Universal trees grow inside separating automata: Quasi-polynomial lower bounds for parity games

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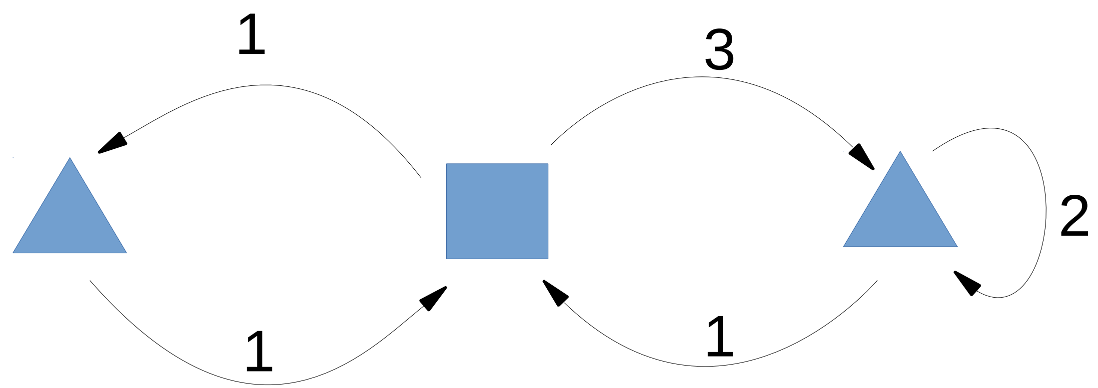
Highlights 2018

Parity games



- Priorities on edges
- Player owning the current vertex choses the next vertex
- Player ☐ wins if the biggest priority seen infinitely often is even.

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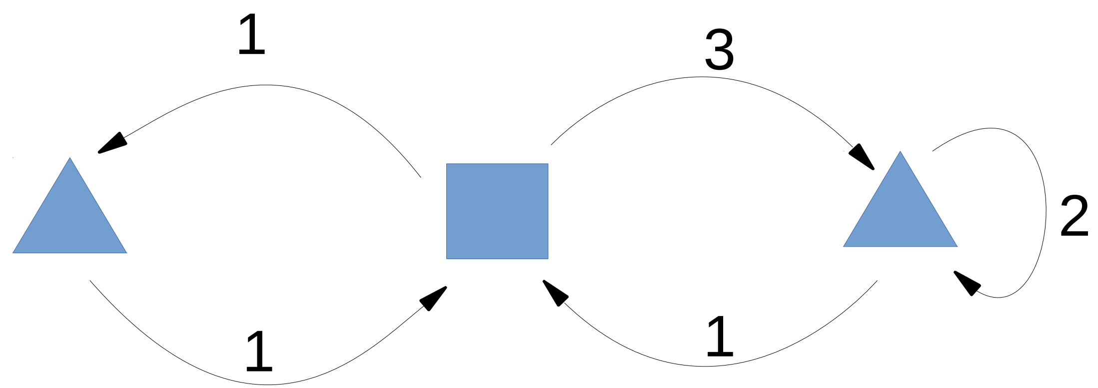

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Decide in PTIME which player has a winning strategy.

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This can be decided in quasipolynomial time, i.e. $n^{log(n)+O(1)}$

Several algorithms achieving this:
- Calude, Jain, Khoussainov, Li, Stephan 2017
- Fearnley, Jain, Schewe, Stephan, Wojtczak 2017
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Our contributions:
1) All these algorithms use „the separation approach”
2) Quasipolynomial lower bound for the separation approach

