ITÔ-NISIO'S THEOREM REVISITED AND RELATED TOPICS

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Itô-Nisio's theorem describes a situation where the pointwise convergence of stochastic processes implies the uniform convergence pathwise a.s. It can be stated as follows

Theorem 1 (Itô-Nisio)**.** *Let $\{S_n(t)\}_{t\in T}$ be partial sums of independent symmetric stochastic processes with continuous paths over a compact metrizable set T such that*

$$\lim_{n\to\infty} S_n(t) = S(t) \quad a.s. \quad \text{for each } t \in T,$$

where $\{S(t)\}_{t\in T}$ is a processes with continuous paths. Then

$$\lim_{n\to\infty} \|S_n - S\|_{C(T)} = 0 \quad a.s.$$

This theorem covers all cases when paths of processes live in separable Banach spaces. But many processes have paths in non-separable Banach spaces, such as $D([0,1]^d)$ with the uniform norm, spaces of bounded p-variation $BV_p[0,1]$, Hölder spaces $C^{0,\alpha}[0,1]$. It is desirable to consider approximation and convergence of processes in the corresponding strong norms.

It has been known for a while that the Itô-Nisio's theorem fails in $BV_p[0,1]$ for any $p > 1$ and it fails in $C^{0,\alpha}[0,1]$ for any $\alpha \in (0,1]$. However, it holds in $D[0,1]$ under the uniform norm and it holds in $BV_1[0,1]$, see [1]. This raises the question what property of a Banach spaces makes the Itô-Nisio theorem valid?

In this talk we will give the framework for the extension of Itô-Nisio's theorem and present the necessary and sufficient condition for its validity of in any Banach space of countable type. This framework covers all non-separable spaces mentioned above, and some others that will be mentioned in the talk, such as Wiener classes.

Wiener classes are certain non-separable subspaces of $BV_p[0,1]$, $p > 1$ (more generally, of $BV_\phi[0,1]$ spaces), where our criterion for Itô-Nisio's theorem holds (but it fails in $BV_p[0,1]$). Lévy processes without Gaussian part have paths in Wiener classes while Brownian motion may not be in it. The optimal ϕ_2-variation formula for a general Lévy process (extending the S.J. Taylor formula) will be given. As an application, we will use Itô-Nisio's theorem in Wiener classes to prove a strong mode of convergence in series expansions of Lévy process.

This talk is based on joint work with Andreas Basse-O'Connor and Jørgen Hoffmann-Jørgensen.

References

[1] Andreas Basse-O'Connor and Jan Rosiński, On the uniform convergence of random series in Skorohod space and representations of cadlag infinitely divisible processes, *Ann. Probab.* 41 (2013), no. 6, 4317-4341. MR 316147