Problem 1.1. (6 pt) A single-tape Turing machine M is called *predictable* if its head moves in the same way for every input word (formally: if there exists a function $pos : \mathbb{N} \to \mathbb{N}$ such that after k steps of every run of M, regardless of the input word, the head is over the tape cell number $pos(k)$). Is it the case that for every nondeterministic Turing machine M there exists a predictable nondeterministic Turing machine M' recognizing the same language as M and working at most polynomially slower?

Problem 1.2. (6 pt) A function $f : \Sigma^* \to \Gamma^*$ is called a *morphism* if $f(w \cdot v) = f(w) \cdot f(v)$ for all words $w, v \in \Sigma^*$ (the symbol "$\cdot$" denotes concatenation of words). A morphism f is *nonabbreviating* if $|f(w)| \geq |w|$ for all $w \in \Sigma^*$ (i.e., f does not decrease the length of words). For a set of words $L \subseteq \Sigma^*$ we define $f(L) = \{f(w) \mid w \in L\}$. We say that a class $\mathscr{C}$ is closed under images of nonabbreviating morphisms if for every L in $\mathscr{C}$, and for every nonabbreviating morphism f, also $f(L)$ belongs to $\mathscr{C}$.

Prove that the complexity class P is closed under images of nonabbreviating morphisms if and only if $\mathsf{P} = \mathsf{NP}$.