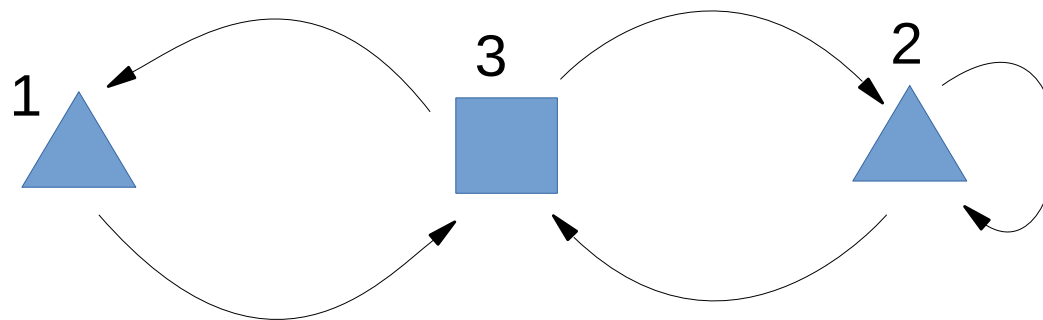


# Parity Games: Zielonka's Algorithm in Quasi-Polynomial Time

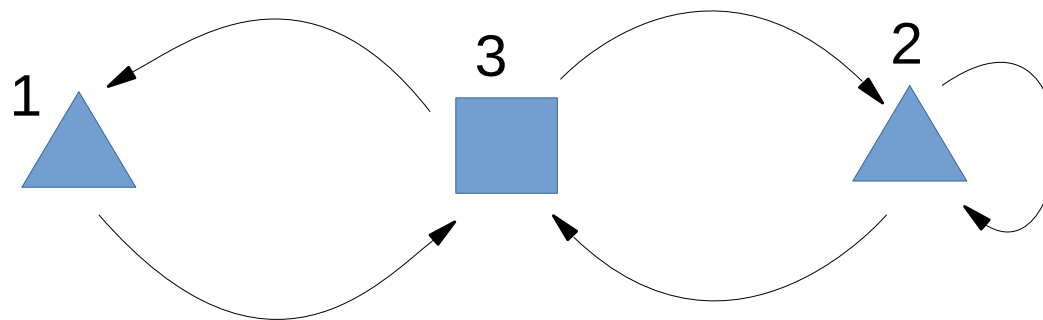
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**This can be decided in quasi-polynomial time, i.e.  $n^{\log(n)+O(1)}$**

A few algorithms achieving this:

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*Older results:*

- multiple (sub-)exponential algorithms
- among them: **Zielonka's algorithm 1998**
  - very simple recursive algorithm
  - exponential in the worst case
  - behaves quite well in practice

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Idea of the recursion in the Zielonka's algorithm:

- for  $k=0,1,2,3,\dots$  find regions where Odd can win while visiting the maximal even priority at most  $k$  times

Idea of the modification:

- at most one of these regions can have more than  $n/2$  nodes (because they are disjoint)

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Why our algorithm is interesting?

- simplicity
- different approach (all the other quasi-polynomial-time algorithms follow so-called separation approach)

Thank you!