XPath Evaluation in Linear Time

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Considered problem

Input:

XML document

XPath query

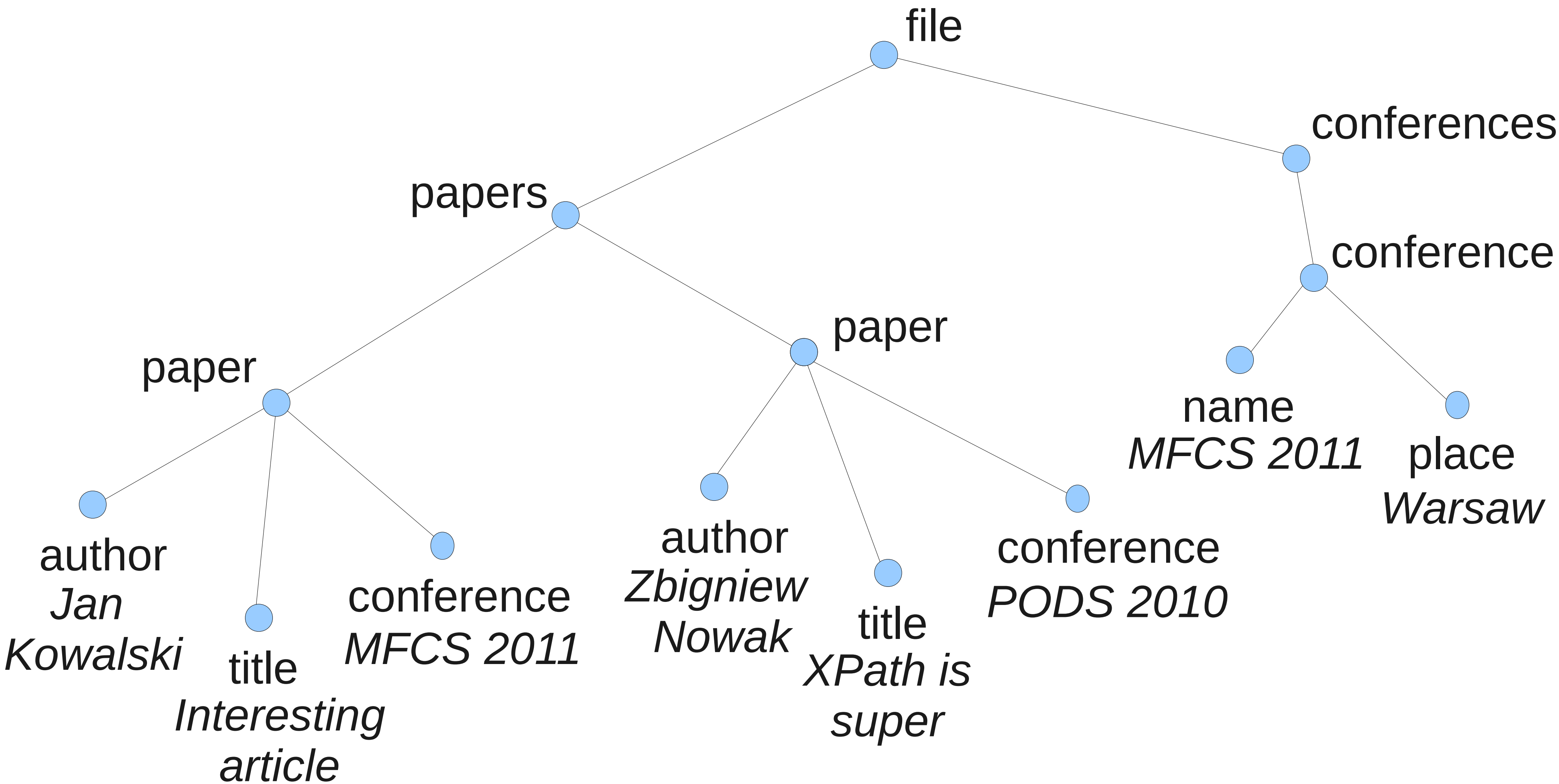
Output:

