Computational Complexity

10.12.2019

Problem 1. (8 pt) Prove that the following problem is NP-complete (wrt. polynomial-time reductions):
Input: weights $w_1, \ldots, w_n$ of n items, a number k of boxes, a number $w_{\max}$ (all the numbers are given in binary);
Question: is it possible to distribute items between k boxes (each item in exactly one box) so that the total weight of items in each box is at most $w_{\max}$?

Problem 2. (8 pt) Prove that the following problem belongs to the class NL:
Input: a finite directed graph G;
Question: is the length of every cycle in G divisible by 2019?

Problem 3. (8 pt) The first-order logic over directed graphs has the following syntax:

$$\varphi ::= \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi_1 \mid \exists x.\varphi_1 \mid \forall x.\varphi_1 \mid E(x_1, x_2)\,.$$

We evaluate a sentence (i.e., a formula without free variables) of first-order logic in a given graph: variables range over vertices of the graph, and $E(x_1, x_2)$ says that there is an edge from x_1 to x_2. For example, the sentence $\forall x.\exists y.E(x, y)$ is true in graphs in which there is an outgoing edge from every vertex.

Prove that the following problem is PSPACE-hard (wrt. polynomial-time reductions):
Input: a finite directed graph G, a sentence φ of first-order logic;
Question: is φ true in G?