Computational Complexity
Exam
5.02.2019

Problem 1. (0.6 pt) Consider the following problem:
INPUT: a directed graph G such that every vertex has at most one successor (i.e., in every vertex there starts at most one edge), and vertices s, t.
QUESTION: is there a path from s to t in G?
Prove either that this problem is in L, or that it is NL-complete (under log-space reductions).

Problem 2. (0.6 pt) Prove that the following problem is NP-complete:
INPUT: a list of pairs of words $(v_1, w_1), (v_2, w_2), \ldots, (v_k, w_k)$, and a word y of length m.
QUESTION: is there a list of indices $i_1, \ldots, i_m$ such that $v_{i_1} v_{i_2} \ldots v_{i_m} = w_{i_1} w_{i_2} \ldots w_{i_m}$?
Note. The word y is irrelevant, only its length is important. By $v_{i_1} v_{i_2} \ldots v_{i_m}$ we mean the concatenation of the words $v_{i_1}, v_{i_2}, \ldots, v_{i_m}$.
Example. For $(v_1, w_1) = (ba, a)$, and $(v_2, w_2) = (a, ab)$, and $m = 2$ the answer is YES: we can take $i_1 = 2, i_2 = 1$, and the concatenated word is aba. For $(v_1, w_1) = (ba, a)$, and $(v_2, w_2) = (a, ab)$, and $m = 1$ the answer is NO.

Problem 3. (0.6 pt) We say that $L \in$ RPSpace if there exists a probabilistic Turing machine M such that:

- M uses polynomial space;
- M stops in exponential time;
- $\forall w \in L \; \mathbb{P}_r[M(x,r) = YES] \geq \frac{1}{2}$;
- $\forall w \notin L \; \mathbb{P}_r[M(x,r) = YES] = 0$.

Prove or disprove: RPSpace = PSpace.