Nodes of the document satisfying the query

Example

```
<file>
  <papers>
    <paper>
      <author>Jan Kowalski</author>
      <title>Interesting article</title>
      <conference>MFCS 2011</conference>
    </paper>
    <paper>
      <author>Zbigniew Nowak</author>
      <title>XPath is super</title>
      <conference>PODS 2010</conference>
    </paper>
  </papers>
  <conferences>
    <conference>
      <name>MFCS 2011</name>
      <place>Warsaw</place>
    </conference>
  </conferences>
</file>
```

Example


file
papers
conferences
conference
paper
paper
name
MFCS 2011
place
Warsaw
author
Jan
Kowalski
title
Interesting
article
conference
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XPath is
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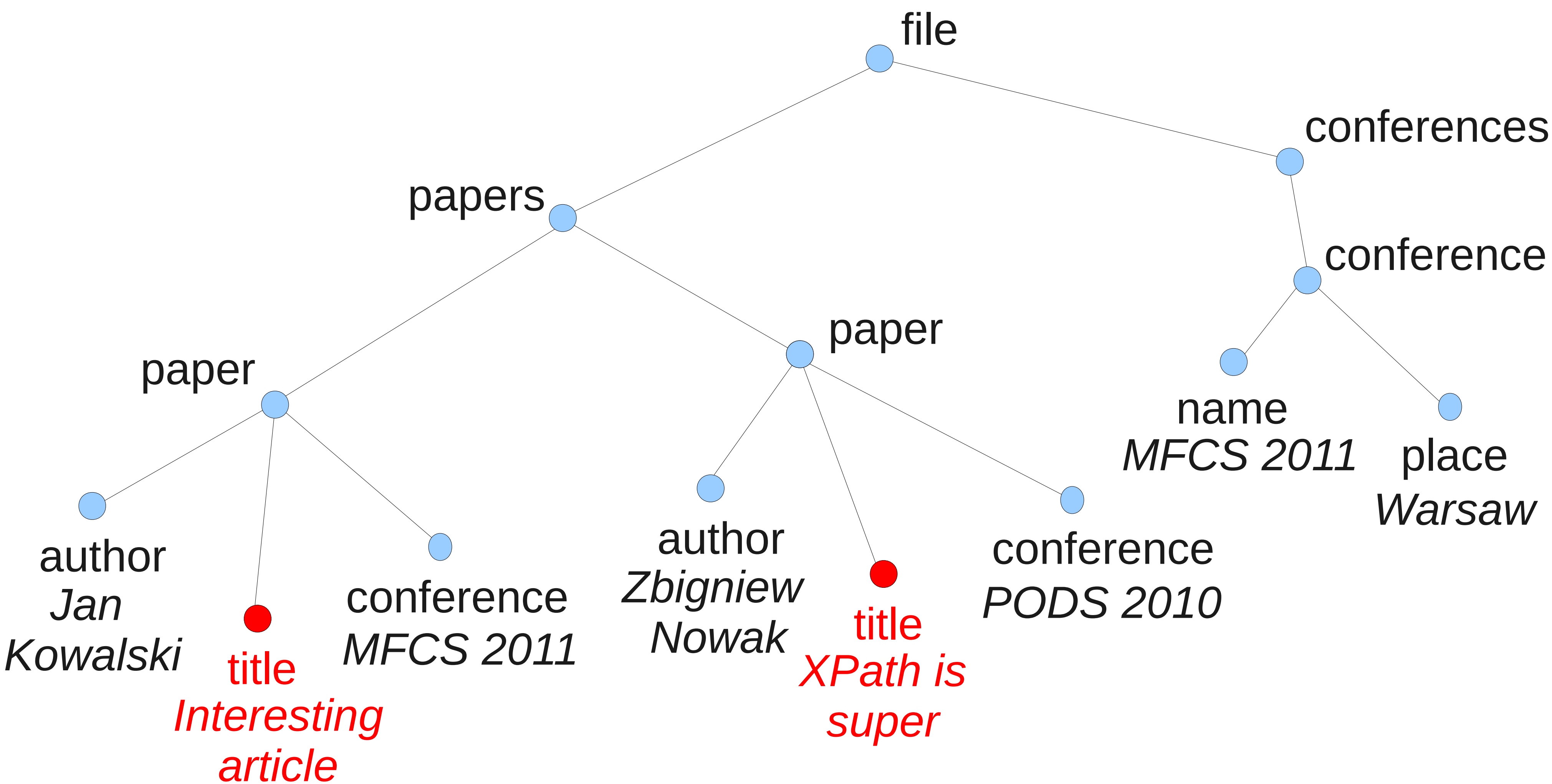
conference
PODS 2010

