Logic homework #2

due October 22, 2019

Zadanie 1. A transversal for a family of sets $\mathcal{F}$ is a one-to-one choice function, i.e., a one-to-one function f with domain $\mathcal{F}$ such that for every $X \in \mathcal{F},\ f(X) \in X$.

With the help of the compactness theorem of propositional logic show that if $\mathcal{F}$ is a family of finite sets such that for every finite $\mathcal{F}' \subseteq \mathcal{F}$, $\mathcal{F}'$ has a transversal, then $\mathcal{F}$ has a transversal.

Is this result true if $\mathcal{F}$ contains infinite sets?

Zadanie 2. A binary relation E (called the edges) on a set V (called the vertices) is a graph iff E is:

a. (irreflexive) $\forall x \in V \neg xEx$; and

b. (symmetric) $\forall x, y \in V\ (xEy \Rightarrow yEx)$.

We say x and y are adjacent iff xEy.

For a graph (V, E) an n coloring is a map $c : V \to \{1, 2, \ldots, n\}$ satisfying $\forall x, y \in V(xEy \Rightarrow c(x) \neq c(y))$, i.e. adjacent vertices have different colors. A graph (V, E) has chromatic number $\leqslant n$ iff there is a n coloring on its vertices.

With the help of the compactness theorem for propositional logic show that a graph has chromatic number $\leqslant n$ iff every finite subgraph of it has chromatic number $\leqslant n$.