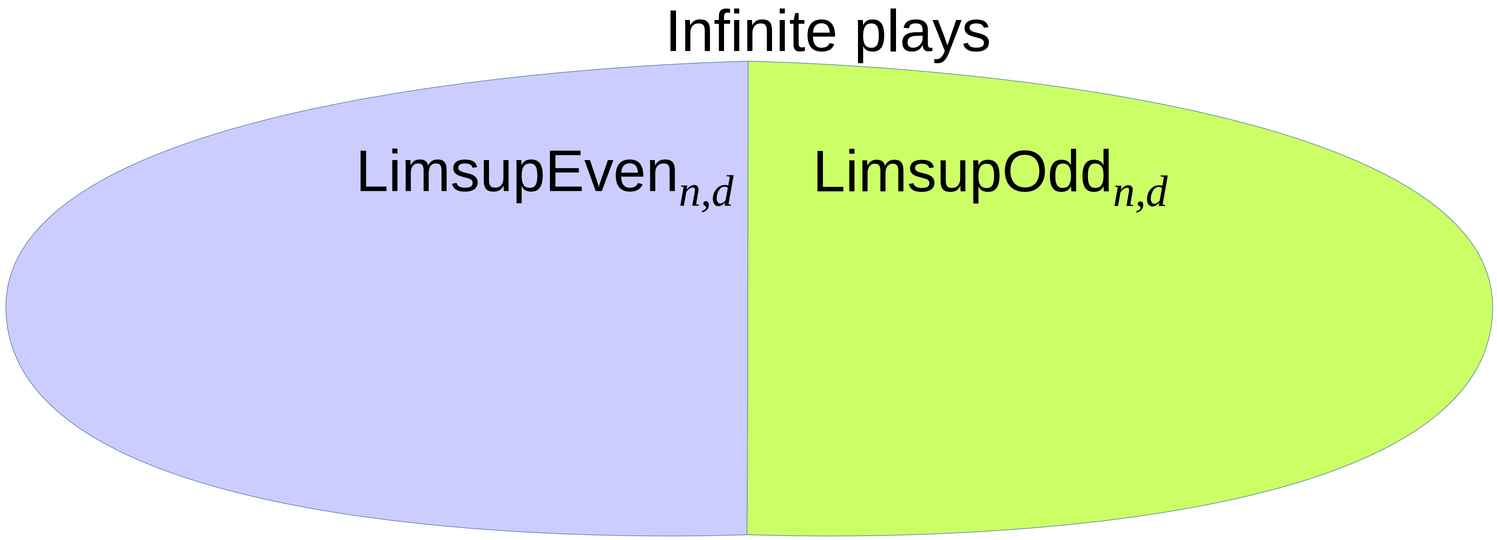
Corollary: **A polynomial algorithm has to work differently.**

The separation approach

Encoding of infinite plays – a sequence of pairs:

- vertex number
- the priority read from this vertex

➔(1,1),(2,2),(2,3),(3,1),(2,3),...



