## Exam in Computational Complexity, 19 June 2017

## Problems

Please write down solutions of problems 1,2,3 on separate signed sheets of paper. For the theoretical part you will obtain a special leaf. All problems are valued equally, and the theoretical part amounts to two problems.

1. Prove that the following problem is complete in the class NL w.r.t. reductions in logarithmic space. Given a directed graph, is this graph strongly connected?
(We assume that the graph is given by an incidence matrix.)
2. Suppose there exists a deterministic polynomial algorithm $A$, which approximates (with error $2 / 5$ ) the probability that a given circuit $C$ accepts a random input. More precisely, for a circuit $C\left(x_{1}, \ldots, x_{n}\right)$, the algorithm computes a rational number $A(C)$, such that

$$
\left|\operatorname{Pr}\left(C\left(U_{n}\right)=1\right)-C(A)\right| \leq \frac{2}{5}
$$

(Here the random variable $U_{n}$ assumes values in $\{0,1\}^{n}$ with uniform probability distribution.) Prove that then $\mathbf{P}=\mathbf{B P P}$.
3. Let $M$ be a non-deterministic Turing machine working in polynomial space. Show that there exists a deterministic machine working in exponential time (i.e., $2^{n^{k}}$, for some $k$ ), which for an input $w$ computes the number of accepting computations of the machine $M$ on input $w$.
(You can assume that $M$ has no infinite computations.)

For each question, give answer: YES, NO, or NOT KNOWN. The third possibility means that the current state of knowledge allows for both possibilities. All questions are equally valued, there are no negative points for wrong answer.

1. If $L_{1}, L_{2} \in \mathbf{N P} \cap \mathbf{c o} \mathbf{- N P}$, then $L_{1} \dot{-} L_{2} \in \mathbf{N P} \cap \mathbf{c o}-\mathbf{N P}$, where $L_{1} \dot{-} L_{2}$ (symmetric difference) contains words that are in exactly one of $L_{1}, L_{2}$.
2. There exists an algorithm solving the following problem.

Decide if a given Turing machine $M$ works in time bounded by $5 \cdot n$.
3. There is an NP-complete problem in the class NSPACE $(n)$.
4. There is a PSPACE-complete problem, which is a regular language.
5. $\mathbf{N P} \neq \mathrm{DTIME}\left(2^{n}\right)$.
6. $\mathbf{B P P} \cap \mathbf{N P} \subseteq \mathbf{P}$.
7. $\mathbf{C F L} \cap \mathbf{c o} \mathbf{- C F L} \subseteq \mathbf{P}$ (where $\mathbf{C F L}$ is the class of context-free languages).
$\qquad$
8. NSPACE $(n \log n)$ is strictly included in DSPACE $\left(n^{3}\right)$.
9. There exists a decidable language in the class $\mathbf{P} /$ poly, which is not in $\mathbf{P}$.
10. co-RP $\subseteq \mathbf{P} /$ poly.
11. The problem of reachability in directed graphs is complete in the class co-NL (w.r.t. reductions in logarithmic space).
12. $\mathbf{A C}^{1} \subseteq \mathbf{N C}^{2}$.

