Logic homework #1

due October 15, 2019

Exercise 1. Let α be a formula of propositional logic, built with the help of variables and the connective $\Leftrightarrow$. Prove that α is a tautology if and only if the number of occurrences of each variable in α is even.

Exercise 2. Define a new binary connective $*$ by defining its truth table so that the one-element set $\{*\}$ is a complete set of connectives.

Exercise 3. Let us recall that a formula is in *disjunctive normal form* (DNF) if it is equal to

$$\alpha_1 \vee \ldots \vee \alpha_n,$$

with

$$\alpha_i = l_1^{(i)} \wedge \ldots \wedge l_{k_i}^{(i)},$$

and each $l_j^{(i)}$ is a *literal*, i.e., p or $\neg p$ for some propositional variable p.

Complete the proof that for each formula α there is a formula in DNF logically equivalent to α.

Exercise 4. We say that $S \subseteq Form$ is a *finitely satisfiable* set of propositional formulas, if for every finite subset S_0 of S there is a valuation under which all the formulas from S_0 are true.

A set $T \subseteq Form$ is called *complete*, if for every formula α precisely one of the following holds: either $\alpha \in T$ or $\neg\alpha \in T$.

Prove that if $S \subseteq Form$ is finitely satisfiable but for no formula $\beta \notin S$ the set $S \cup \{\beta\}$ is finitely satisfiable, then S is complete.