

# Shelah-Stupp's and Muchnik's Iterations Revisited

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## Decidability of MSO

The MSO logic is decidable over:

- finite structures
  - natural numbers with successor  
(= infinite words)
  - infinite complete binary tree
- } MSO defines  
regular languages

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  - natural numbers with successor + additional unary predicate
  - transition graphs of pushdown automata
  - transition graphs of higher-order pushdown automata
  - HR-equational hypergraphs
  - VR-equational hypergraphs
  - prefix-recognizable graphs
  - trees generated by higher-order recursion schemes
- } MSO defines regular languages

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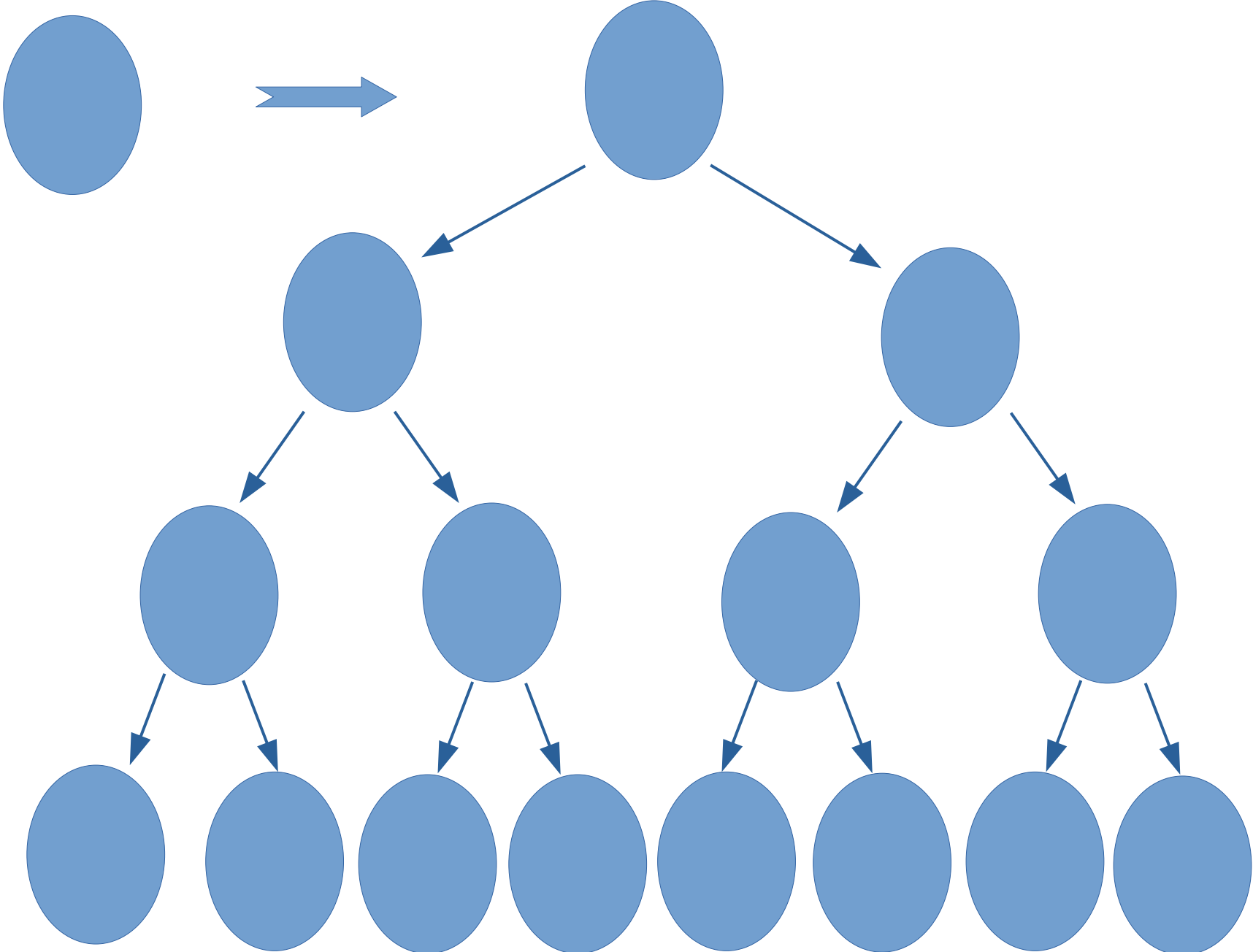
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MSO-decidability preserved by:

