

Problem 1.1. (0.25 pt) Consider multitape Turing machines in which before moving a head left, it is necessary to write a blank symbol. Does machines fulfilling this restriction recognize the same class of languages as standard Turing machines?

Problem 1.2. (0.25 pt) A finite set $C \subseteq \mathbb{N}^k$ (where $k \geq 2$) is called a *rectangle* if it can be represented as $C = A \times B$, for some $A \subseteq \mathbb{N}^i$ and $B \subseteq \mathbb{N}^j$, where $i + j = k$ and $i, j \geq 1$. We assume the standard representation of finite sets $C \subseteq \mathbb{N}^k$ in words over $\{0, 1, \$, \#\}$: a vector $(a_1, \dots, a_k) \in \mathbb{N}^k$ is represented as $a_1\$a_2\$ \dots \a_k (where a_i is written in binary), and a set of vectors $\{v_1, \dots, v_m\}$ is represented as $v_1\#v_2\# \dots \#v_m$ (notice that a set may allow multiple representations).

Prove that the set of words representing rectangles can be recognized by a deterministic Turing machine working in logarithmic space.

Example: the following two words should be accepted:

0\$0#1\$1#0\$1#1\$0

and

0\$0\$0#0\$0\$1#0\$1011\$0.