Computational Complexity
Exam
5.02.2020

Problem 1. (8 pt)

Prove that the following problem is coNP-complete (wrt. polynomial-time reductions):
Input: Boolean circuits C, D with the same number of input gates;
Question: do the two circuits compute the same function?

Problem 2. (8 pt)

Prove that the following problem belongs to the class L:
Input: undirected graph G;
Question: is there a cycle in G?

Remark. You are allowed to use Reingold's theorem: ST-CONNECTIVITY (given an undirected graph H and its two nodes s, t check whether s and t are connected) is in L.

Problem 3. (8 pt)

We say that a language K is *length-decreasing self-reducible* if there is a polynomial-time Turing machine M with oracle K, such that

$$w \in K \iff M^K(w) = 1,$$

and the computation of $M^K(w)$ only queries K on strings of length strictly less than $|w|$. Let K be a language over an unary alphabet (i.e., $K \subseteq \{1\}^*$). Prove that K is in P if and only if K is length-decreasing self-reducible.

Remark. A Turing machine with oracle for $X \subseteq \{0,1\}^*$ is a Turing machine equipped with an additional query tape. After writing some word to the query tape, it can enter a special query state, and then it instantly receives a (binary) answer whether the word belongs to X. The machine continues its computation, maybe asking further questions to the oracle.