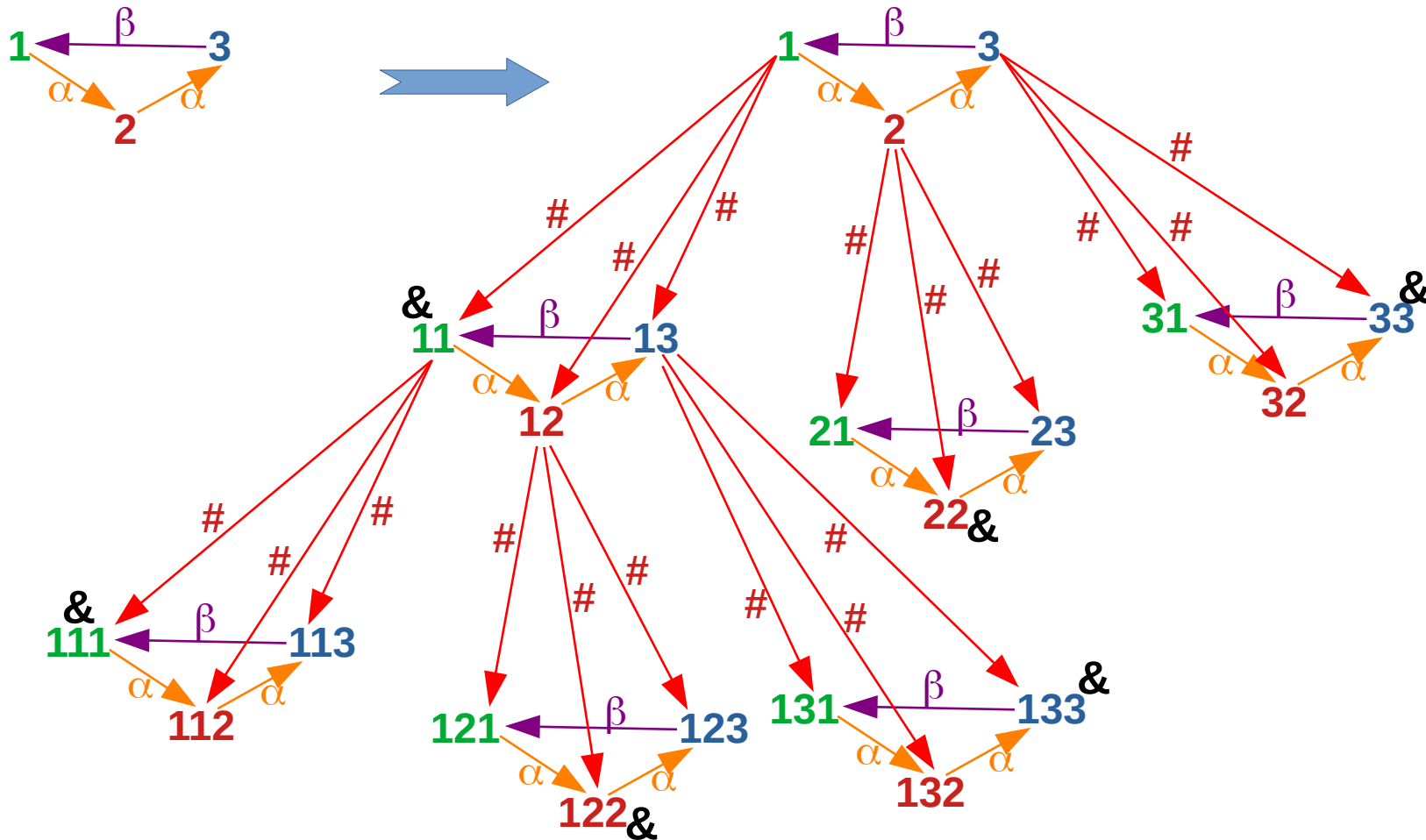
MSO decidable over  $A \Rightarrow$  MSO decidable over  $f(A)$

- generalised unions (Shelah)
- MSO-interpretations
- unfolding of directed graphs into trees
- iteration