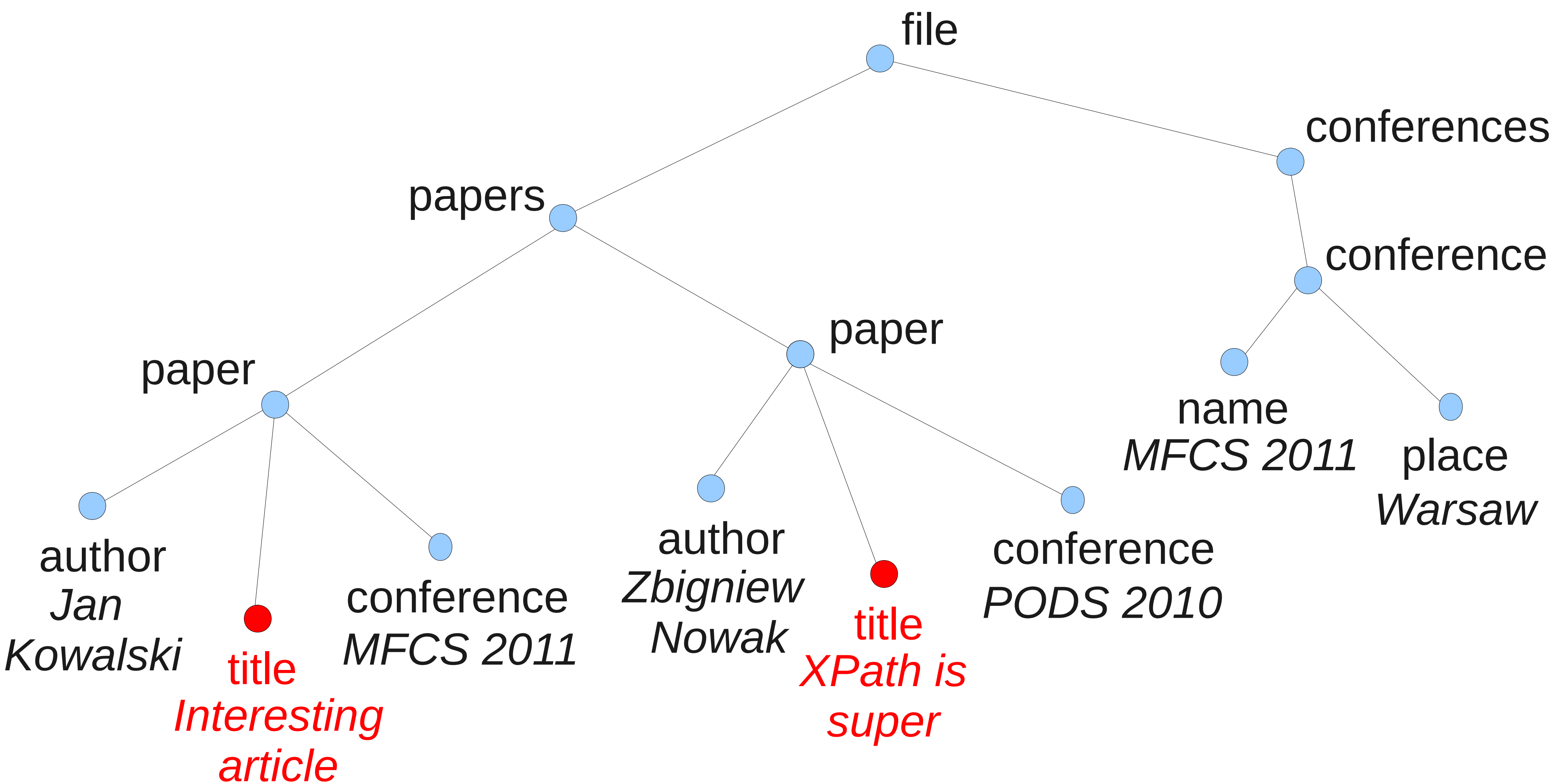
Example



Query:

file/papers/paper/title

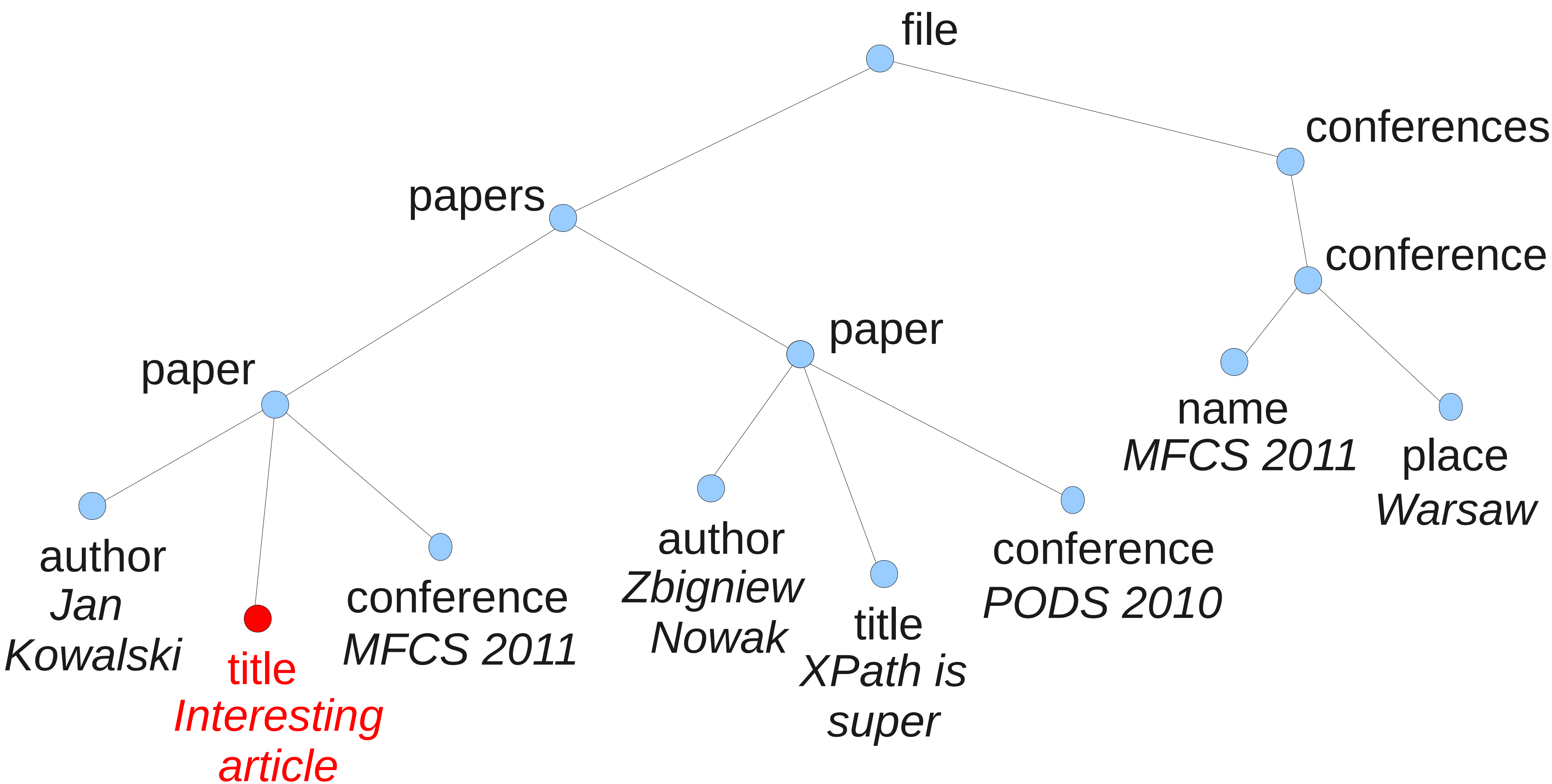
Example



Query:

file/papers/paper/../paper/../paper/../paper/../paper/
../paper/../paper/../paper/../paper/../paper/title

