Parity Games:
Zielonka's Algorithm in Quasi-Polynomial Time

Paweł Parys (University of Warsaw)

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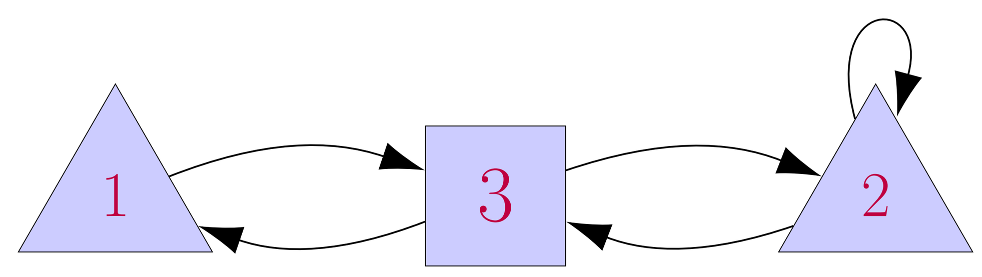
Parity Games

- Each vertex carries a **priority**.
- Player □ wins if the biggest priority seen infinitely often is even.



Long-standing open problem:

Decide in PTIME which player has a winning strategy.

State of the Art

Classical results:

- multiple (sub-)exponential algorithms
- among them: *Zielonka's algorithm* (1998)
  - very simple recursive algorithm
  - exponential in the worst case
  - behaves quite well in practice

Recent results:

- *quasi-polynomial* running time: $n^{O(\log n)}$
- several algorithms achieving this:
  - play summaries [Calude, Jain, Khoussainov, Li, Stephan 2017]
  - ordered play summaries [Fearnley, Jain, Schewe, Stephan, Wojtczak 2017]
  - succinct progress measure [Jurdziński, Lazić 2018]
  - register games [Lehtinen 2018]
- all are special instances of the separation approach
  [Czerwiński, Daviaud, Fijalkow, Jurdziński, Lazić 2019]

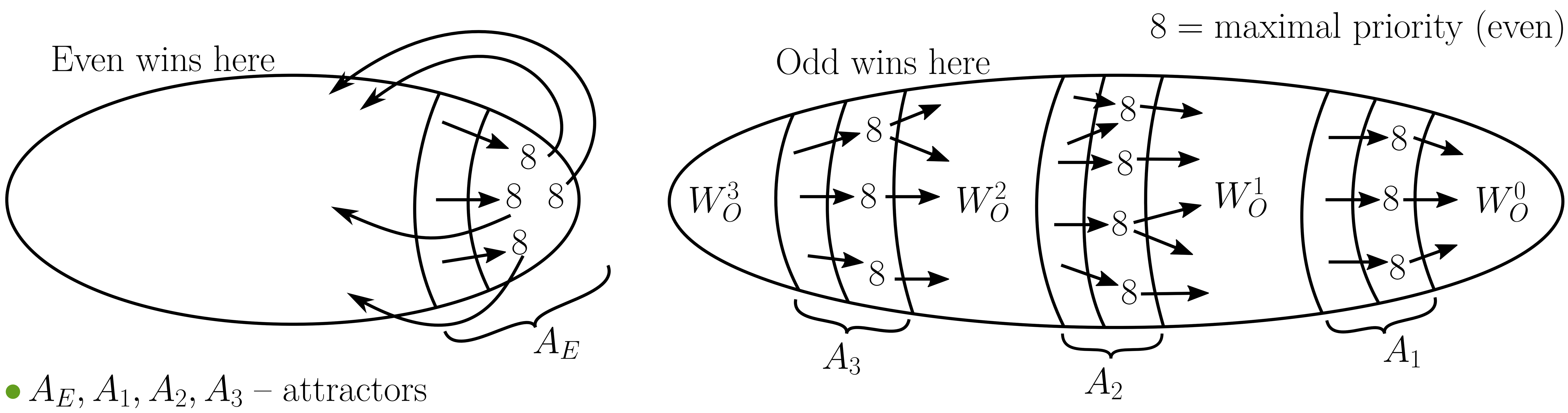
Results:

We present a small modification of the simple, recursive Zielonka's algorithm; the new algorithm works in quasi-polynomial time, i.e., $n^{O(\log n)}$.

Highlights:

- simplicity
- a different approach (we do not follow the separation approach)

Idea of the Algorithm



- A_E, A_1, A_2, A_3 – attractors
- W_O^k – a region where Odd can win while seeing priority 8 at most k times

Idea of a recursion in the Zielonka's algorithm:

- For $k = 0, 1, 2, 3, \ldots$ find the region W_O^k and remove it (together with its attractor A_{k+1}).
- To this end, for each k we need to solve a game having less priorities than the original one.

Idea of our modification:

- At most one of the regions W_O^k can have more than $n/2$ vertices (because they are disjoint).