Iteration

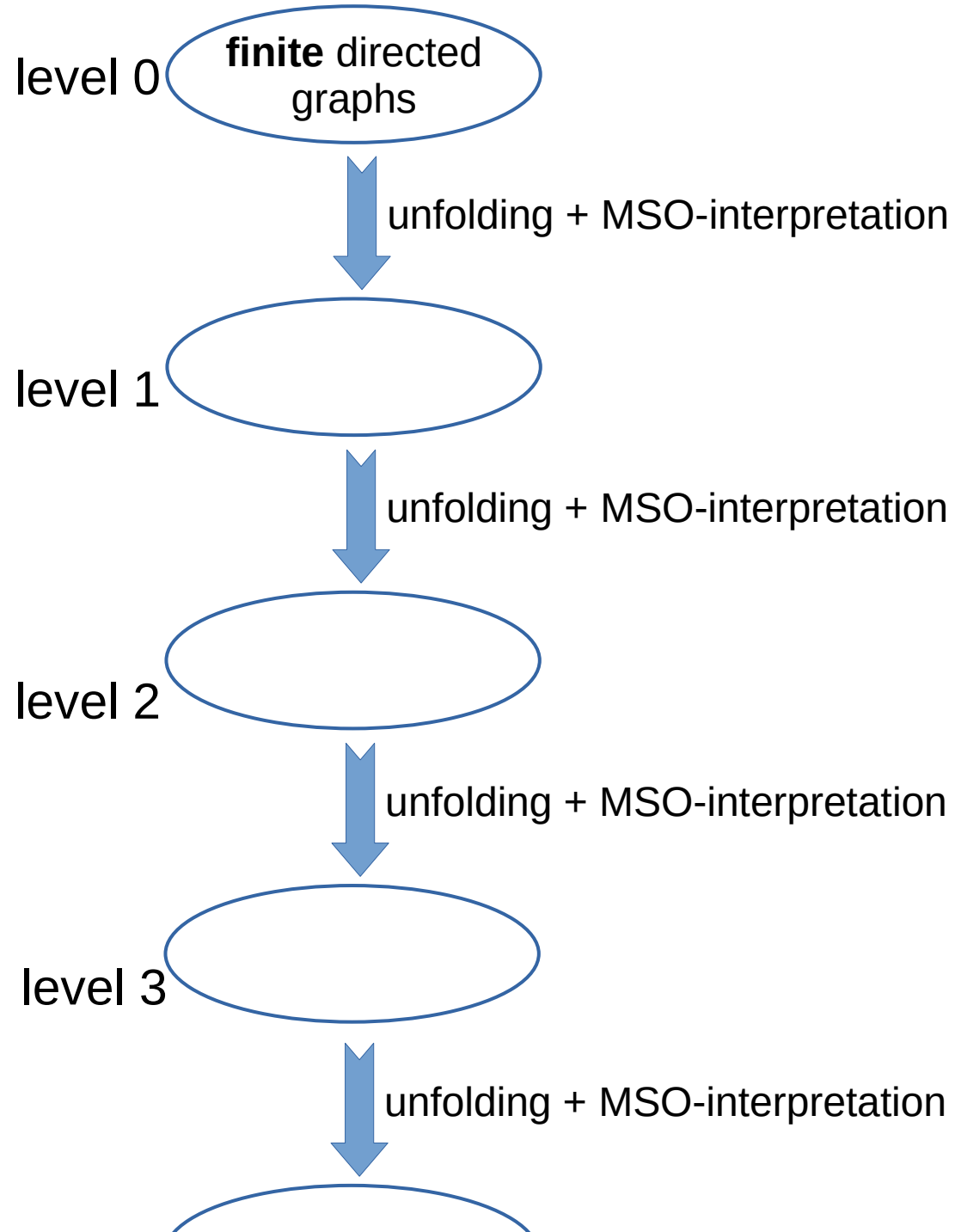


# Iteration

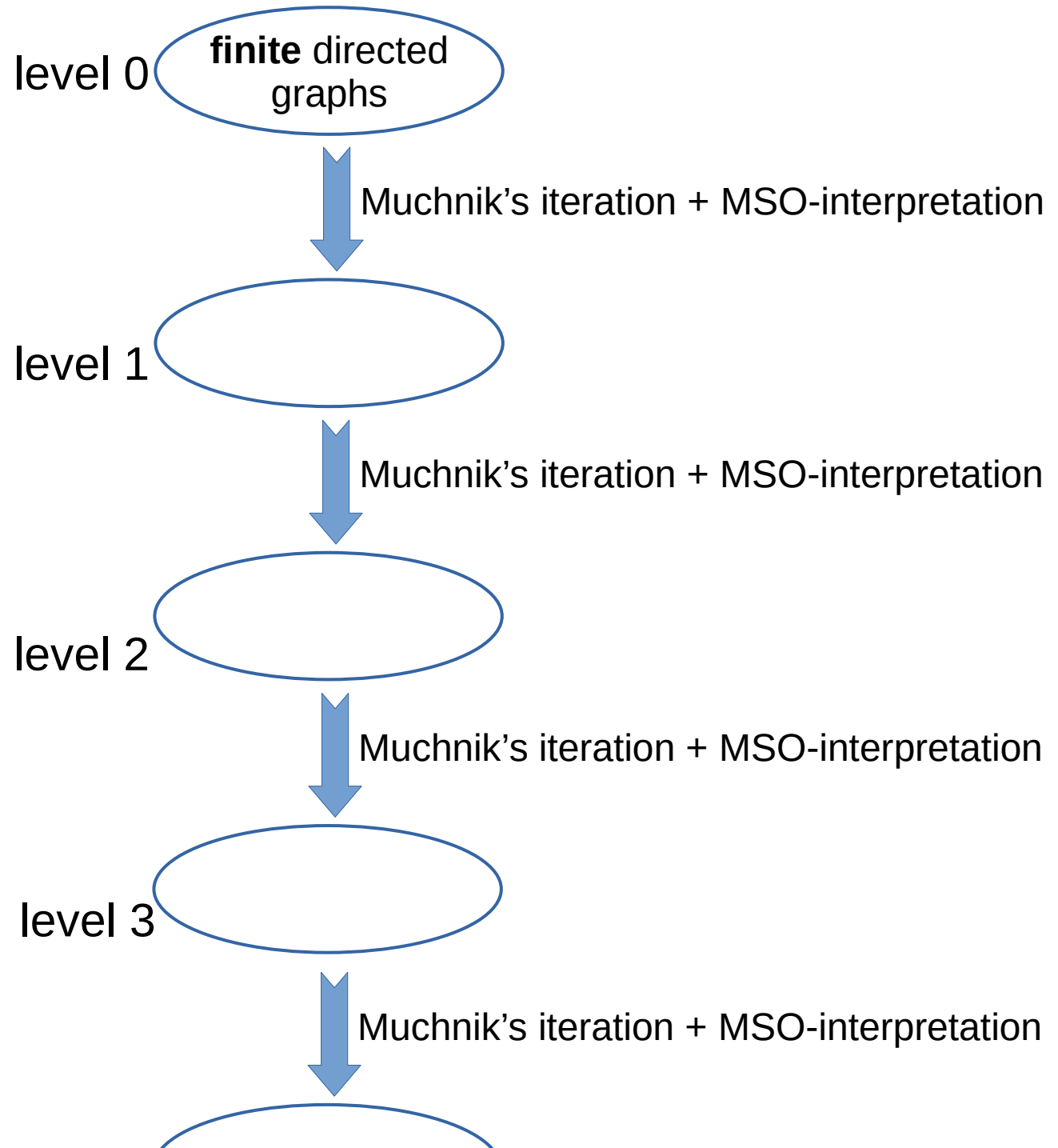


- Shelah-Stupp's iteration (basic iteration) – predicate #
- Muchnik's iteration – predicates # and &

