

On a Model for the Initiation of Cell Movement

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In [1] we proposed a one dimensional model for the polarization of the actin cytoskeleton in initially symmetric cells upon some external stimulus. The model was comprised of a minimum of four hyperbolic conservation laws describing the motion of actin filament ends and at least one reaction diffusion equation for the actin monomer concentration. We showed by simulations how a very concise model for the cytoskeleton is sufficient to account for the initiation of an asymmetric distribution of actin filaments which may drive the cell into directed motion.

Now we describe the further polarization and the actual initiation of movement by a free boundary problem for the cell body spanned by the cytoskeleton. Here we assume that at an initial stage of motion the cell membrane exerts only negligible forces on the growing filaments which allows for characteristic boundary conditions. We will comment on the well posedness of the resulting model for small time assuming physically plausible initial conditions. Furthermore there will be some remarks made on possible gradient blow up within finite time.

Moreover we have investigated a reduced system to understand the hyperbolic dynamics of the proposed model. We will demonstrate some interesting stability properties of the resulting system of no more than 2 equations. In particular a genuine difference is observed between linear and nonlinear stability indicating sensitivity to the shape and size of perturbations of the trivial steady state. Furthermore various types of long time behavior will be shown by numerical simulations. Finally we will comment on the existence of traveling wave solutions to the reduced model depending on the parameters. Some numerical results on traveling waves will also be presented.

[1] J. Fuhrmann, J. Ks, A. Stevens: Initiation of cytoskeletal asymmetry for cell polarization and movement. *Theor Biol*, 249, 2 , p. 278-288, 2007.