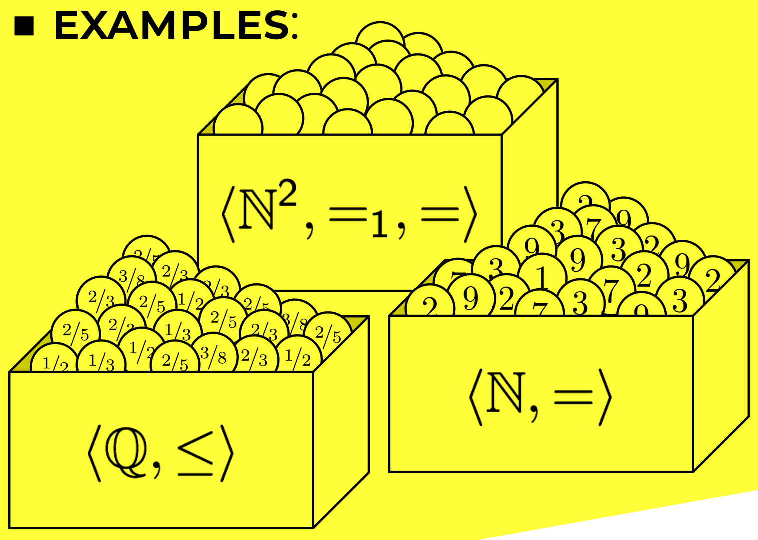
S. Lasota R. Piórkowski University of Warsaw

WQO DICHOTOMY CONJECTURE

Petri nets with data

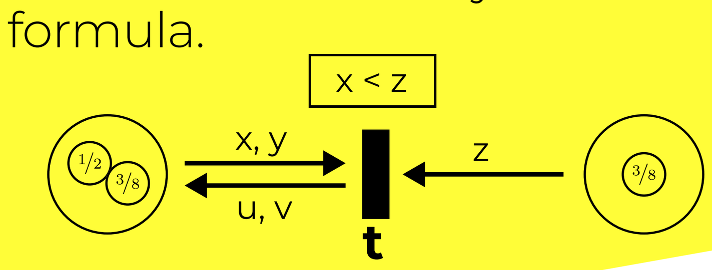
Unlike in the regular Petri nets, here the tokens are labeled with "data", i.e. elements of some countably infinite set **A** called the data domain.

▪ **EXAMPLES:**



By labeling some transition **t** with a FO formula, one can specify a condition that should be met before **t** can be used.

▪ **EXAMPLE**: Below, **t** is inactive, even though there are enough tokens — the given data values cannot satisfy the formula.



assumptions:

A IS HOMOGENEOUS

Def.: every finite partial automorphism of **A** extends to a full one.

It is a reasonable restriction, as:
▪ it is satisfied by many domains investigated by other authors,
▪ it implies some nice properties (e.g. amalgamation)

Ask me for a nice example!

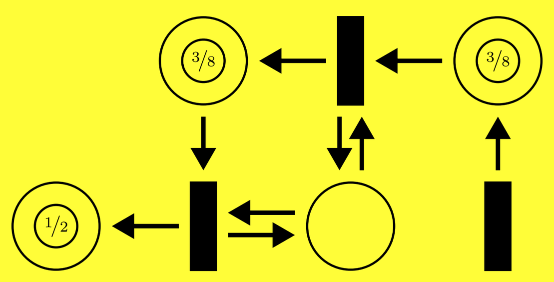
A IS EFFECTIVE

Def.: the membership in Age(**A**) is decidable, i.e.:

Given a finite structure **S**, one may decide whether **S** is an induced substructure of **A**.

FOR PETRI NETS WITH DATA **A** STANDARD PROBLEMS ARE **DECIDABLE** iff **A** ADMITS WQO

standard decision problems



Input: a Petri net with data **A** and its initial configuration **c**.

▪ **BOUNDEDNESS**
is the set of reachable configurations finite up to isomorphism?

▪ **COVERABILITY**
does some reachable config. cover a given configuration **c**' up to isomorphism?

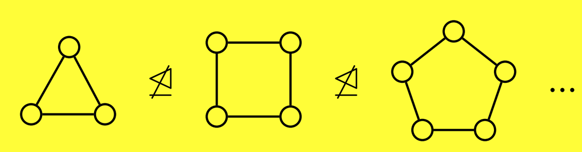
▪ **PLACE NON-EMPTINESS**
does some reachable configuration has a token at a given place?

▪ **TERMINATION**
are all runs finite?

Def.: Age(**A**) — set of all finite induced substructures of **A**.

