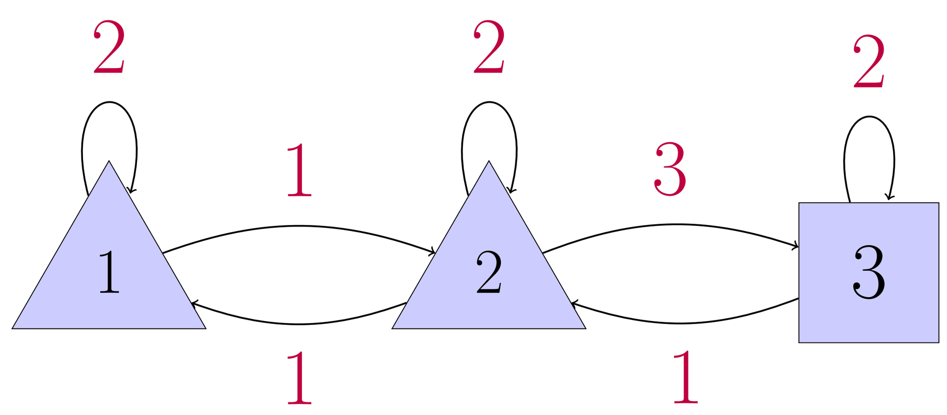
Universal trees grow inside separating automata: Quasi-Polynomial lower bounds for parity games

W. Czerwiński, L. Daviaud, N. Fijalkow, M. Jurdziński, R. Lazić, P. Parys

Parity Games

- Each edge carries a **priority**.
- Player □ wins if the biggest priority seen infinitely often is even.



Long-standing open problem:

Decide in PTIME which player has a winning strategy.

Encoding of infinite plays:

Letters:

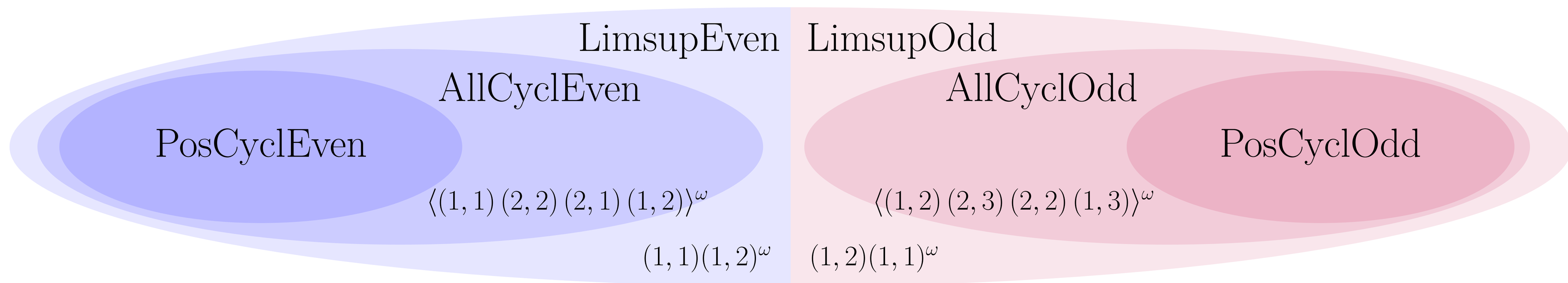
- a vertex
- a priority read from this vertex

$\rightarrow (1,1)(2,2)(2,3)(3,1)(2,1)\cdots$

The Separation Approach

Construct a ***safety automaton*** *which separates:*

- **PosCyclEven**: plays compatible with a positional strategy winning for □
- from **PosCyclOdd**: the ones for △.



A Simple Separating Safety Automaton: [Bernet, Janin, Walukiewicz]

States: sequences $\langle c_{d-1}, c_{d-3}, \ldots, c_1 \rangle$ of integers between 0 and the number of states and `reject`

Transitions:

$$\langle c_{d-1}, c_{d-3}, \ldots, c_1 \rangle \xrightarrow{(v,\text{even } p)} \langle c_{d-1}, c_{d-3}, \ldots, c_p, n, \ldots, n \rangle$$

$$\xrightarrow{(v,\text{odd } p) \text{ and } c_p>0} \langle c_{d-1}, c_{d-3}, \ldots, c_p - 1, n, \ldots, n \rangle$$

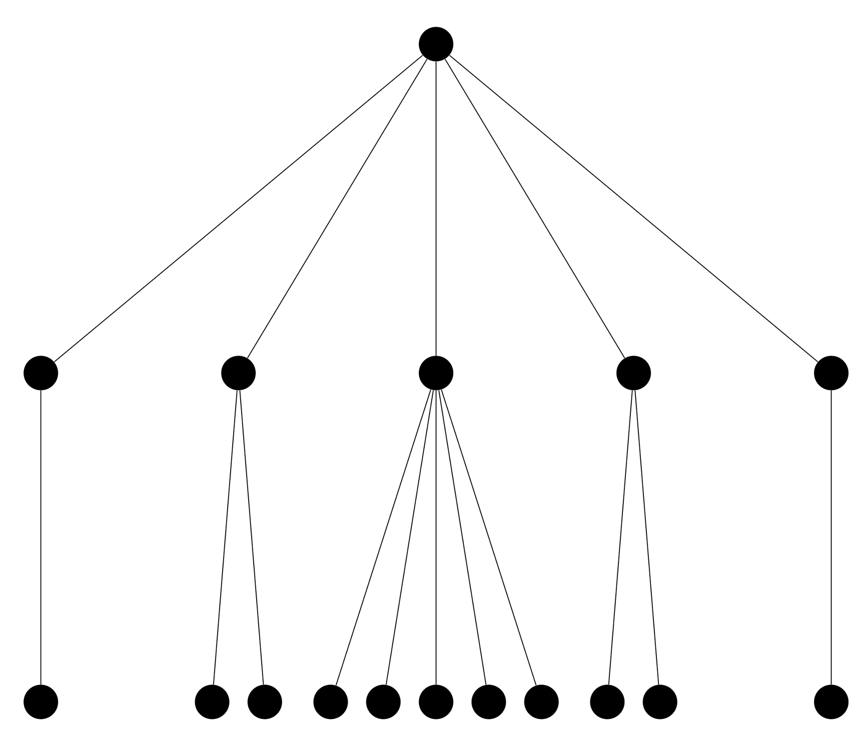
$$\xrightarrow{(v,\text{odd } p) \text{ and } c_p=0} \texttt{reject}$$

Results:

- Quasi-polynomial lower bound for the size of a separating automata.
- The following algorithms constructs (implicitely or explicitely) a separating automaton:
  - play summaries [Calude, Jain, Khoussainov, Li, and Stephan]
  - progress measures [Jurdziński, Lazić]
  - register games [Lehtinen]

Universal trees

(n, h)-universal tree:
ordered tree of height h,
such that
every ordered tree of height at most h and with at most n leaves can be isomorphically embedded into it.



1. Universal trees are of quasi-polynomial size (upper and lower bounds).
2. Embedding of the leaves of universal trees in any safety automata separating PosCyclEven from LimsupOdd.