

Complexity theory 2017/18

Mid-term exam

12 December 2017

Problem 1. A two-way two-head deterministic finite automaton (22-DFA) is defined as a deterministic Turing machine with two read-only input tapes (the same input word is written on both tapes), and without any working tapes.

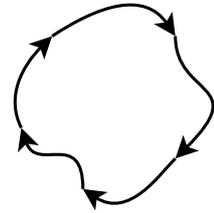
a) (0.25 pt) Prove that every language recognized by a 22-DFA belongs to **L**, and to **P**.

b) (0.25 pt) Prove that **P** contains a language which cannot be recognized by any 22-DFA.

Problem 2. (0.5 pt) For a directed graph $G = (V, E)$ with vertices $V = \{1, \dots, n\}$, let $tab(G) = a_1 \dots a_{n^2}$, where $a_{i+n(j-1)}$ is 1 if in E there is an edge (i, j) , and 0 otherwise. Let

$$C = \{tab(G) \mid G \text{ is a simple cycle}\}.$$

Prove that $C \in \mathbf{NC}$.



Problem 3. (0.5 pt) Prove that the following problem is in **L**:

input: a directed graph G (given as a list of edges);

question: is G a tree, in which all edges are directed down the tree?