# Causal hierarchy (directed graphs)

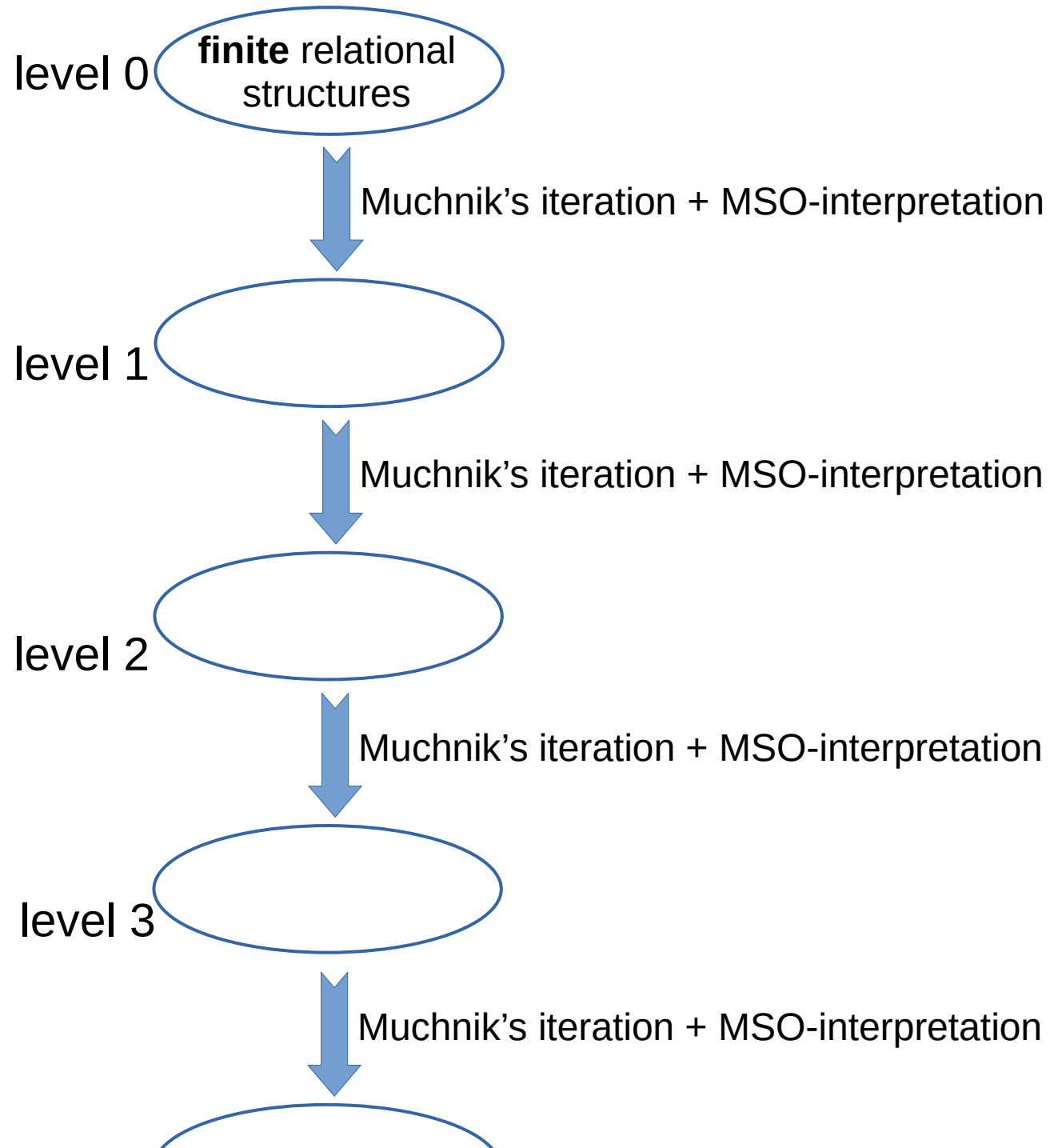


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