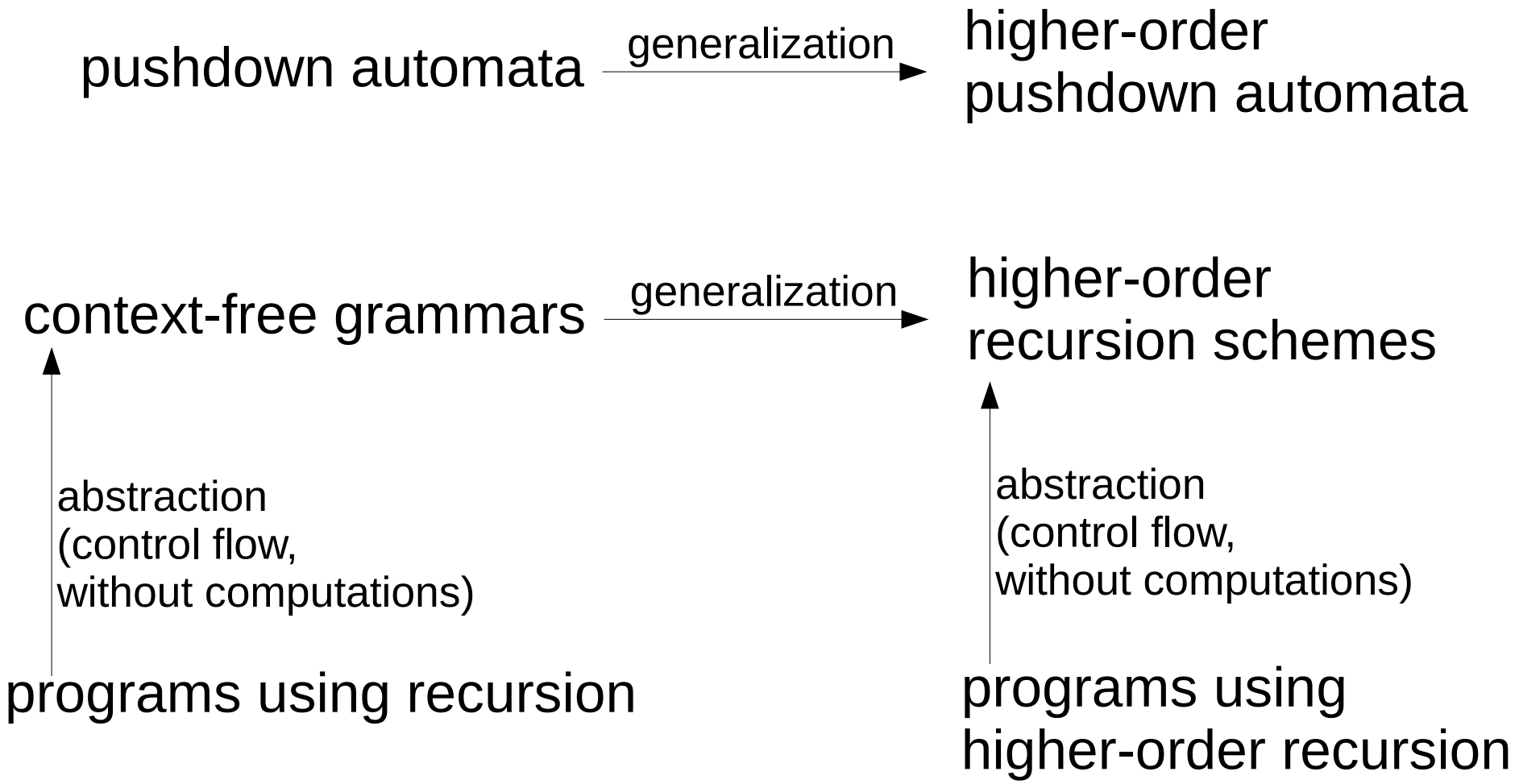
Automaty a rekurencja

Paweł Parys

Informacje ogólne

1)Materiały do wykładu będą na stronie:
http://www.mimuw.edu.pl/~parys/teaching/aar2016/
(slajdy / streszczenie / zeskanowane zapiski / linki)
2)Konsultacje: wtorek: 8:30-12:00, środa 8:30-12:00
(najlepiej dać znać na email: parys@mimuw.edu.pl)
3)Zachęcam do zadawania pytań, gdy coś nie jest jasne.
4)Ćwiczenia zaczynamy o 14:00.
5)Zaliczanie:
– zadania domowe (do 2 pkt)
– notatki do wykładu (do 2 pkt, po 1 pkt za wykład)
w Latexu, do 2 tyg. po wykładzie, proszę się zgłaszać
– egzamin z teorii (do 2 pkt)
ocena = 2+liczba_punktów

Higher-order recursion schemes



Higher-order pushdown automata - definition

A *1*-stack is an ordinary stack. A *2*-stack (resp. *(n+1)*-stack) is a stack of *1*-stacks (resp. *n*-stack).

[[*abc*][*de*][*fghij*]] – order 2
[[[*abc*][*de*][*xy*]][[*ab*][*fghij*]]] – order 3

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Operations on 2-stacks: (top of stack is on right)

$push_1 x$: $[[abc][de][fghij]] \rightarrow [[abc][de][fghijx]]$

pop_1 : $[[abc][de][fghij]] \rightarrow [[abc][de][fghi]]$

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An **order-n PDA** has an order-n stack, and has $push_i$ and pop_i for each $i \in \{1, ..., n\}$.

The next operation depends on the topmost stack symbol, the state, and the next letter on the input.