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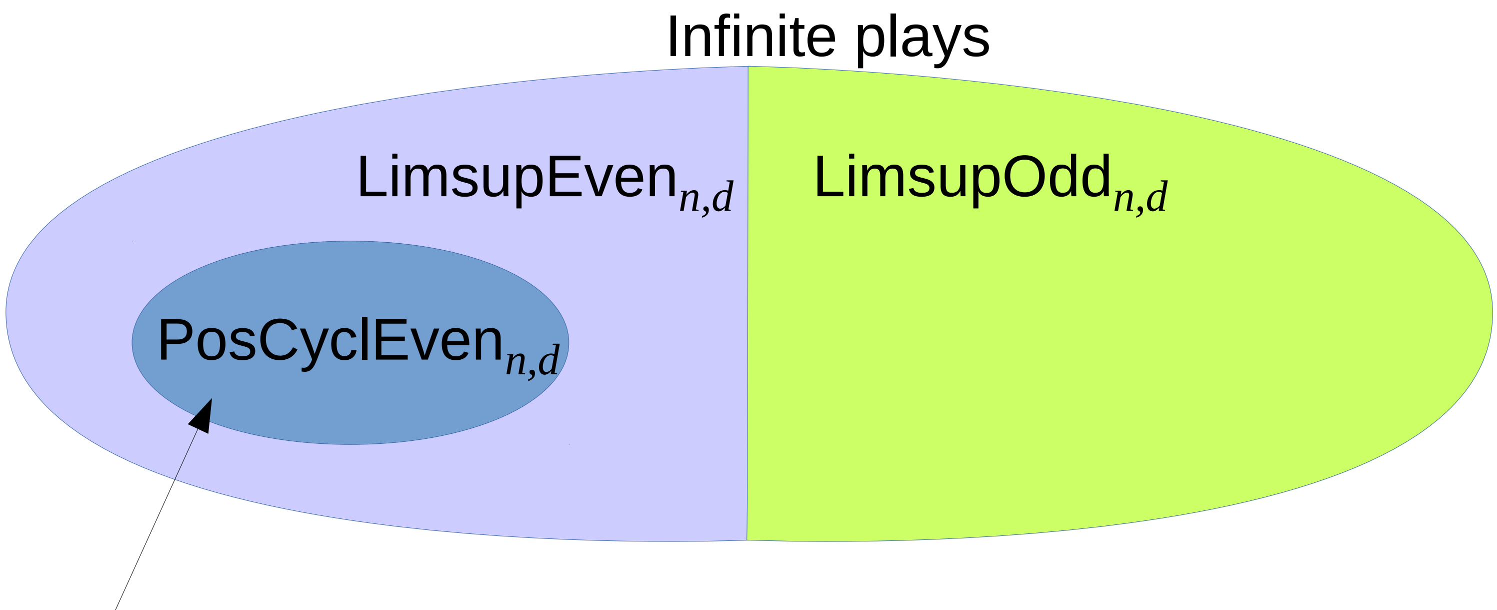
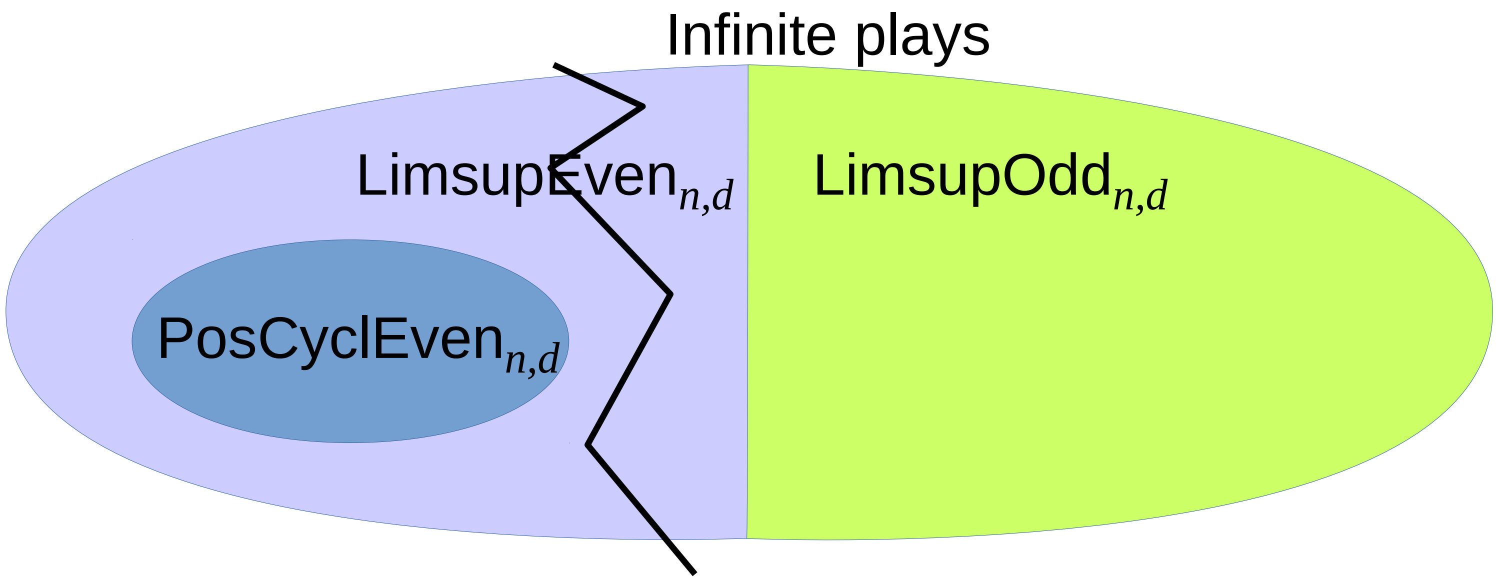


plays consistent with a <u>positional</u> winning strategy (in some game graph)

Theorem

If a player has a winning strategy, then it has a positional winning strategy (a move does not depend on the history, only on the current vertex)

