Computational Complexity
Exam (Theory Test)
9.02.2018

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your name & index number

For each question, give answer: YES, NO, or NOT KNOWN. The third possibility means that the current state of knowledge allows for both possibilities. All questions are equally valued, there are no negative points for wrong answer.

1. The following implication holds for every language L: if there is an algorithm (an infinite loop) that prints all words from L (every word from L exactly once; the order in which the words are printed is not specified), then L is decidable. NO

   Languages satisfying the assumption are called semidecidable or recursively enumerable. An example language that is semidecidable, but not decidable, is

   $$L = \{M \mid \text{Turing machine } M \text{ stops on the empty input}\}.$$

   Clearly it is undecidable whether a given Turing machine stops on the empty input. But we can generate all such machines: we simulate all machines in paralel (we start machine number i after simulating i steps of machine number 1), and when some machine stops, we output it.

2. SAT $\in \mathbf{NTIME}(n^2)$. YES

   A nondeterministic algorithm can guess a valuation and check whether it satisfies a formula (this can be done in time $O(n^2)$).

3. REACHABILITY is **NP**-complete. NOT KNOWN

   Because REACHABILITY is **NL**-complete, the question is equivalent to $\mathbf{NL} = \mathbf{NP}$, which is an open problem.

4. $\mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}$. NOT KNOWN

   This is an open problem ($\mathbf{NP}^{\mathbf{NP}}$ is the second level of the polynomial hierarchy). It can be also seen that $\mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}$ is equivalent to $\mathbf{NP} = \mathbf{coNP}$, which is a more known open problem. Indeed, if $\mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}$, then $\mathbf{coNP} \subseteq \mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}$, and hence $\mathbf{NP} = \mathbf{coNP}$. If $\mathbf{NP} = \mathbf{coNP}$, then $\mathbf{NP}^{\mathbf{NP}} = \mathbf{NP}^{\mathbf{NP} \cap \mathbf{coNP}} = \mathbf{NP}$.

5. The problem "is there a perfect matching in a given undirected graph" is **NP**-complete. NOT KNOWN

   If yes, then $\mathbf{P} = \mathbf{NP}$, because the problem is in **P**. If not, then $\mathbf{L} \neq \mathbf{NP}$, because the problem is **L**-hard (as almost every problem). Both $\mathbf{P} = \mathbf{NP}$ and $\mathbf{L} \neq \mathbf{NP}$ are open problems.

6. HamiltonianCycle can be solved in polynomial space. YES

   We know that HamiltonianCycle is in **NP**, so even more in **PSPACE**.

7. Horn-SAT $\in \mathbf{NC}$. NOT KNOWN

   Horn-SAT is **P**-complete, so the question is equivalent to $\mathbf{NC} = \mathbf{P}$, which is an open problem.

8. $\mathbf{coRP} \cap \mathbf{RP} \subseteq \mathbf{BPP} \cap \mathbf{coBPP}$. YES

   We know that $\mathbf{RP} \subseteq \mathbf{BPP}$, so $\mathbf{coRP} \subseteq \mathbf{coBPP}$ as well.

9. VertexCover admits a polynomial time approximation with factor $\frac{1}{2}$ (finding a cover twice larger than the optimal one). YES

   The algorithm was presented during the lecture.

10. $\mathbf{P} = \mathbf{IP}$. NOT KNOWN

    We know that $\mathbf{IP} = \mathbf{PSPACE}$, so this is equivalent to the open problem $\mathbf{P} = \mathbf{PSPACE}$.

11. If $\mathbf{L} = \mathbf{P}$, then $\mathbf{PSPACE} = \mathbf{EXP}$. YES

    Standard padding argument.

12. The following implication holds for every problem X with a parameter k: if X has an $O(n^{k^k})$ time algorithm, then X (with parameter k) is in **FPT**. NO

    Consider the problem

    $$X = \{(k, M, w) \mid M \text{ accepts } w \text{ in } \sqrt{(|w| + 2)^{k^k}/|M|} \text{ steps}\}.$$

    It can be solved in time $O(n^{k^k})$: we simply simulate the machine from the input (simulating t steps of a machine M can be done in time $|M| \cdot t^2$). Suppose X is in **FPT**. Then, by definition, there exists an algorithm solving X in time $O(n^c \cdot f(k))$, for some constant c and function f. Every problem solvable by a machine M working in polynomial time $p(n)$ can be solved in time $O(n^c)$ by such an algorithm, used for this M and for a constant k such that $\sqrt{(n + 2)^{k^k}/|M|} > p(n)$ for all $n \in \mathbb{N}$. This contradicts the hierarchy theorem, so X is not in **FPT**.