Higher-order pushdown automata - example

Language: $\{b^{2^k} : k \in \mathbb{N}\}$

- order 2
- 3 stack symbols: $\perp$, x, #

$(\_, q_1) \xrightarrow{\varepsilon} (q_1, \mathrm{push}_1(x))$

dowolny symbol (skrót)

q_1 $\boxed{\perp}$

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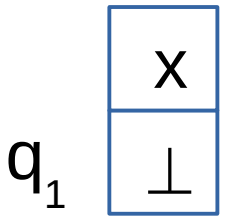

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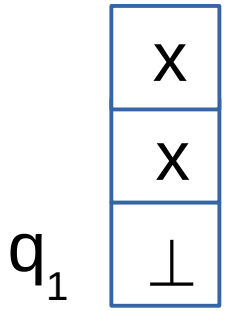

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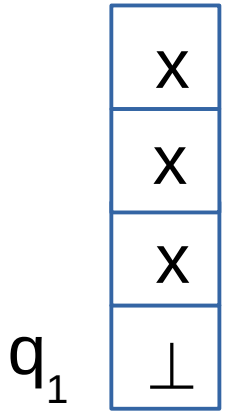

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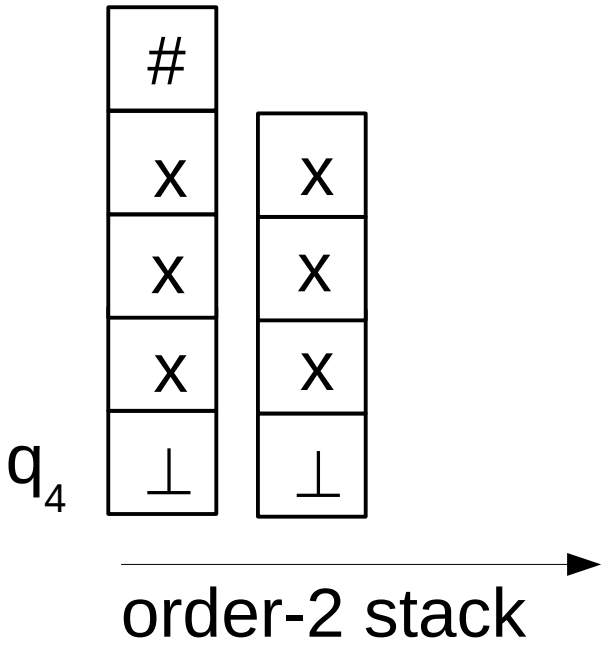

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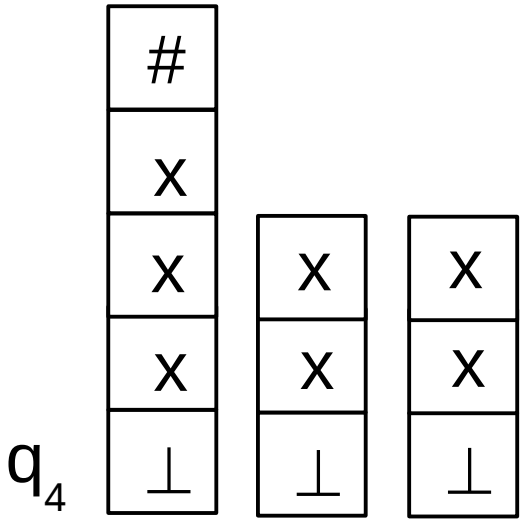

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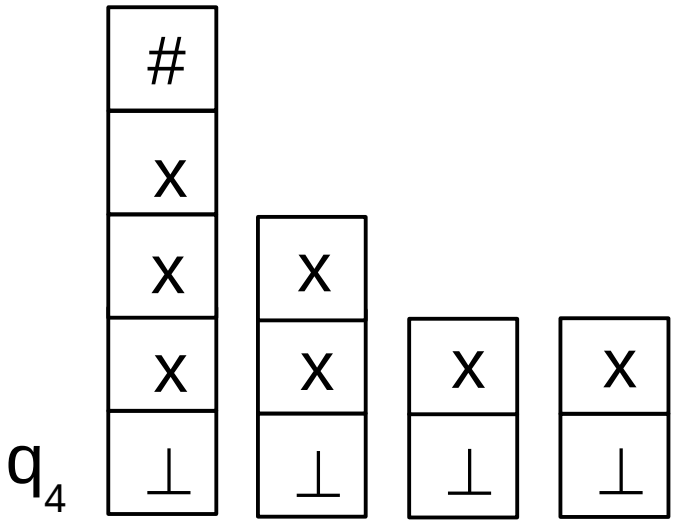

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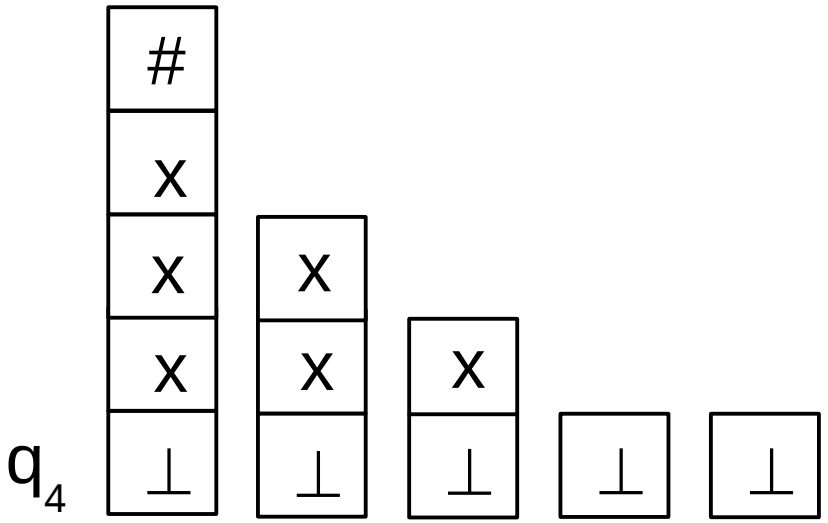