The separation approach

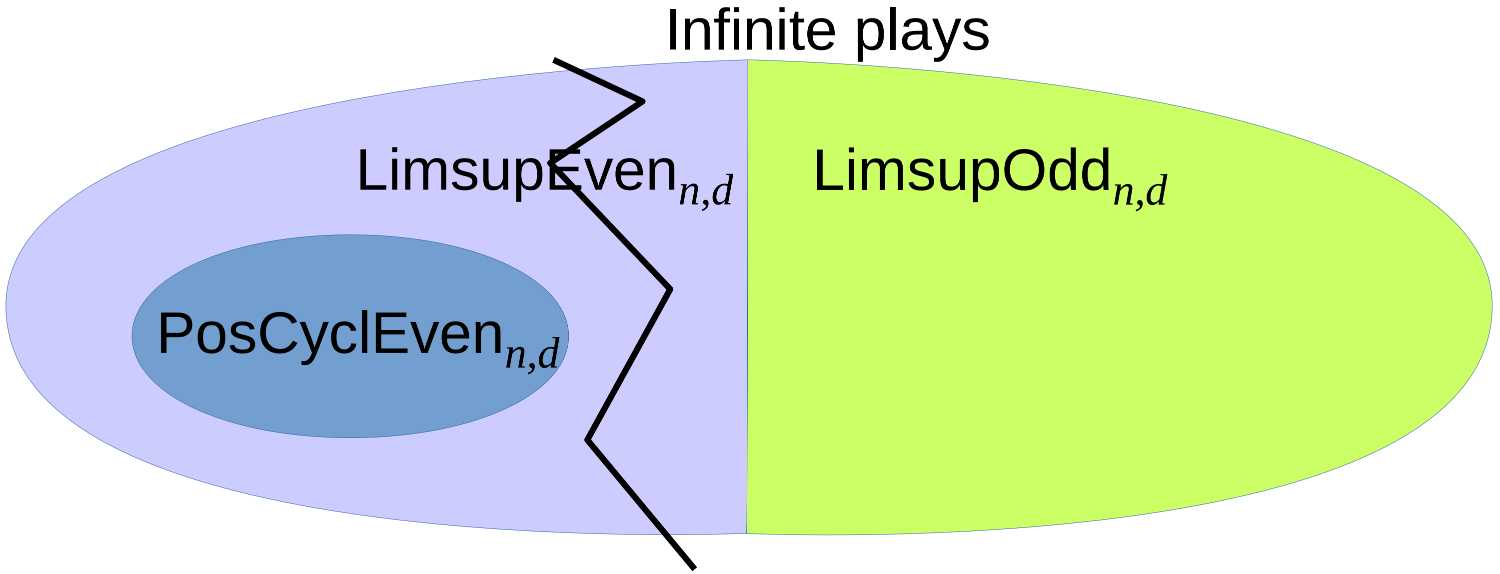


1) Construct a safety automaton A which
   - accepts plays compatible with a positional strategy for Even
   - rejects plays lost by Even
2) Consider the product game $G \times A$ (safety game)
3) Solve this safety game

(running time $\approx$ size of A)

