Transport inequalities on the Poisson space

Ronan Herry

(joint work with N. Gozlan & G. Peccati)

In this talk, I will explain how to derive transport-entropy inequalities for the law of a Poisson point process. The investigation of transport-entropy inequalities goes back to works by Marton and Talagrand in the nineties, in connection with concentration of measure for product measures. A transport-entropy inequality for a probability measure $\mu \in \mathscr{P}(E)$ is of the form:

$$\mathcal{T}_c(\nu_1|\nu_2) \leq a_1\mathcal{H}(\nu_1|\mu) + a_2\mathcal{H}(\nu_2|\mu),$$

where $\mathcal{T}_c\colon \mathscr{P}(E) \times \mathscr{P}(E) \to \mathbb{R}$ is the transport cost associated with the cost function c, $\mathcal{H}$ is the relative entropy (or Kullback-Leibler divergence), and the a_i's are constants. For instance, when E is a Euclidean space and c is the square of the Euclidean distance, the previous transport-entropy inequality gives back the infamous Talagrand inequality on $\mathbb{R}^d$, first established for the Gaussian measure. The work of Marton also shows that for a properly chosen cost c every probability measure satisfies a transport-entropy inequality. This universal transport-entropy inequality gives back concentration of measure with respect to the so-called Talagrand convex distance. For Marton's inequality, we need to take $c\colon E \times \mathscr{P}(E) \to [0, \infty]$ and $\mathcal{T}_c$ is a generalized transport cost in the sense of Gozlan et al.

Recall that a Poisson point process η is a completely independent random discrete measure. We write $\nu = \mathbb{E}\eta$ for the intensity measure of η. The law of η is completely determined by ν and is denoted by Π_ν. Here, we are interested in transport-entropy inequalities when E is the space of discrete measures and $\mu = \Pi_\nu$ is the law of a Poisson point process. Our work highlights a general principle: any transport-entropy inequality for the intensity measure ν lifts to a transport-entropy inequality for Π_ν. In particular, we show that Marton's universal transport-entropy inequality lifts at level of the Poisson process yielding an inequality for the Poisson space. Our strategy uses various stability results for transport-entropy inequalities and the fact that Π_ν admits a closed expression in terms of ν. From our result, we recover with a simple argument deviation inequalities previously obtained by Bachmann & Reitzner. We also show that our inequality yields a modified logarithmic Sobolev inequality.