Computational Complexity
Exam (Theory Test)
5.02.2019

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your name & index number

For each question, give answer: YES, NO, or NOT KNOWN. The third possibility means that the current state of knowledge allows for both possibilities. All questions are equally valued, there are no negative points for wrong answer. You should assume that completeness is defined using log-space reductions.

1. Given a deterministic Turing machine M and an input word w, is it decidable whether M accepts w in space $15 \cdot |w|^{100}$? YES

   Existence of a universal Turing machine—we can simply simulate M on w, halting whenever a configuration repeats.

2. Is the following implication true for all complexity classes C, D (i.e. for all sets of languages that are closed under log-space reductions): if $\mathsf{C} \subseteq \mathsf{D}$ then $\mathsf{coD} \subseteq \mathsf{coC}$? NO

   False e.g. for $\mathsf{C} = \mathsf{L}$ and $\mathsf{D} = \mathsf{PSPACE}$. We rather have that $\mathsf{C} \subseteq \mathsf{D}$ implies $\mathsf{coC} \subseteq \mathsf{coD}$.

3. Does REACHABILITY $\in \mathsf{coNL}$? YES

   We have REACHABILITY $\in \mathsf{NL}$, and $\mathsf{NL} = \mathsf{coNL}$ by the Immerman–Szelepcsényi theorem.

4. Does $\mathsf{NP} \subseteq \mathsf{DTIME}(n^{2019})$? NO

   By the time-hierarchy theorem $\mathsf{DTIME}(n^{2020}) \setminus \mathsf{DTIME}(n^{2019})$ is nonempty, and $\mathsf{DTIME}(n^{2020}) \subseteq \mathsf{NP}$.

5. Does QBF $\in \mathsf{NP}$ (where QBF = "quantified Boolean formula")? NOT KNOWN

   We know that QBF is PSPACE-complete, and it is an open problem whether $\mathsf{NP} = \mathsf{PSPACE}$.

6. Does there exist languages L and O such that L can be recognized in polynomial time by a non-deterministic Turing machine with oracle O, but L cannot be recognized in polynomial time by a deterministic Turing machine with oracle O? YES

   This is a part of the statement of the Baker-Gill-Solovay theorem.

7. Is it true that for every complexity class C (i.e. for every set of languages that is closed under log-space reductions) there exists a C-complete problem? NO

   This is false e.g. for the class $\mathsf{C} = \bigcup_{k \in \mathbb{N}} \mathsf{DSPACE}(\log^k n)$. Indeed, a C-complete problem would belong to $\mathsf{DSPACE}(\log^k n)$ for some particular k, and using a log-space reduction to this problem we could solve every problem from C in $\mathsf{DSPACE}(\log^k n)$. But, by the space hierarchy theorem there are problems in $\mathsf{DSPACE}(\log^{k+1} n) \subseteq \mathsf{C}$ that are not in $\mathsf{DSPACE}(\log^k n)$.

8. Is the following implication true for every language L: if L can be recognized by a sequence of circuits of exponential size, then $L \in \mathsf{EXPTIME}$? NO

   There is no uniformity assumption. Every language can be recognized by a sequence of circuits of exponential size, in particular some undecidable languages.

9. Does $\mathsf{L} \subseteq \mathsf{u\text{-}AC}^2$ (where $\mathsf{u\text{-}AC}^2$ = "log-space uniform AC^2")? YES

   We have that $\mathsf{L} \subseteq \mathsf{NL} \subseteq \mathsf{u\text{-}AC}^1 \subseteq \mathsf{u\text{-}AC}^2$. The inclusion $\mathsf{NL} \subseteq \mathsf{u\text{-}AC}^1$ holds because we can solve REACHABILITY by a circuit of logarithmic depth (and polynomial size).

10. Does $\mathsf{RP} \subseteq \mathsf{P/poly}$? YES

    We have that $\mathsf{RP} \subseteq \mathsf{BPP} \subseteq \mathsf{P/poly}$. The second inclusion is the Adleman's theorem.

11. Is the following implication true for every problem X with a parameter k: if X has an $O(7^{8^k+9\ln n})$-time algorithm, then X (with parameter k) is in FPT? YES

    By definition of FPT; we have that $7^{8^k+9\ln n} = 7^{8^k} \cdot n^{9\cdot\ln 7}$.

12. Does $\mathsf{NPSPACE} \subseteq \mathsf{IP}$? YES

    We have $\mathsf{NPSPACE} \subseteq \mathsf{PSPACE}$ by the Savitch's theorem, and $\mathsf{PSPACE} \subseteq \mathsf{IP}$ by the Shamir's theorem.