# Single use register automata for data words 

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# Deterministic register automata 

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\mathbb{A}=\{\square \square \ldots\}
$$

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There are at most 3 different letters in the input word


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When reading input letter $x$ in location $q_{2}$ the automaton does this:


## Not robust:

The following recognise different languages:

- Deterministic register automata
- 2-way deterministic register automata
- Nondeterministic register automata
- 2-way nondeterministic register automata


## Single use restriction



## Single use restriction



## An example

## There are at most 3 different letters in the input word

Natural construction doesn't work with the single use restriction, but there is another one that does:


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## An example

## There are at most 3 different letters in the input word

Natural construction doesn't work with the single use restriction, but there is another one that does:

First letter appears again
Cannot be expressed with the single use restriction

## Main result

Two way single use deterministic register automata

One way single use deterministic register automata

## Orbit-finite semigroups

## Main result

Two way single use deterministic register automata

One way single use deterministic register automata

## Main result

Two way single use deterministic register automata

One way single use deterministic register automata

Orbit-finite semigroups
M. Bojańczyk 2013
T. Colcombet, C. Ley, G. Puppis 2015

Rigidly guarded MSO~
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One way single use deterministic register automata

Orbit-finite semigroups
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