Computational Complexity
Exam (Theory Test)
5.02.2020

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your name & index number

For each question, give answer: YES, NO, or NOT KNOWN. The third possibility means that the current state of knowledge allows for both possibilities. Correct answer gives 1 pt, incorrect answer gives -0.5 pt.

1. Does there exist an undecidable language L and a deterministic Turing machine that (in an infinite loop) outputs all words from L? YES

   The statement says that L is semidecidable but not decidable; such languages exist.

2. Is there an algorithm that given a Turing machine M and a number $k \in \mathbb{N}$ answers whether for every input the machine M stops after at most k steps? YES

   We can check all possibilities (only the first k letters of the input can be read).

3. Is there an NP-complete (w.r.t. polynomial-time reductions) language containing only finitely many words? NOT KNOWN

   Equivalent to $\mathsf{P} = \mathsf{NP}$.

4. Does SAT $\in$ coNP? NOT KNOWN

   Equivalent to $\mathsf{NP} = \mathsf{coNP}$.

5. Is $\mathsf{NTIME}(n^2)$ closed under logarithmic-space reductions? NO

   This would contradict the time hierarchy theorem: every language in $\mathsf{NTIME}(n^4)$ can be reduced to a language in $\mathsf{NTIME}(n^2)$ (we add padding).

6. Is L closed under polynomial-time reductions? NOT KNOWN

   Equivalent to $\mathsf{L} = \mathsf{P}$.

7. Is it true that either $\mathsf{P} = \mathsf{NP}$, or there is a language $L \in \mathsf{NP} \setminus \mathsf{P}$ that is not NP-complete? YES

   This is the statement of the Ladner's theorem (lecture).

8. Is it true that if $\mathsf{P} = \mathsf{PSPACE}$ then $\mathsf{EXPTIME} = \mathsf{EXPSPACE}$? YES

   Can be shown using the padding technique.

9. Does $\mathsf{AC}^0 = \mathsf{AC}^5$? NO

   Parity (i.e., XOR of input bits) cannot be recognized in AC^0 (lecture).

10. Does QBF $\in$ uniform-NC^1 (where QBF = "quantified Boolean formula")? NO

    Impossible because QBF is PSPACE-complete and uniform-$\mathsf{NC}^1 \subseteq \mathsf{L} \neq \mathsf{PSPACE}$.

11. Does $\mathsf{P/poly} \subseteq \mathsf{PSPACE}$? NO

    Impossible, because P/poly contains some undecidable languages.

12. Is RP closed under polynomial-time reductions? YES

    Direct from definition.

13. Does $\mathsf{BPP} \cap \mathsf{coBPP} \subseteq \mathsf{NP}$? NOT KNOWN

    Notice that $\mathsf{BPP} \cap \mathsf{coBPP} = \mathsf{BPP}$; it is an open problem whether $\mathsf{BPP} \subseteq \mathsf{NP}$.

14. Current best polynomial approximation algorithm for VERTEX-COVER gives 2-approximation. Does there exist a PTAS (polynomial time approximation scheme) for VERTEX-COVER? NOT KNOWN

    By definition PTAS requires $(1+\varepsilon)$-approximation for every ε, but we only have 2-approximation. But $\mathbf{P} = \mathbf{NP}$ implies existence of PTAS.

15. Current best algorithm for k-clique works in $O(n^{0.8k})$ time. Is k-clique (with parameter k) in **FPT**? NOT KNOWN

    This algorithm is not **FPT**, as for **FPT** we need $O(f(k) \cdot n^c)$ for a constant c. But $\mathbf{P} = \mathbf{NP}$ implies that the problem is in **FPT**.

16. Does $\mathsf{NPSPACE} \cap \mathsf{coNPSPACE} = \mathsf{IP}$? YES

    $\mathsf{NPSPACE} \cap \mathsf{coNPSPACE} = \mathsf{PSPACE} = \mathsf{IP}$.