Log concavity and concentration of measure on the discrete hypercube.

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It is well known that if $V : \mathbb{R}^n \to \mathbb{R}$ is a potential such that $Hess(V) \preceq (1-\epsilon)Id$ (hence it is not too convex) then the measure whose density is e^V with respect to the Gaussian satisfies several concentration properties (for instance, Lipschitz functions have a bounded variance). In this talk, we try to find analogs of this fact when the Gaussian measure is replaced by the uniform measure on the Boolean hypercube. In this case, it is not clear what should be the correct definition of log-concavity, and even when V is quadratic (which is trivial in the Gaussian case) proving concentration becomes a difficult question (with implications to spin-glasses, for example). We'll present two results in this direction: First, we will suggest a natural definition of log-concavity which attains such concentration, namely, in terms of the (semi) log-concavity of the multilinear extension. Second, we will present a result which gives sufficient conditions for concentration of quadratic forms, and in particular implies that the Gibbs measure on the Sherrington-Kirkpatrick model admits concentration when the temperature is higher than some universal constant. These are joint works with Koehler, Shamir and Zeitouni.