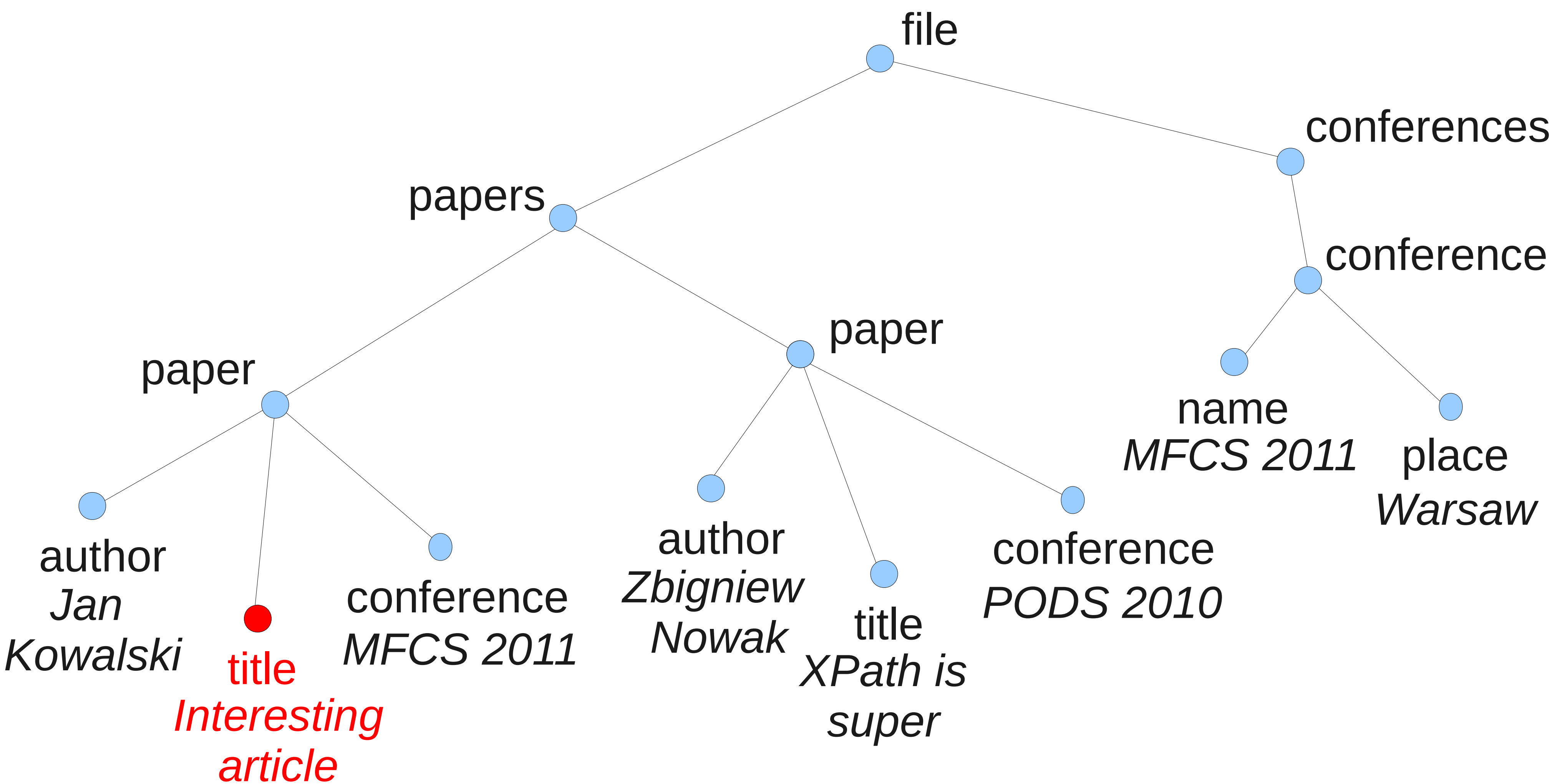
Example



Query:

file/papers/paper[author='Jan Kowalski']/title

Example



Query:

file/papers/paper[conference=
../../conferences/conference[place='Warsaw']/name]/title

Results summary

XML document
XPath query

→ Nodes of the document
Satisfying the query

XPath not refering to data

$O(D \cdot Q)$ - Gottlob, Koch, Pichler 2002

XPath with data (but without counting)

$O(D^2 \cdot Q)$ - Gottlob, Koch, Pichler 2002

$O(D \cdot Q^3)$ - our contribution

Where: D - document size
Q - query size

Subproblem:

Fix a regular language L. A word $u = a_1 \ldots a_n$ is given.

First, in time linear in n, we can prepare ourselves.

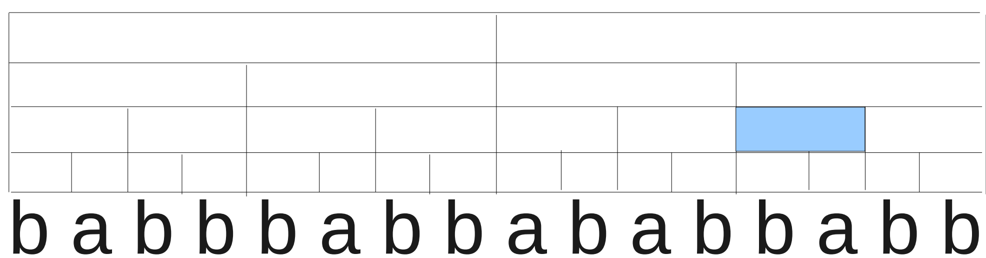
Then, in constant time we want to aswer queries:

$$a_i \ldots a_j \in L\,?$$

Subproblem:

Fix a regular language L. A word $u = a_1 \dots a_n$ is given.

Preprocessing: divide and conquer



For each subword remeber all possible automaton transitions: pairs of states p, q such that

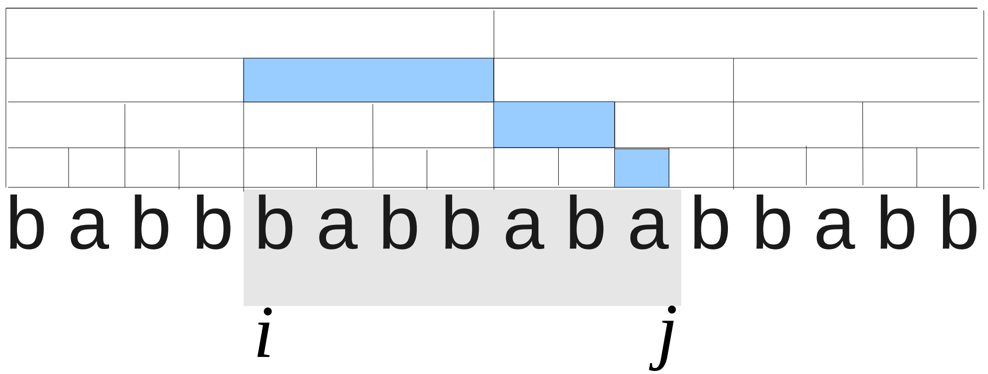
$$p \xrightarrow{a_i \dots a_j} q$$

time: $O(n)$

Subproblem:

Fix a regular language L. A word $u = a_1 \ldots a_n$ is given.