Def.: WQO (well quasi order) is a quasi order that:
— is well founded,
— has no infinite antichains.

▪ **EXAMPLE**: Let **A** = ⟨V, E⟩ be an infinite graph. Sequence of all cycles forms an antichain



in $\langle \mathrm{Age}(A), \trianglelefteq \rangle$.

admitting WQO

Def.: structure **A** admits WQO if for any WQO X:

$$\langle \{\mathcal{A} \to X \mid \mathcal{A} \in \mathrm{Age}(A)\}, \trianglelefteq_X \rangle$$

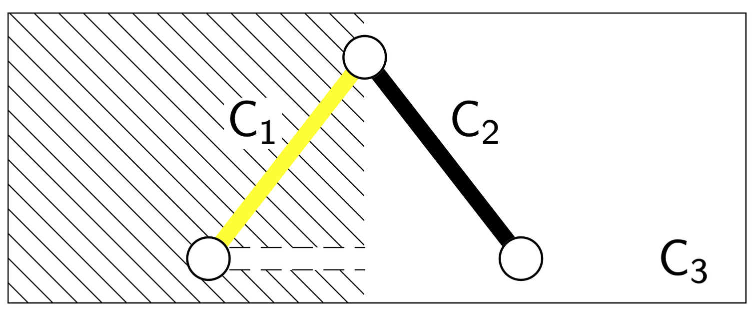
is a WQO too.

Usually, the hardest task is to show that $\langle \mathrm{Age}(A), \trianglelefteq \rangle$ is a WQO, and then the extension to labeled case is straightforward.

PROOF FOR 3-GRAPHS

3-graphs

Def.: a relational structure over $\langle V, C_1, C_2, C_3 \rangle$, where the three relations are a partition of the edge set of an infinite clique.



3-graphs may also be seen as 2-edge coloured graphs — after considering one of the 'colours' as a lack of an edge (or background).

proof structure

Direction ⟵ is already proven in the paper, in which the WQO dichotomy was stated.

First and also the most involved part of the proof for ⟶ is the main theorem.

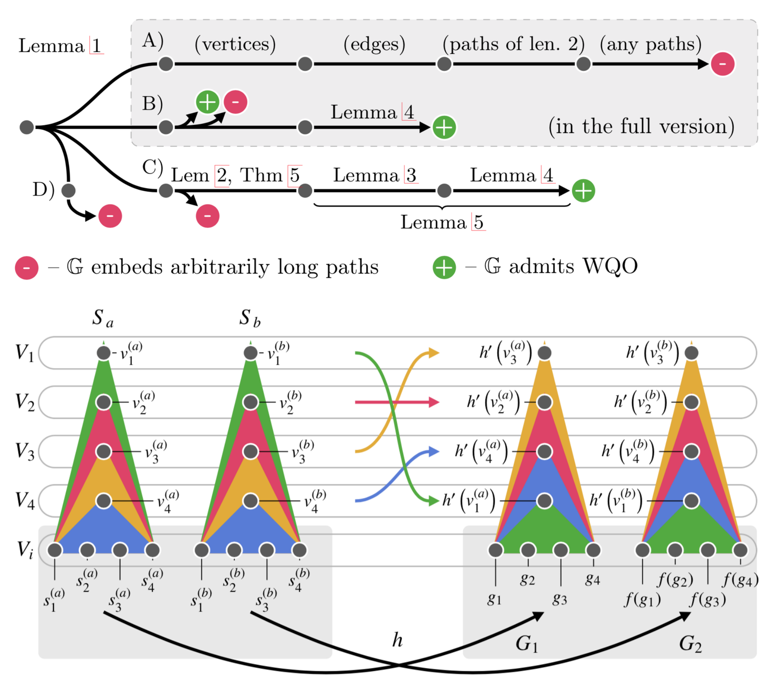
Having that, it only remains to show that existence of an infinite path implies undecidability. We prove that using those paths one can easily implement zero-testable counters in a Petri net — that makes all of the standard problems undecidable.

main theorem

LET **A** BE A STRONGLY HOMOGENEOUS 3-GRAPH. **A** EITHER ADMITS WQO, OR EMBEDS AN INFINITE PATH.

Proof of the theorem provides a content for two more posters of this size.

It begins with a Ramsey style argument and splits into four separate branches — each one being a chain of lemmas — that conclude with one of the two disjuncts.
The proof uses recent result in classification of homogeneous bipartite graphs.



HIGHLIGHTS of Logic, Games and Automata

https://arxiv.org/abs/1802.07612