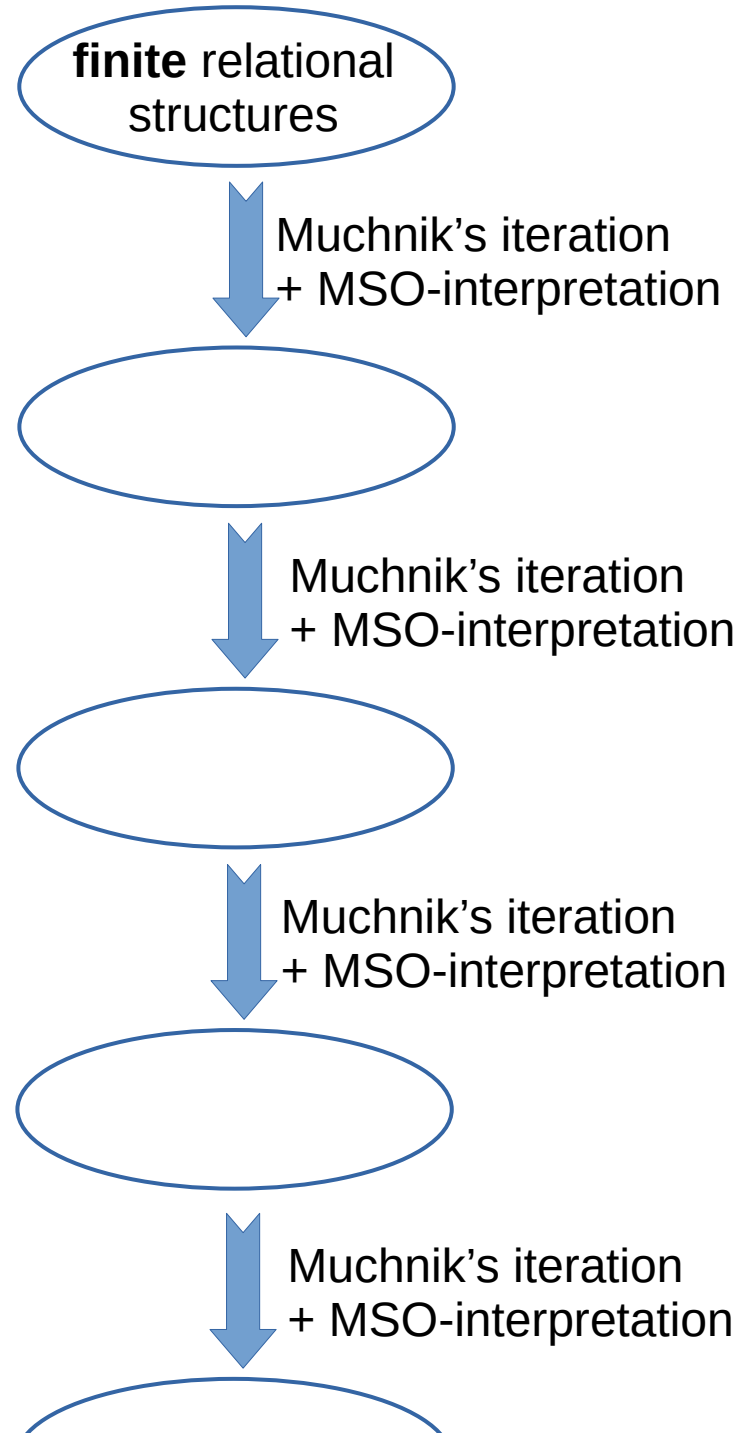
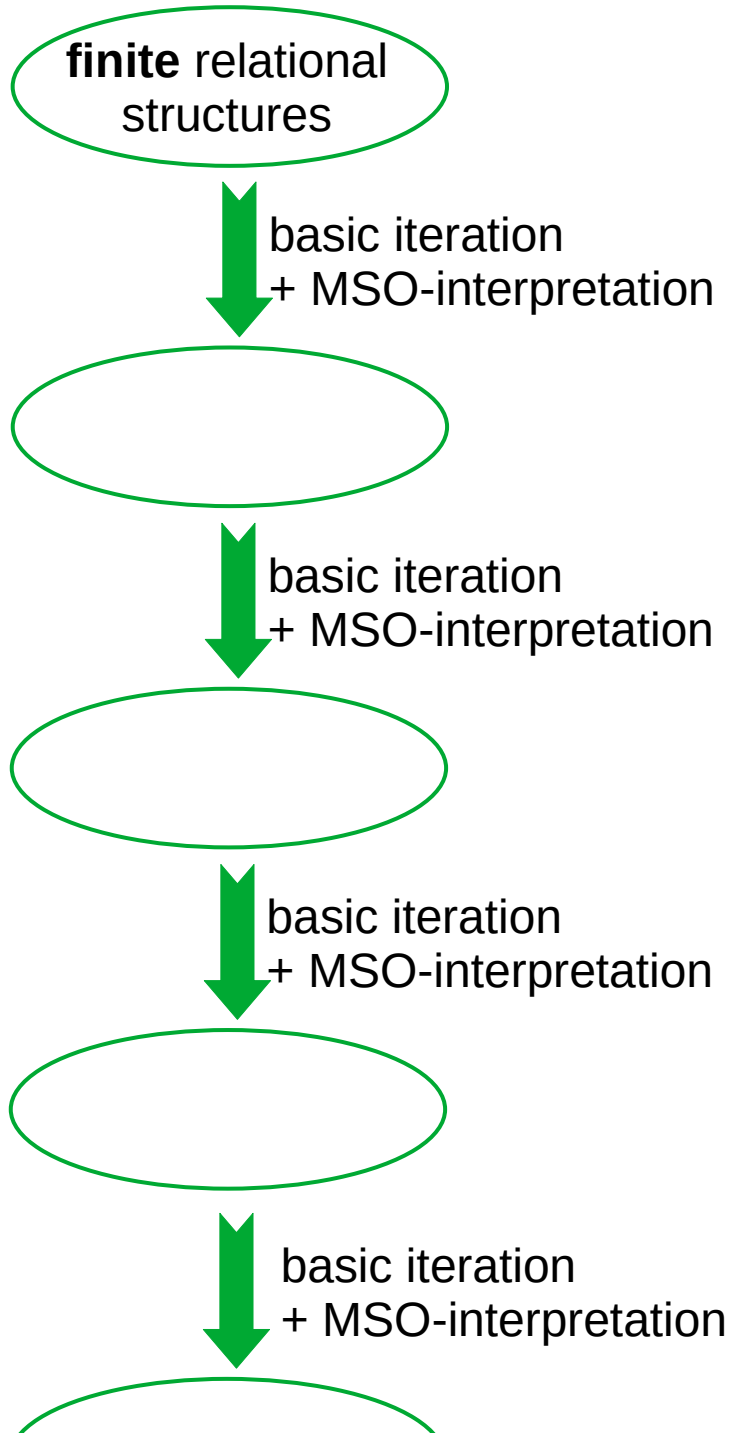