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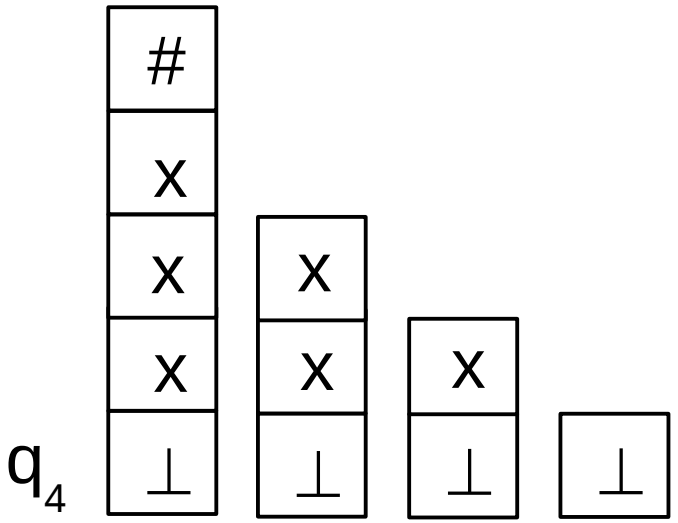

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Input: b

Higher-order pushdown automata - example

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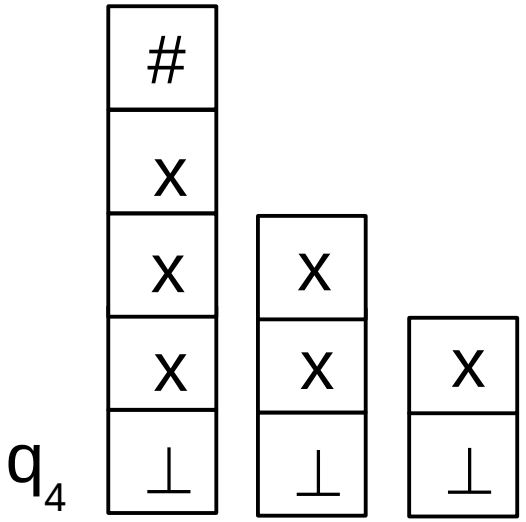

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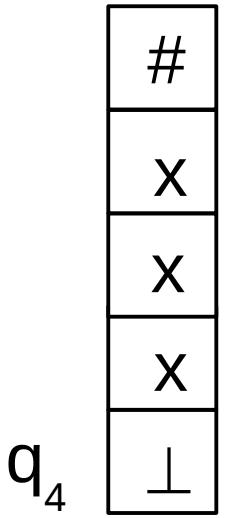

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$(\perp,q_4) \xrightarrow{b} (q_4,pop_2)$

$(\#,q_4) \xrightarrow{\varepsilon} (q_{acc},id)$

Input: b b b b b b b b

1) Dlaczego to dobrze działa?
2) Ten język nie jest bezkontekstowy.

Higher-order pushdown automata

“Traditional” view:

- a nondeterministic HOPDA recognizing a language of words, as on previous slides

“Modern” view:

- a deterministic HOPDA generating a single tree (node-labeled, ranked, ordered, usually infinite)

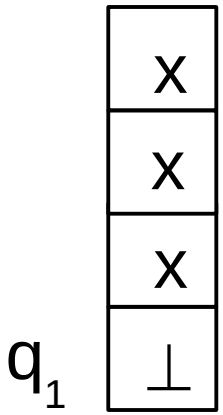
“Graph” view:

- configuration graph of a (deterministic) HOPDA; ε-contraction of this graph

Higher-order pushdown automata - example

nondeterminism – what to do next?

- order 2
- 3 stack symbols: ⊥, x, #



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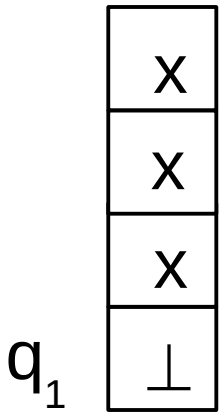
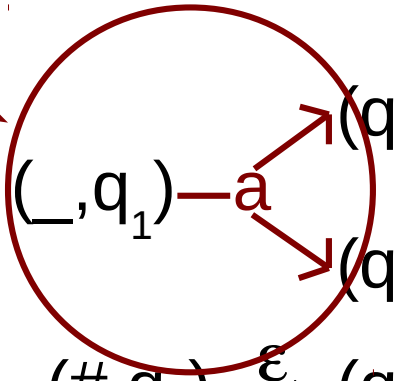


letter a of rank 2



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Higher-order pushdown automata - example

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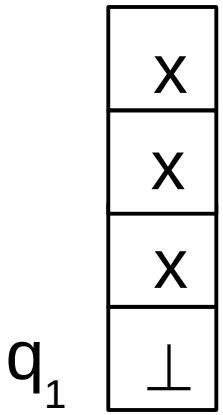

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$(\#,q_4) \xrightarrow{c}$

letter c of rank 0, instead of an accepting state

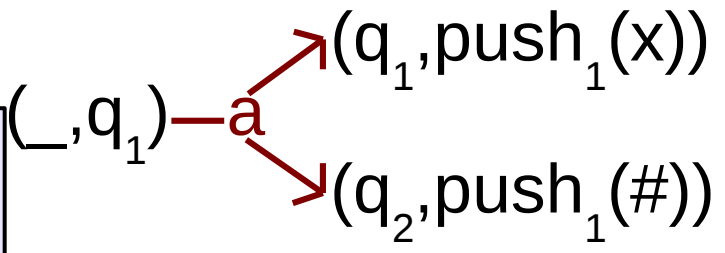
Higher-order pushdown automata - example

- order 2
- 3 stack symbols: ⊥, x, #

Tree-generating HOPDA - definition

From every pair of stack symbol & state there is either:

- one ε-transition
- one transition reading a letter of rank k, resulting in k (ordered) pairs of state & operation.



