

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We look for ground state solutions to the following nonlinear (fractional) Schrödinger equation

$$(-\Delta)^{\alpha/2}u + V(x)u = f(x, u) - \Gamma(x)|u|^{q-2}u \text{ on } \mathbb{R}^N, \quad 0 < \alpha \leq 2,$$

where  $V = V_{per} + V_{loc} \in L^\infty(\mathbb{R}^N)$  is the sum of a periodic potential  $V_{per}$  and a localized potential  $V_{loc}$ ,  $\Gamma \in L^\infty(\mathbb{R}^N)$  is periodic and  $\Gamma(x) \geq 0$  for a.e.  $x \in \mathbb{R}^N$  and  $2 \leq q < 2_\alpha^*$ . We assume that

$$\begin{cases} \inf \sigma(-\Delta + V) > 0, & \text{for } \alpha = 2, \\ \text{ess inf } V(x) > 0, & \text{for } 0 < \alpha < 2, \end{cases}$$

where  $\sigma(-\Delta + V)$  stands for the spectrum of  $-\Delta + V$ , and  $f$  has the subcritical growth but higher than  $\Gamma(x)|u|^{q-2}u$ , however the nonlinearity  $f(x, u) - \Gamma(x)|u|^{q-2}u$  may change sign. Although a Nehari-type monotonicity condition for the nonlinearity is not satisfied we investigate the existence of ground state solutions being minimizers on the Nehari manifold. We also investigate the existence of infinitely many geometrically distinct solutions.

## References

- [1] B. Bieganowski, J. Mederski: *Nonlinear Schrödinger equations with sum of periodic and vanishing potentials and sign-changing nonlinearities*, to appear in *Commun. Pure Appl. Anal.* (2017).
- [2] B. Bieganowski: *Solutions of the fractional Schrödinger equation with sign-changing nonlinearity*, *J. Math. Anal. Appl.*, Vol. **450**, Issue 1, (2017), p. 461-479.

# (NON)UNIQUENESS OF MINIMIZERS IN LEAST GRADIENT PROBLEM

WOJCIECH GÓRNY

The least gradient problem, which is related to conductivity imaging in medical scans and to free material design models, is a variational problem of the form

$$\min\left\{\int_{\Omega} |Du| : u \in BV(\Omega), Tu = f\right\},$$

where  $f \in L^1(\Omega)$  and  $T$  denotes the trace operator. It is well known that for continuous boundary data the solution exists and is continuous up to the boundary. Here, we focus on the two-dimensional case and extend the class of functions for which the solution exists to  $BV(\partial\Omega)$  and discuss how the set of minimizers looks like. However for discontinuous  $f$  uniqueness of solutions may fail, it turns out that the structure of superlevel sets of all minimizers is very similar and we may characterize all of them.

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**THE INTEGRAL REPRESENTATION FOR THE  
EFFECTIVE ENERGY FUNCTIONAL FOR THIN FILMS  
WITH BENDING MOMENT IN THE ORLICZ-SOBOLEV  
SPACE SETTING**

WŁODZIMIERZ LASKOWSKI

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Let  $\omega$  be a bounded open subset of  $\mathbb{R}^2$  with Lipschitz boundary and  $\Omega := \omega \times (-\frac{1}{2}, \frac{1}{2}) \subset \mathbb{R}^3$ . We consider the functional  $\bar{J}_\varepsilon : W^{1,M}(\Omega; \mathbb{R}^3) \times L^M(\Omega; \mathbb{R}^3) \rightarrow \mathbb{R} \cup \{+\infty\}$  defined by

$$\bar{J}_\varepsilon(u, \bar{b}) := \begin{cases} \int_\Omega W(D_1 u | D_2 u | \frac{1}{\varepsilon} D_3 u) dx & \text{if } \frac{1}{\varepsilon} D_3 u = \bar{b} \\ +\infty & \text{otherwise,} \end{cases}$$

where  $W : \mathbb{R}^{3 \times 3} \rightarrow \mathbb{R}$  is a continuous function satisfying the growth and coercivity conditions

$$\frac{1}{C}(M(\|F\|) - 1) \leq W(F) \leq C(1 + M(\|F\|)) \quad (\forall F \in \mathbb{R}^{3 \times 3})$$

for some  $C > 0$ . Here  $M : \mathbb{R} \rightarrow [0, \infty)$  is some non-power Orlicz-Young convex function.

We present the integral estimation for the  $\Gamma$ -lower and  $\Gamma$ -upper limits for functional  $\bar{J}_\varepsilon$ , by studying the equivalent re-scaled integral functional

$$\bar{F}_\varepsilon(H) := \begin{cases} \frac{1}{\varepsilon} \int_{\Omega_\varepsilon} W(H(x)) dx & \text{if } \text{curl } H = 0 \text{ in } \Omega_\varepsilon \text{ (distributionally)} \\ +\infty & \text{otherwise,} \end{cases}$$

with  $H = Du \in L^M(\Omega_\varepsilon; \mathbb{R}^{3 \times 3})$ ,  $\Omega_\varepsilon := \omega \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2})$ , as the thickness  $\varepsilon$  goes to 0.

The results are extended also for the case  $\mathcal{A} = \text{div}$  (which appears in the context of functional on solenoidal vector fields) or  $\mathcal{A} = \begin{pmatrix} \text{div} & \text{div} \\ 0 & \text{curl} \end{pmatrix}$  (which appears in the context of micro-magnetic functional) instead of  $\mathcal{A} = \text{curl}$ . All results were obtained in collaboration with prof. Hong Thai Nguyen from University of Szczecin.

# Infinite semipositone problems with a falling zero and nonlinear boundary conditions

Mohan Kumar Mallick\*

## Abstract

We consider the boundary value problem

$$\begin{cases} -u'' = h(t)\left(\frac{au - u^2 - c}{u^\alpha}\right), & t \in (0, 1), \\ u(0) = 0, u'(1) + g(u(1)) = 0, \end{cases}$$

where  $a > 0, c \geq 0, \alpha \in (0, 1), h : (0, 1] \rightarrow (0, \infty)$  is a continuous function which is allowed to be singular at  $t = 0$ , but belongs to  $L^1(0, 1) \cap C^1(0, 1)$ , and  $g : [0, \infty) \rightarrow [0, \infty)$  is a continuous function. We discuss existence, uniqueness, and non existence results for positive solutions for certain ranges of  $a$  and  $c$ .

This is a joint work with R. Shivaji (University of North Carolina at Greensboro, USA), S Sundar (IIT Madras, India), L Sankar (IIT Palakkad, India).

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**WELL-POSEDNESS OF FULLY NONLINEAR PDES WITH  
CAPUTO'S TIME-FRACTIONAL DERIVATIVE**

TOKINAGA NAMBA

ABSTRACT. We will find a proper extended notion of viscosity solutions for (first order) Hamilton-Jacobi equations with Caputo's time-fractional derivative of order less than one. As for the integer-order case, the unique existence is established by the comparison principle and the Perron's method. Stability with respect to the order of time derivative as well as the standard one is also proved by the half-relaxed limit method. If time permits, we will touch on an extension to second-order equations. This talk is partially based on the joint work with Professor Yoshikazu Giga (U.Tokyo, Japan).

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