# Generalization to arbitrary relational structures



# Two hierarchies (how are they related?)



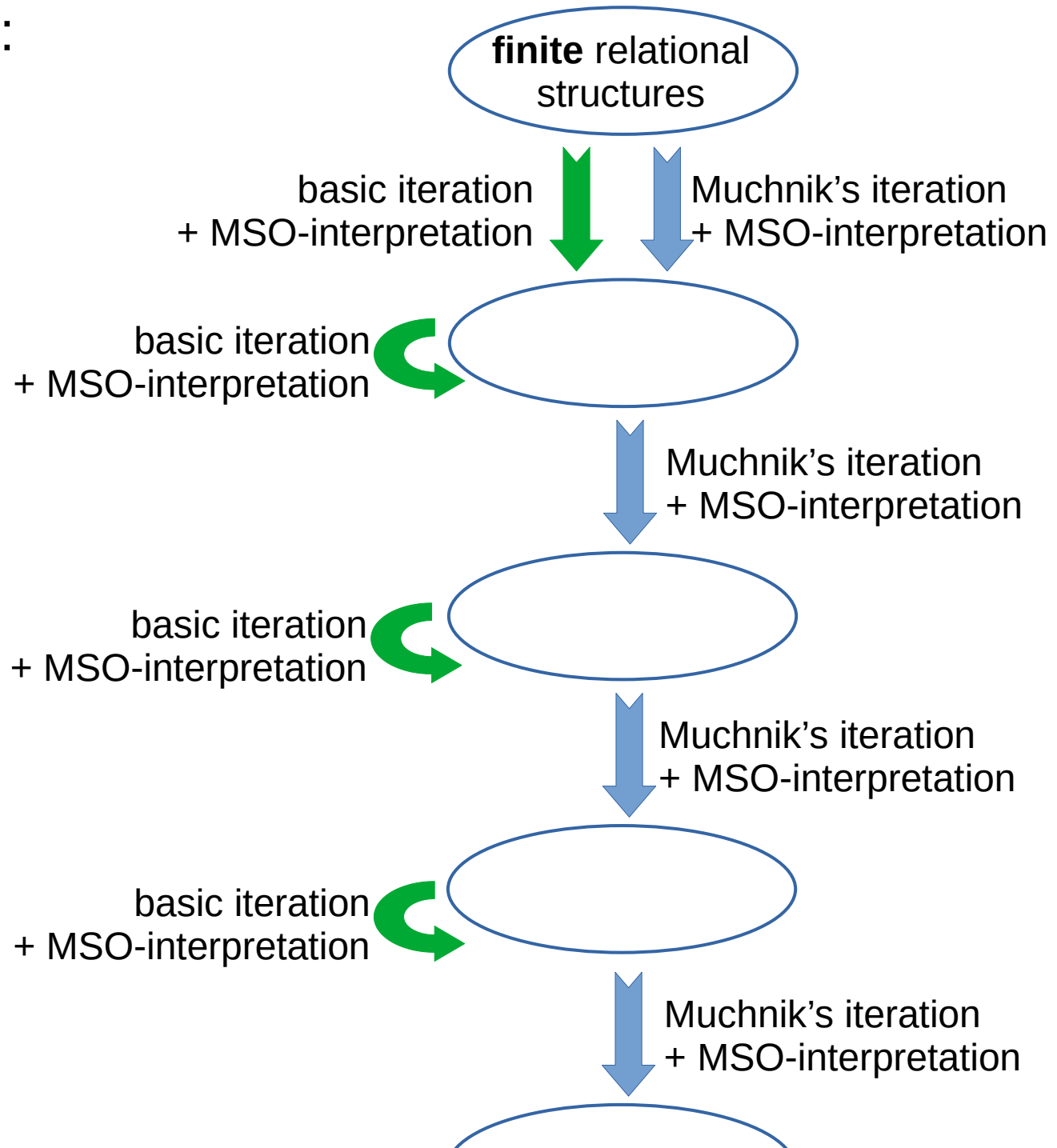
## Two hierarchies (how are they related?)

Caucal & Knapik (2018):