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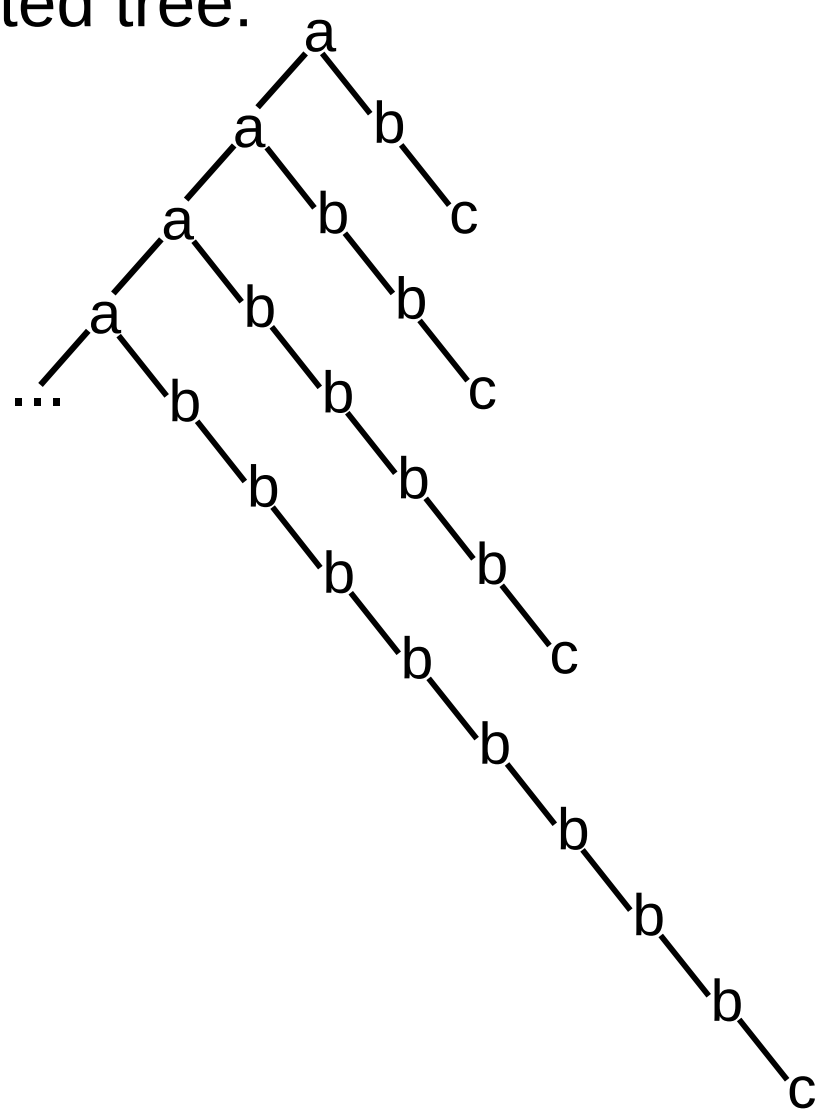
Generated tree:



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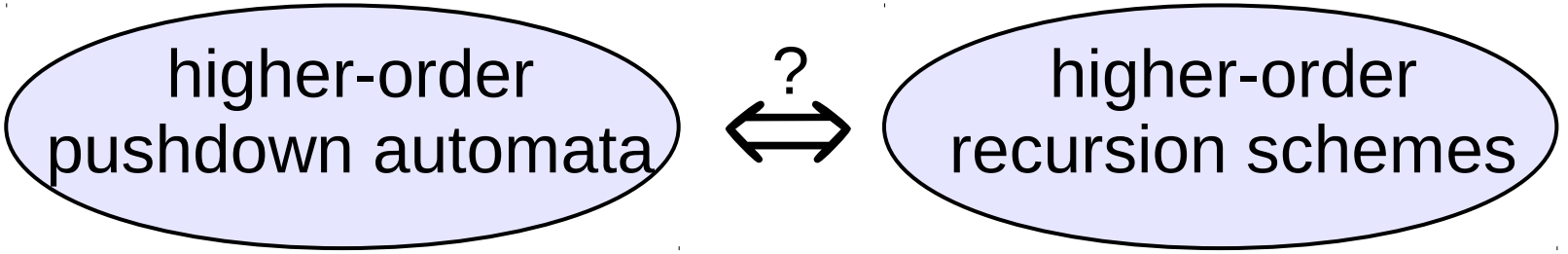
Higher-order recursion schemes - definition

Trochę jak gramatyka bezkontekstowa,
ale nieterminale mogą brać argumenty
i używać ich po prawej stronie produkcji.
Dokładna definicja później.

Inna definicja: otypowany rachunek λ + rekurencja

HOPDA vs HORS

Are these two formalisms equivalent?



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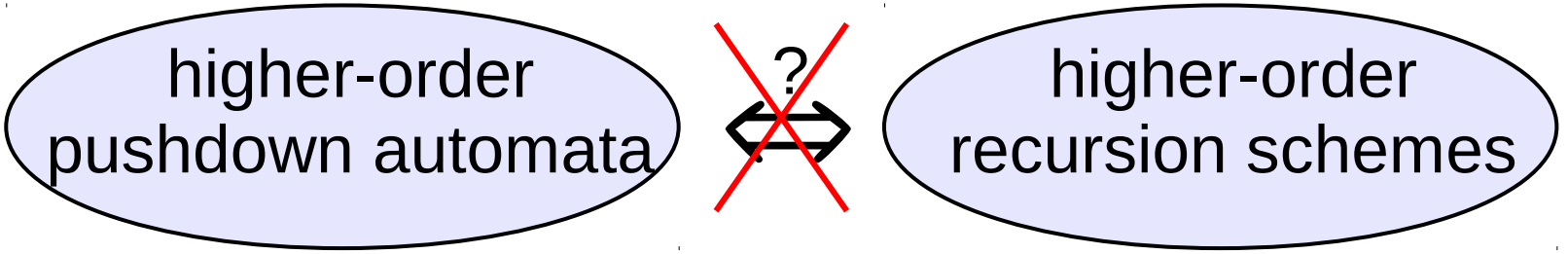

Not exactly!

