Computational Complexity
Exam
9.02.2018

Problem 1. (0.6 pt) Let

$$\text{DIST} = \{(G, s, t, d) \mid d \text{ is the length of the shortest path from } s \text{ to } t \text{ in directed graph } G\}.$$

In other words, $(G, s, t, d) \in \text{DIST}$ when there is no path from s to t in G of length smaller than d, but there is such a path of length d. Show that DIST is **NL**-complete. (Don't forget to show that DIST $\in$ **NL**. Note that because d is given in binary, the working memory should be $O(\log(\log(d) + |G|))$.)

Problem 2. (0.6 pt) We say that a language $L \subseteq \{0,1\}^*$ has $\mathbf{AC}^0$ witnesses if there exists a polynomial $p : \mathbb{N} \to \mathbb{N}$ and a uniform sequence of circuits of polynomial size and constant depth $(C_n)_{n\in\mathbb{N}}$, where C_n has $n + p(n)$ input gates, such that for every $v \in \{0,1\}^*$,

$$\big(v \in L\big) \Leftrightarrow \big(\exists w \in \{0,1\}^{p(|v|)} \text{ such that } C_{|v|}(v, w) = 1\big).$$

Prove that the class of languages that have $\mathbf{AC}^0$ witnesses equals **NP**.

Problem 3. (0.6 pt) For a word $w \in \{0,1\}^*$, consider the following randomized process:

- we randomly choose a pair of positions a, b such that $1 \le a \le b \le |w|$ and $b - a \le \frac{|w|}{2}$ (every such a pair is equally probable);
- we reverse all bits of w on positions $i \in \{a, a+1, \dots, b\}$ (all 0's are changed to 1's, and all 1's are changed to 0's).

For a language $L \subseteq \{0,1\}^*$, let

$$\text{robust}(L) = \left\{ w \in L \mid \textit{Prob}\big(\text{the process applied to } w \text{ gives a word in } L\big) > \frac{3}{4} \right\}.$$

Show that if $L \in \mathbf{RP}$, then $\text{robust}(L) \in \mathbf{RP}$.