Remark: A does not depend on G, only on n and d

The lower bound

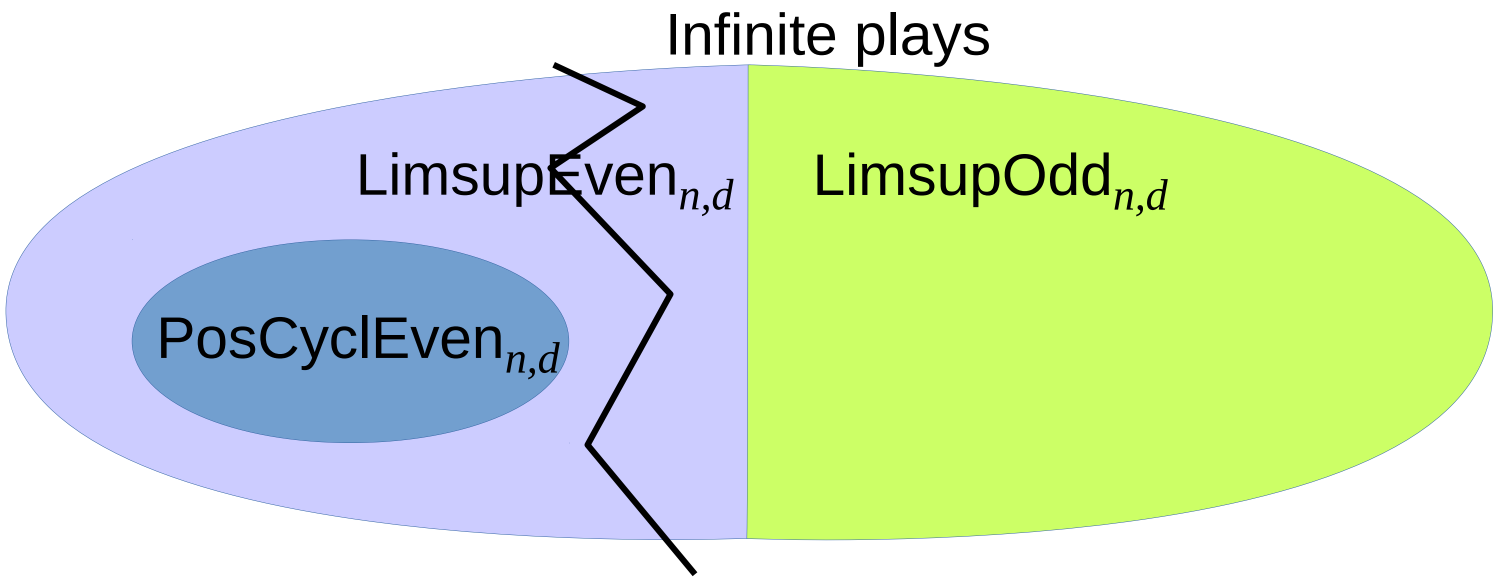


Theorem
Every safety automaton A which

- accepts plays compatible with a positional strategy for Even
- rejects plays lost by Even

has at least quasipolynomial size

The lower bound



Theorem

Every safety automaton A which

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has at least quasipolynomial size