HOPDA vs HORS

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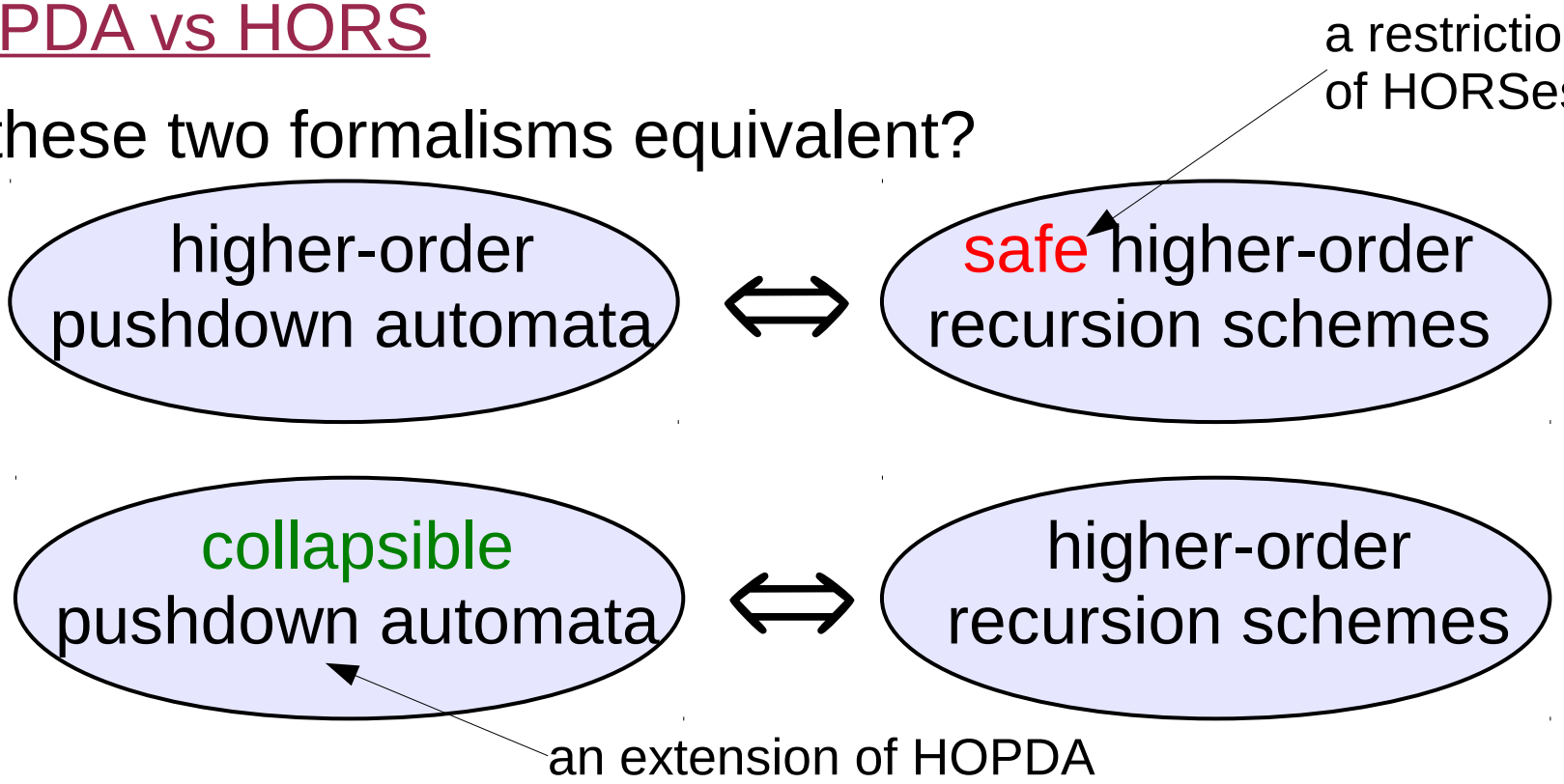

Theorem

For every n, HOPDA of order n and safe HORSes of order n generate the same trees (recognize the same word languages);