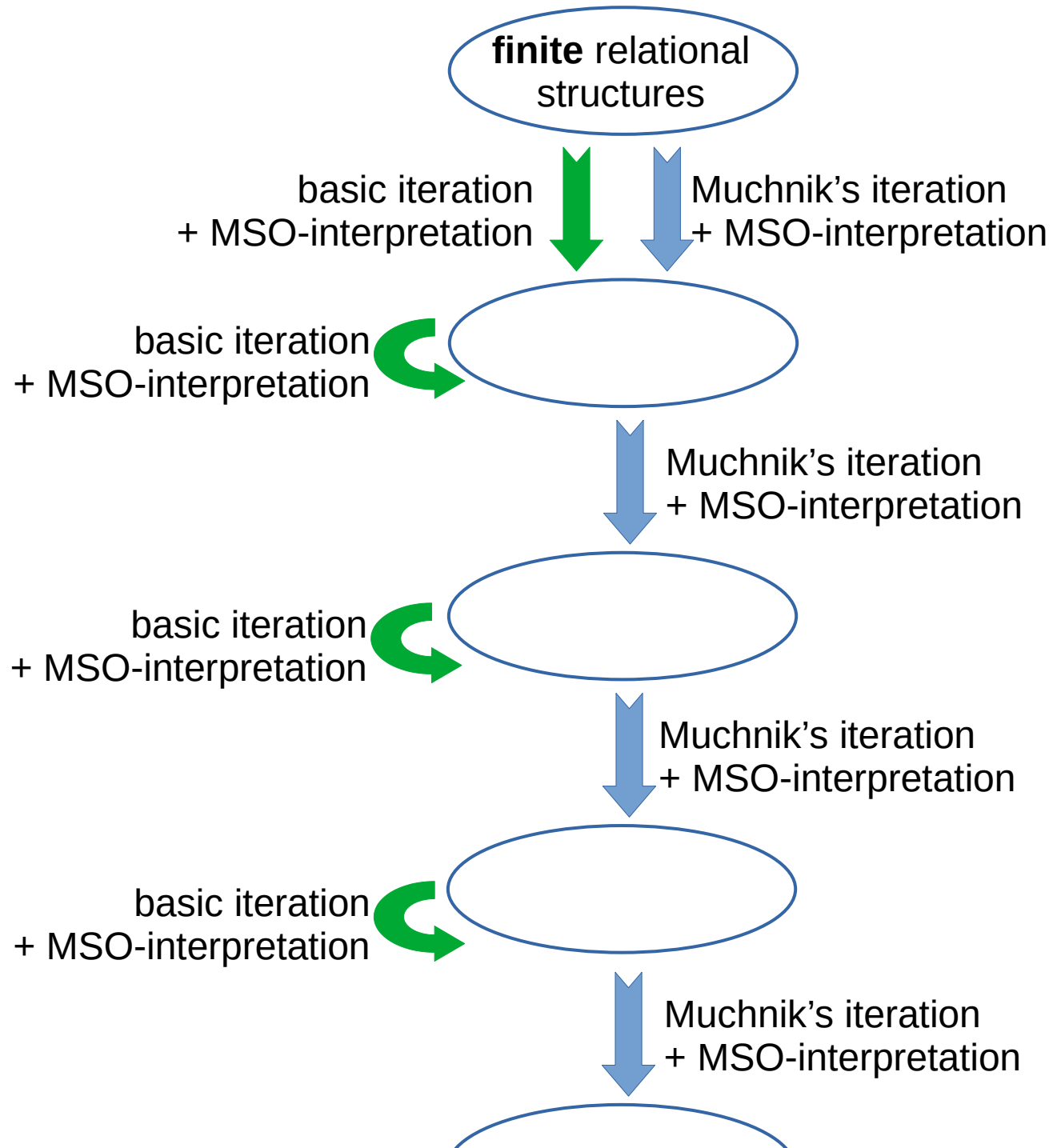
- level 1 coincides
- positive levels closed under basic iterations

This paper:

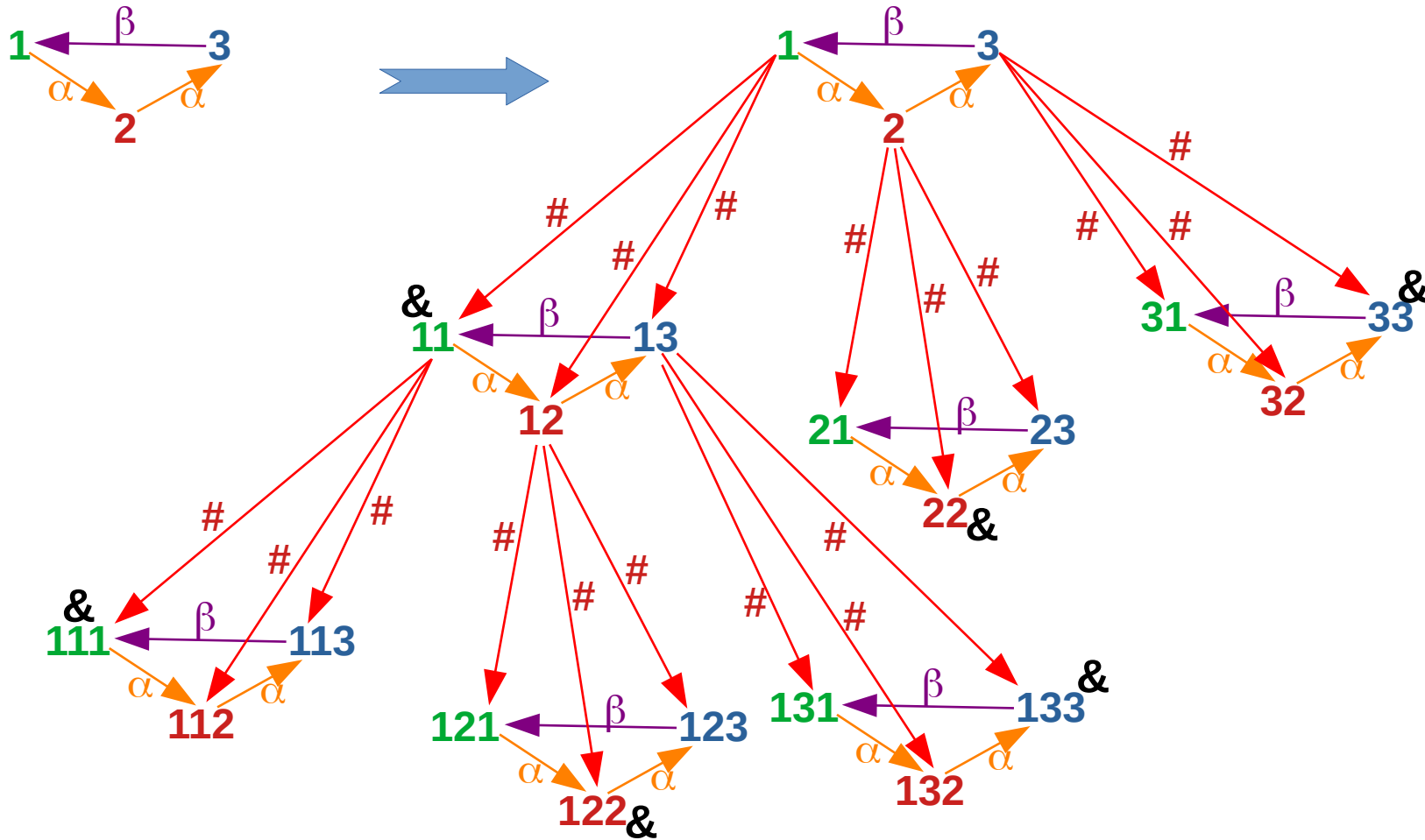
- the same (simpler proofs)



