Set Theory homework #2

due October 16, 2019

Exercise 1. Prove that there exists an uncountable family of subsets of the set of natural numbers linearly ordered by inclusion.

Exercise 2. Prove that the set of all well-orderings of $\mathbb{N}$ has cardinality the continuum.

Exercise 3. Prove that every countable linearly ordered set is isomorphic to a subset of $\mathbb{Q}$ (with the natural ordering).

Exercise 4. Prove that there exists a collection of continuum many pairwise non-isomorphic linear orderings of $\mathbb{Q}$.