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HOPDA vs HORS

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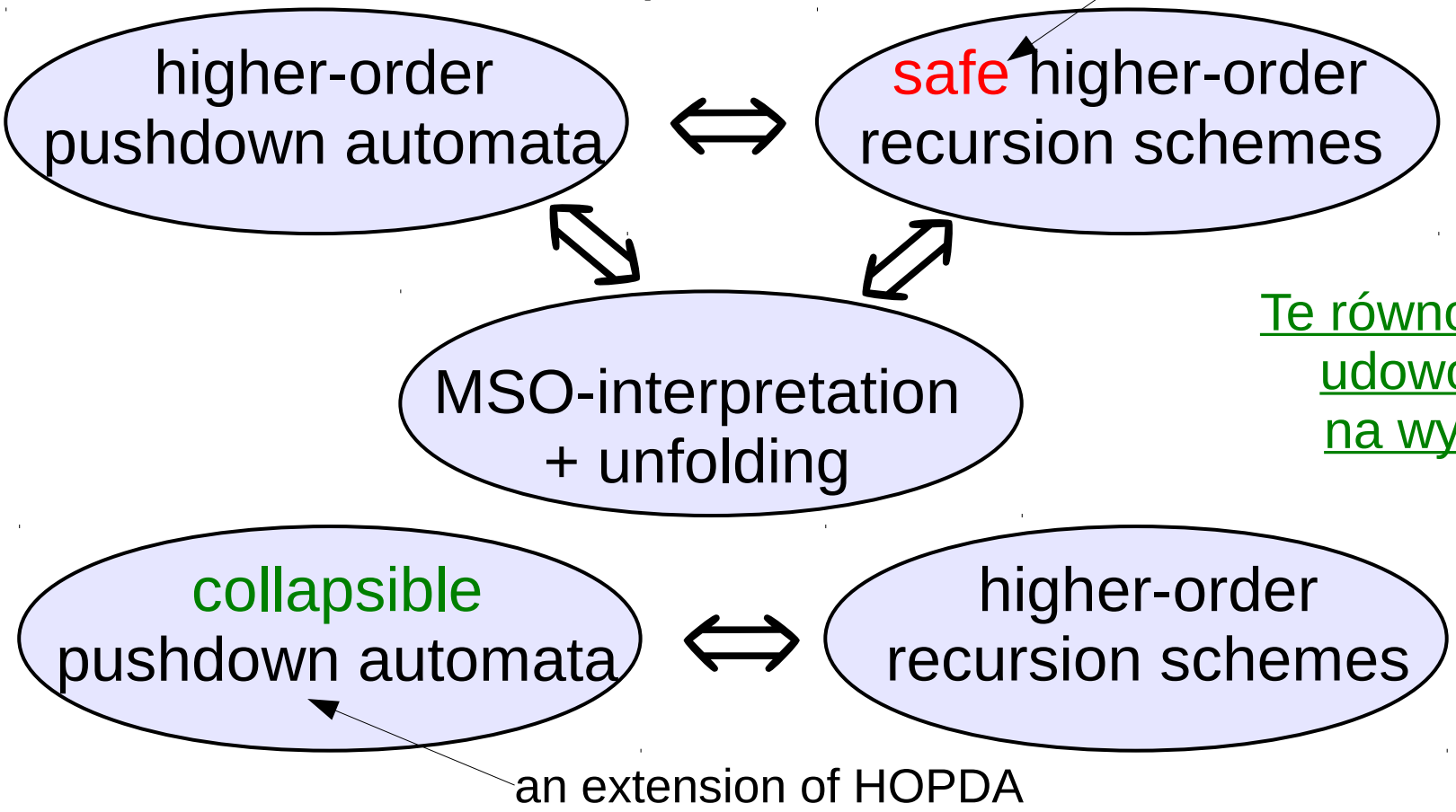

Te równoważności udowodnimy na wykładzie

Theorem

For every n, HOPDA of order n and safe HORSes of order n generate the same trees (recognize the same word languages).

These are trees from the Caucal hierarchy, defined by iterating MSO interpretations and unfolding of graphs into trees.

Expressivity questions

Tree(n)= trees generated by HORSes (CPDA) of order n
SafeTree(n) = trees generated by safe HORSes (HOPDA) of order n

$$\begin{array}{ccccccccc} \text{SafeTree}(0) & \subseteq & \text{SafeTree}(1) & \subseteq & \text{SafeTree}(2) & \subseteq & \text{SafeTree}(3) & \subseteq & \ldots \\ \cap & & \cap & & \cap & & \cap & & \\ \text{Tree}(0) & \subseteq & \text{Tree}(1) & \subseteq & \text{Tree}(2) & \subseteq & \text{Tree}(3) & \subseteq & \ldots \end{array}$$

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\| & & & & & & & & \\
\text{regular} & & & & & & & & \\
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= & & = & & & & & & \\
\text{regular trees} & & \text{context-free trees} & & & & & &
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= & & = & & \cap & & \cap & & \\
\text{Lang}(0) & \subseteq & \text{Lang}(1) & \subseteq & \text{Lang}(2) & \subseteq & \text{Lang}(3) & \subseteq & \ldots \\
= & & = & & & & & & \\
\text{regular languages} & & \text{context-free languages} & & & & & &
\end{array}$$

Expressivity questions

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\| & & \| & & \cap\!\| & & \cap\!\| & & \\
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Are these hierarchies strict?

Expressivity questions

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\end{array}
$$

Are these hierarchies strict?

Theorem

For every n, $\mathrm{SafeLang}(n) \neq \mathrm{SafeLang}(n+1)$ & $\mathrm{Lang}(n) \neq \mathrm{Lang}(n+1)$, and thus also $\mathrm{SafeTree}(n) \neq \mathrm{SafeTree}(n+1)$ & $\mathrm{Tree}(n) \neq \mathrm{Tree}(n+1)$.

Separating language: correct sequences of operations of order-$(n+1)$ HOPDA (including the topmost stack symbol after every step).

Będzie na wykładzie.

Expressivity questions

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Are these hierarchies strict?

Another separator:

T_n = tree with branches $a^k b^{exp_n(k)} c$, where $exp_n(k) = 2^{2^{\cdots^{2^k}}}$ (a tower of n twos)

We have SafeTree($n+1$) $\ni T_n \notin$ Tree(n).

Expressivity questions

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\begin{array}{ccccccccc}
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$$

Is safety really a restriction?

Expressivity questions

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\mathrm{SafeTree}(0) & \subsetneq & \mathrm{SafeTree}(1) & \subsetneq & \mathrm{SafeTree}(2) & \subsetneq & \mathrm{SafeTree}(3) & \subsetneq & \dots \\
\| & & \| & & \not\| & & \not\| & & \\
\mathrm{Tree}(0) & \subsetneq & \mathrm{Tree}(1) & \subsetneq & \mathrm{Tree}(2) & \subsetneq & \mathrm{Tree}(3) & \subsetneq & \dots \\
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\end{array}$$

Is safety really a restriction?