Given: i, j



Does $a_i \ldots a_j \in L$?

It is enough to compose remembered transitions!

time: $O(\log n)$

Subproblem:

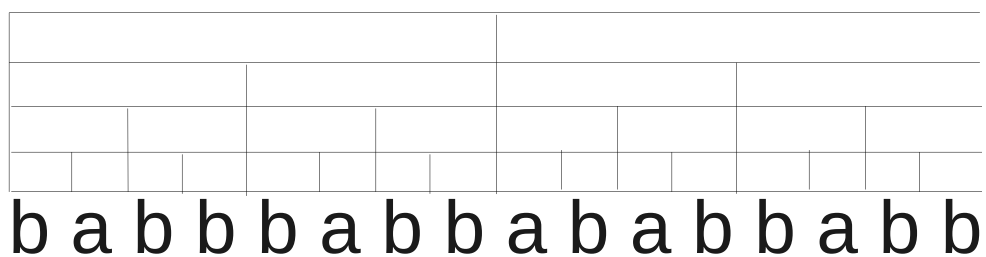
A tool used: Simon's theorem

(I. Simon, Factorization forests of finite height, 1990)

Subproblem:

Fix a regular language L. A word $u = a_1 \ldots a_n$ is given.

In the „logarithmic” decomposition we always split into 2 parts



To achieve a constant height of the decomposition tree
we have to allow splits into arbitrarly many parts
- but then all parts have to be very similar

Simon's decomposition:

Every word u in the decomposition tree we split into

- 2 (arbitrary) parts $u = u_1 u_2$, or
- arbitrarly many parts $u = u_1 \ldots u_k$, where all $u_i \ldots u_j$ are equivalent.

Simon's Theorem:
For every word there exists such a decomposition tree of the same height.

u and v are equivalent, if for any words w_1, w_2 it holds
$w_1 u w_2 \in L \Leftrightarrow w_1 v w_2 \in L$

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Example

L=(a+b)*b

a a a a b

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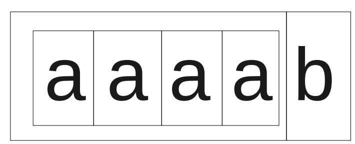

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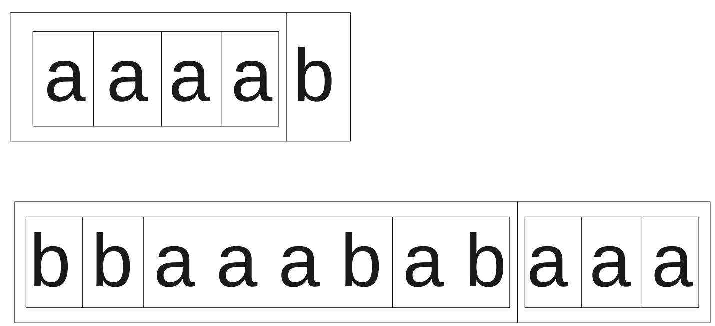

Subproblem:

Fix a regular language L. A word $u = a_1 \dots a_n$ is given.

Preprocessing:

- calculate the Simon's decomposition
- for every subword in the decomposition compute the transitions of the automaton

time: $O(n)$

Does $a_i \dots a_j \in L$?

- It is enough to compose remembered transitions

time: $O(1)$

Subproblem:

Fix a regular language L. A word $u = a_1 \ldots a_n$ is given.

Prepro

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time: (

Does

• It is

time:

Dependance on language L

Height of the decomposition tree is proportional to the number of abstraction classes, which is exponential in the automaton size.

However the tree has at most $2\mathrm{n}-1$ nodes.

<u>Our contribution:</u>
how to deal with this decomposition in time polynomial in the automaton size.

Thank you