# Level 1 coincides



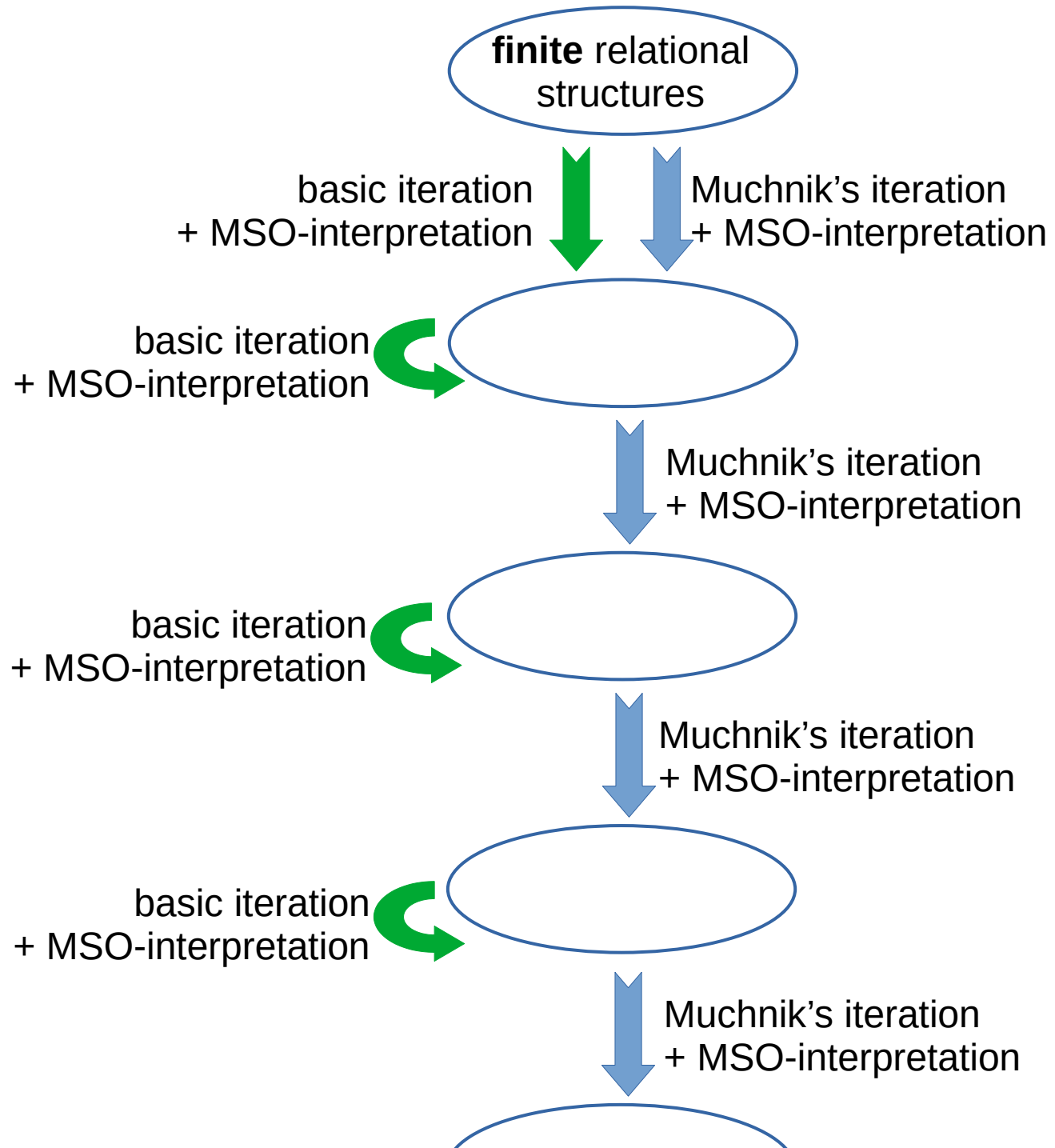
# Level 1 coincides



We want to define & using # and colors.

Possible because the original structure is finite  
(finitely many colors – every element can have a different color)

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Idea: words of words = words (we need a separator)

$$(12311)(1123)(443) = 12311\$1123\$443$$

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- & of the internal iteration = & in result

# Conclusion

We consider hierarchies obtained by basic/Muchnik's iteration + MSO-interpretation

Simple proofs that:

- level 1 coincides
- positive levels closed under basic iterations

Thank you!