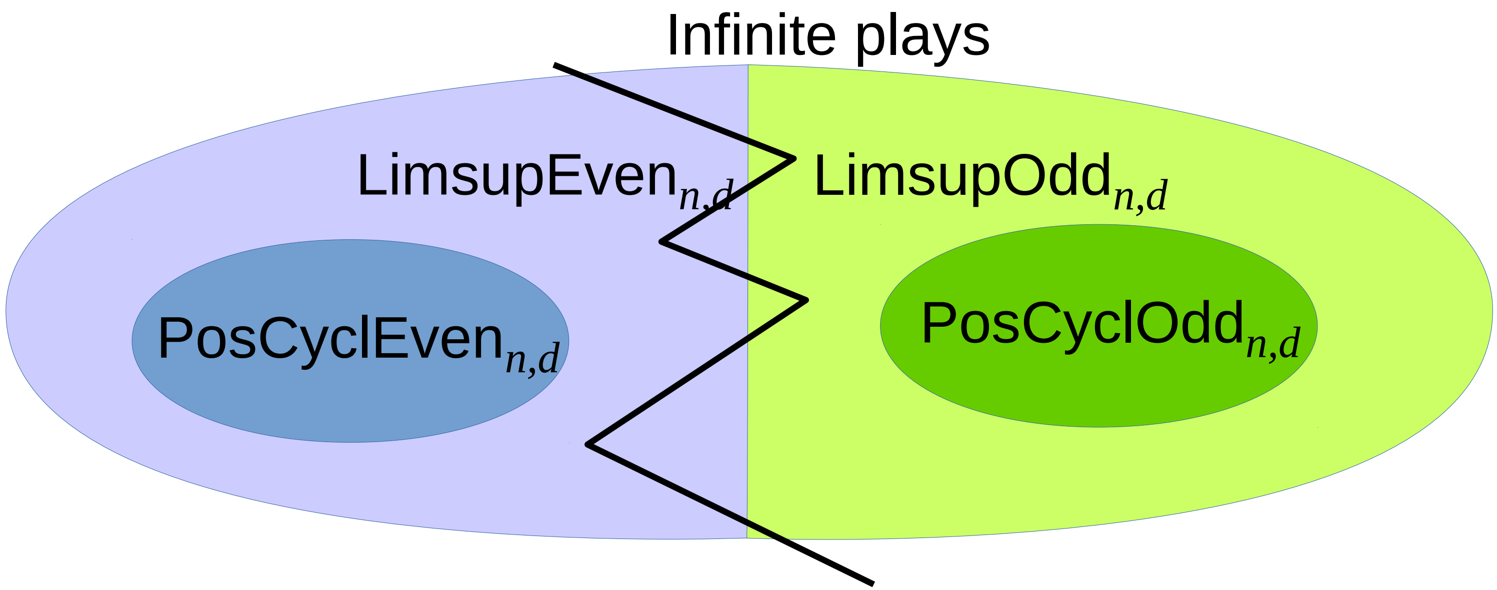
Step 1

Every such automaton has a structure of a universal tree

Step 2

Every universal tree has at least quasipolynomial size

Open problem



Observation:
To solve parity games it is enough to separate PosCyclEven$_{n,d}$ from PosCyclOdd$_{n,d}$.

Open problem:
Does the lower bound apply to automata that separate PosCyclEven$_{n,d}$ from PosCyclOdd$_{n,d}$?

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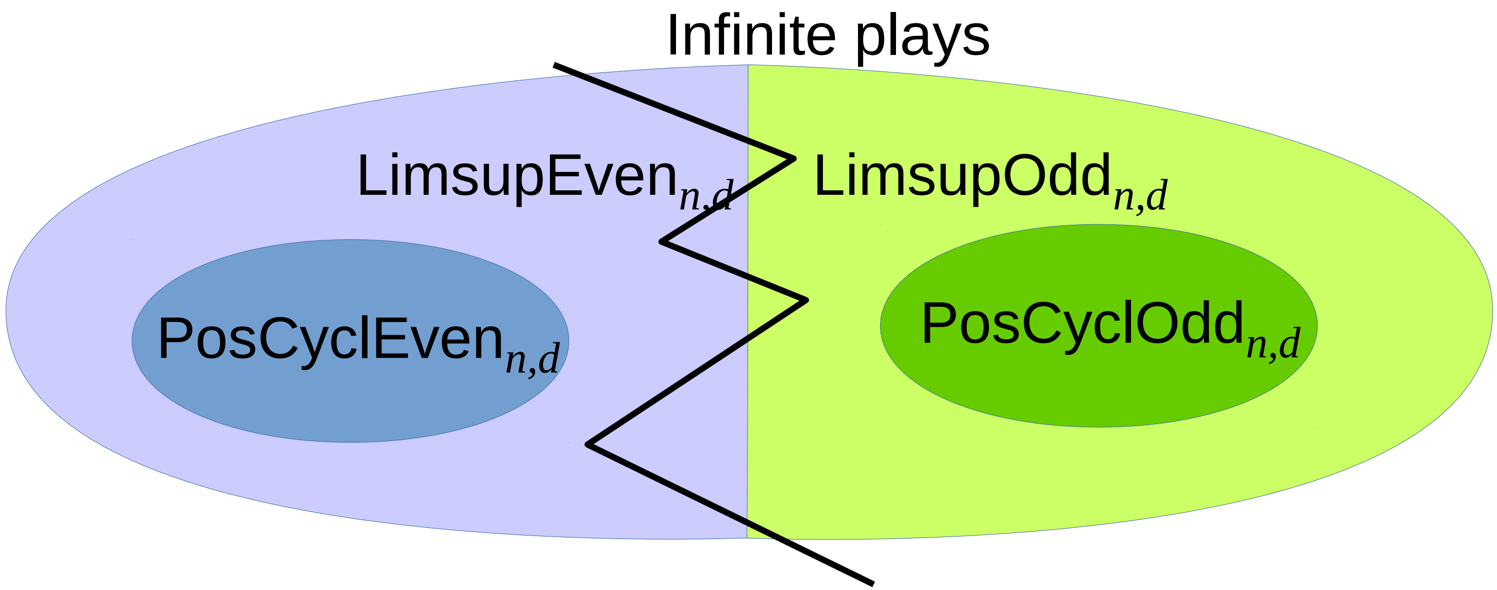

Observation:

To solve parity games it is enough to separate $\text{PosCyclEven}_{n,d}$ from $\text{PosCyclOdd}_{n,d}$.

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Does the lower bound apply to automata that separate $\text{PosCyclEven}_{n,d}$ from $\text{PosCyclOdd}_{n,d}$?

Thank you!