For trees – yes.
Example: Urzyczyn's tree U

$\mathrm{Tree}(2) \ni U \notin \mathrm{SafeTree}(n)$ for every n

For word languages – open problem (e.g. $\mathrm{SafeLang}(3) \stackrel{?}{\neq} \mathrm{Lang}(3)$)

Nie będzie na wykładzie.

Expressivity questions

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Are these languages context-sensitive?

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\text{SafeLang}(0) & \subsetneq & \text{SafeLang}(1) & \subsetneq & \text{SafeLang}(2) & \subsetneq & \text{SafeLang}(3) & \subsetneq & \cdots \subseteq \text{CSens} \\
\| & & \| & & \| & & \cap & & \\
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 & & & & & & \cap & & \\
 & & & & & & \text{CSens} & &
\end{array}$$

Are these languages context-sensitive?

CSens = context-sensitive languages (type-1 in the Chomsky hierarchy)

SafeLang(n) $\subseteq$ CSens, for every n

Lang(3) $\subseteq$ CSens

Lang(n) $\overset{?}{\subseteq}$ CSens for $n \geq 4$ – open problem

Nie będzie na wykładzie.

Algorithmic questions

Problem: MSO model-checking
Input: MSO formula ϕ, HORS S
Output: does ϕ hold in the tree generated by S?

lub równoważnie:

Problem: MSO model-checking
Input: nondeterministic (alternating) parity automaton A,
HORS S
Output: does A accept the tree generated by S?

Można też rozważać słabsze logiki / modele automatów.

Algorithmic questions

Problem: MSO model-checking
Input: MSO formula ϕ, HORS S
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Theorem: MSO model-checking is decidable.

Coś o tym będzie na wykładzie.

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- polynomial when n, ϕ, and maximal arity of a nonterminal fixed

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- (n-1)-EXPTIME-complete for reachability properties (is "a" present in the tree)
- polynomial when n, ϕ, and maximal arity of a nonterminal fixed
- despite high complexity, solvable in practice

Coś o tym będzie na wykładzie.

Algorithmic questions

Trochę silniejsze wyniki:

MSO reflection

Input: MSO formula $\phi(x)$, HORS S (x to wierzchołek)

Output: HORS S' generating the same tree as S, where the nodes x in which $\phi(x)$ holds are marked.

MSO selection

Input: MSO formula $\phi(X)$, HORS S (X to zbiór wierzchołków)

Output: HORS S' generating the same tree as S with some nodes marked so that $\phi(X)$ holds for the set X of marked nodes.

Model-cheking (of programs with higher-order recursion)

Model-checking (weryfikacja modelowa) = automatyczne sprawdzanie, czy program spełnia zadaną specyfikację

NIE polega na uruchamianiu / testowaniu.

Polega na sprawdzeniu wszystkich możliwych przebiegów programu i "udowodnieniu", że specyfikacja jest spełniona – na podstawie kodu źródłowego.

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Teoretycznie problem nierozstrzygalny (problem stopu dla maszyny Turinga), w praktyce jakoś działa.

Stosowane z powodzeniem przy tworzeniu:
- urządzeń sprzętowych
- sterowników (Static Driver Verifier w Windowsie)

Sprawdzane własności:
- brak zakleszczeń, procedura zawsze się kończy
- funkcje systemowe są prawidłowo wywoływane
- asercje są zawsze prawdziwe
- ...

Model-cheking of programs with higher-order recursion

Model-checking dla programów z rekurencją wyższego rzędu.

Obecnie niezbyt rozwinięty,
aczkolwiek istnieją narzędzia umożliwiające zweryfikowanie prostych procedur

Bardzo ogólna zasada działania:

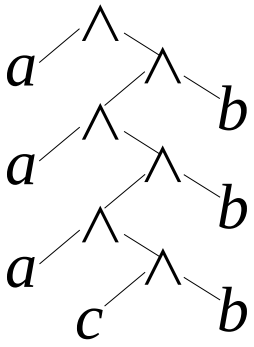
Tłumaczymy program na HORS, pomijając różne szczegóły
[główna trudność – jak to dobrze zrobić]

program błędny ⇒ błąd widoczny w HORS
błąd w HORS ⇒ błąd w programie, lub niedokładność tłumaczenia

Potem sprawdzamy, czy HORS spełnia zadaną własność (w pełni rozstrzygalne)

Coś o tym będzie na wykładzie.

Between trees and words



aaacbbb

regularny język drzew → język w liściach - bezkontekstowy

istnieje regularny język drzew taki, że w liściach jest L ← język L bezkontekstowy

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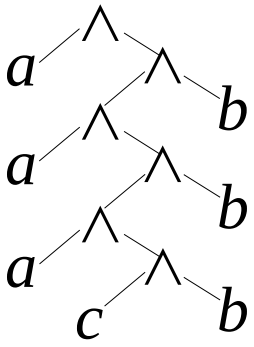

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język drzew rzędu n → język w liściach – rzędu $n+1$

istnieje język drzew rzędu n taki, że w liściach jest L ← język L rzędu $n+1$

Będzie na wykładzie.

Collapsible pushdown automata

- Every stack symbol has an identifier.
- $push_1 x$ pushes symbol x with a fresh identifier.
- $push_k$ for $k \geq 2$ copy symbols with their identifiers.

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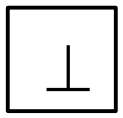

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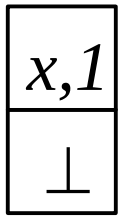

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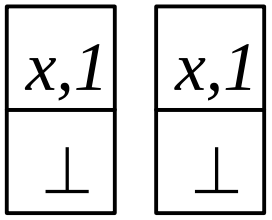
$push_1 x$

$push_2$



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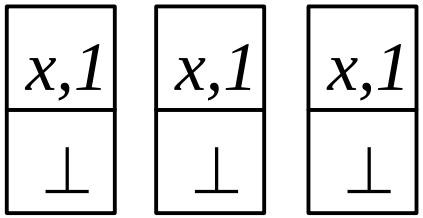
$$push_1 x$$
$$push_2$$
$$push_2$$



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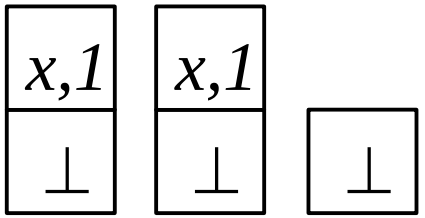
$push_1 x$
$push_2$
$push_2$
pop_1



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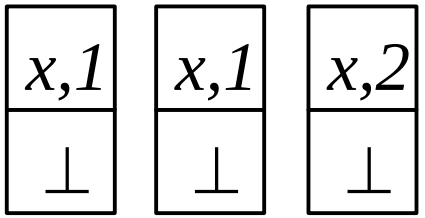

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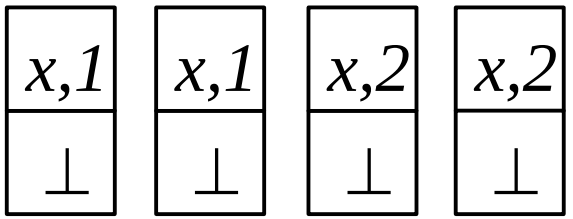

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pop_1
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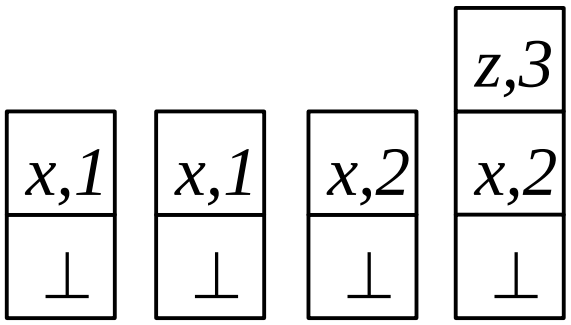

$push_1 x$
$push_2$
$push_2$
pop_1
$push_1 x$
$push_2$
$push_1 z$

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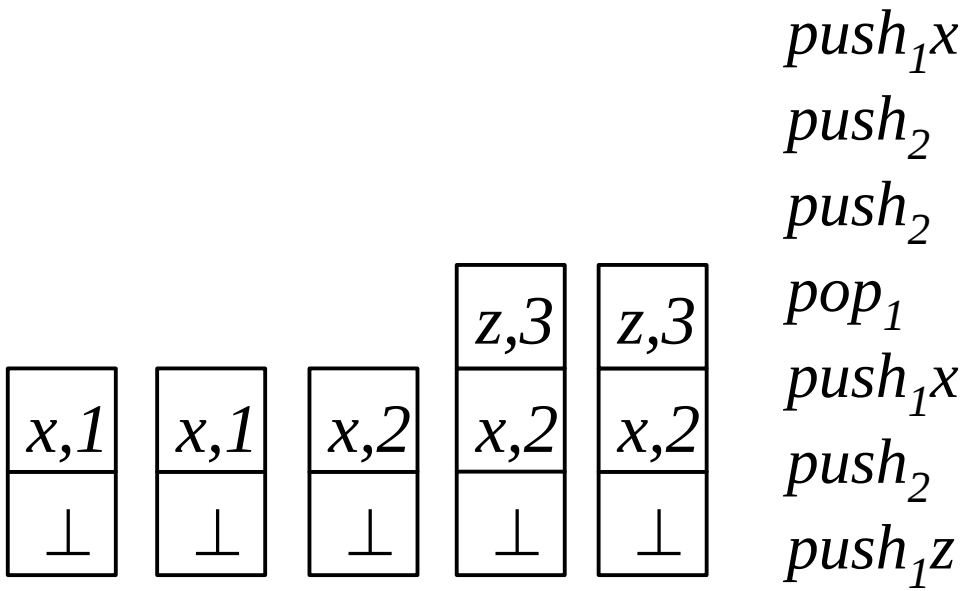

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pop_1
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$push_2$
$push_1 z$
$push_2$

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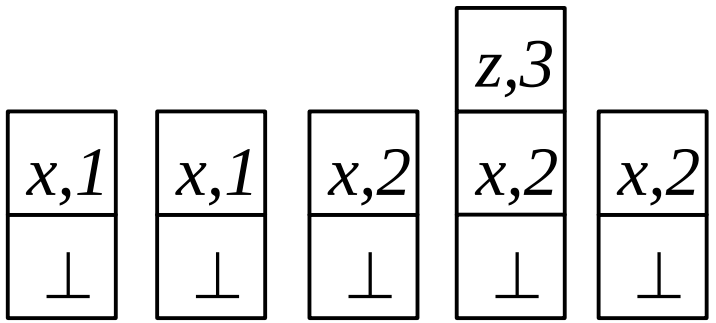

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pop_1
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pop_1

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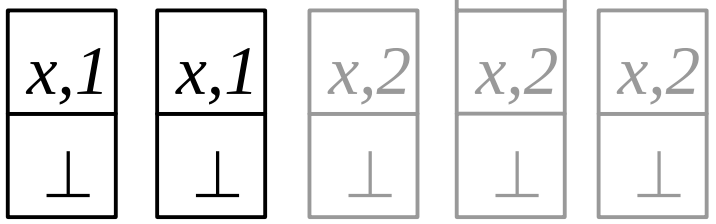
- New operation $collapse_k$:
  remove from the topmost order-k stack all order-(k-1) stacks containing a copy of the topmost stack symbol.



$push_1 x$
$push_2$
$push_2$
pop_1
$push_1 x$
$push_2$
$push_1 z$
$push_2$
pop_1
$collapse_2$