Nonlocal and abstract parabolic equations and their applications

Book of Abstracts

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Preface

The meeting is devoted to a rapidly growing area of nonlocal and abstract parabolic equations. There is a range of phenomena whose models employ such equations, maybe in an implicit way. The presentations are concentrated around the following specific topics.

- The motion of incompressible fluid, the nonlocal character of equations describing its motion is well-known. Namely, this is implied by the constraint $\operatorname{div} u = 0$.
- Equations governing the singular weighted mean curvature flow are another example.
- Models of chemotaxis, which play a significant role in the mathematical models in biology.
- Free boundary problems with the Gibbs-Thomson law, they are in fact mean curvature flows coupled to a diffusion field.
- Recent development of the reaction-diffusion system and their applications. In particular the link between them and the free boundary problems is provided by singular limits of RD systems is exposed.

We plan to publish a volume of the proceedings of the conference in the Banach Center Publications series. This however, depends upon a sufficient number of contributing authors. All participants interested in submitting an original paper (not longer than, say, 20 pages) are invited. However, we request a declaration of intention to be sent over the e-mail to:
Piotr Mucha `mucha@hydra.mimuw.edu.pl` or Piotr Rybka `rybka@hydra.mimuw.edu.pl`.
All papers will be refereed.

Contents

1 On a Diffuse Interface Model for Two-Phase Flows of Viscous, Incompressible Fluids with Matched Densities

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In this presentation we discuss a "diffuse interface model" for the flow of two viscous incompressible Newtonian fluids of the same density in a bounded domain. Such models were introduced to describe the flow when singularities in the interface, which separates the fluids, (droplet formation/coalescence) occur. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed in the model. Moreover, diffusion of both components is taken into account. This leads to a coupled Navier-Stokes/Cahn-Hilliard system, for which we prove existence of weak solutions in two and three space dimensions for a class of physical relevant and singular free energy densities. Moreover, we present some results on regularity and uniqueness of weak solutions. In particular, we obtain that unique "strong" solutions exist in two dimensions globally in time and in three dimensions locally in time. Finally, it is possible to show that any weak solution converges as $t \to \infty$ to a solution of the stationary system.

2 Singular Perturbations of Mean Curvature Flow

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We consider a regularization method for mean curvature flow of a submanifold of arbitrary codimension in the Euclidean space, through higher order equations. The main result shows that such regularized problems converge to the mean curvature flow for all times before the first singularity.

3 Self-similar solutions of a parabolic system of chemotaxis

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We consider solutions of parabolic system of chemotaxis

$$\frac{\partial u}{\partial t} = \nabla \cdot (\nabla u - u\nabla v), \quad x \in \mathbb{R}^2, t > 0, \tag{1}$$

$$\tau \frac{\partial v}{\partial t} = \Delta v + u, \quad x \in \mathbb{R}^2, t > 0, \tag{2}$$

with $\tau \geq 0$, generalizing the parabolic-elliptic Patlak-Keller-Segel system with $\tau = 0$.

Existence of self-similar solutions is studied for control parameter $M \equiv \int_{\mathbb{R}^2} u(x,t)dx$ in an interval $[0, M_\tau)$. The role of self-similar solutions in a description of the long time behavior of general solutions is also investigated.

4 Quasilinear non-uniformly parabolic-elliptic system of chemotaxis, critical exponent for existence of global-in-time solutions

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In my talk I would like to study a system

$$u_t = \nabla \cdot (\alpha(u)\nabla u - u\nabla v)$$
$$0 = \Delta v - M + u \int_U u = 0$$

under Neumann boundary conditions in a bounded domain, $\alpha(u) \to 0$ when $u \to \infty$. We shall show critical exponent for the decay(growth) of α below which there is blow-up in finite time and above which there are global-in-time uniformly bounded solutions. The talk is based on the common paper with M.Winkler.

5 Existence, uniqueness and approximation of a doubly degenerate nonlinear parabolic system

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We consider the following nonlinear parabolic system

$$\begin{aligned} \frac{\partial u}{\partial t} - c\,\Delta u &= -f(u)\,v \quad \text{in } \Omega\times(0,T), \\ \frac{\partial v}{\partial t} - \nabla\cdot(b(u)\,\nabla[\psi(v)]\,) &= \theta f(u)\,v \quad \text{in } \ \Omega\times(0,T) \end{aligned}$$

subject to no flux boundary conditions, and non-negative initial data u^0 and v^0 on u and v. Here we assume that $c > 0$, $\theta \geq 0$ and that $f \in C^{0,1}_{\mathrm{loc}}([0,\infty))$ is increasing with $f(0) = 0$. The system is possibly doubly-degenerate in that $b \in C^{1,1}_{\mathrm{loc}}([0,\infty))$ is only non-negative, and $\psi \in C^1([0,\infty)) \cap C^2((0,\infty))$ is convex, strictly increasing with $\psi(0) = 0$ and possibly $\psi'(0) = 0$. The above models the spatiotemporal evolution of a bacterium on a thin film of nutrient, where u is the nutrient concentration and v is the bacterial cell density. Under some further mild technical assumptions on b and ψ, we prove the existence and uniqueness of a weak solution to the above system. Moreover, we prove error bounds for a fully practical finite element approximation of this system. This is joint work with John Barrett (Imperial College London).

6 On the effective boundary conditions on domains with rough boundaries

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We shall discuss the problem of effective or asymptotic boundary conditions to be satisfied by solutions to a system of partial differential equations posed on domains, the boundary of which varies with a certain small parameter ϵ. The limit problem including the resulting boundary conditions is identified for $\epsilon \to 0$. Several applications, in particular the case of the so-called Navier (partial) slip boundary conditions, are discussed.

7 Evolution Equations with Almost Periodic Initial Data

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It is rather clear that the solution is periodic if it is initially periodic provided that the evolution equation under study is well-posed and translation invariant. It is less obvious to see that almost periodicity is preserved under slightly stronger assumptions. We are interested in evolution of the frequency set when initial data is almost periodic. We consider such a problem for the Navier-Stokes equations and other evolution equations. A typical results is that frequency set is contained in the same vector space over the field of rational numbers as that of initial data.

8 The Leray-Schauder fixed point index as a tool in proving stability of steady states

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We consider an abstract parabolic equation of reaction-diffusion type posed on a domain which geometry is governed by a small parameter δ. We are interested in establishing the main properties of the dynamics as a function of this parameter, and in this aim we study the attractor's properties, in particular stability of steady states. We use a perturbation method, and prove that for small δ the stability properties do not differ from the singular case $\delta = 0$. In the hyperbolic case, we are able to count the steady states.

9 Evolution of polyhedral crystal

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We consider a system modeling evolution of a single crystal grown from vapor. We account for vapor diffusion equation and Gibbs-Thomson relation on the crystal surface. We also assume that the velocity of the growing crystal is determined by the normal

derivative of concentration of vapor at the surface, it the so-called Stefan condition. We show local in time existence of solutions assuming that the initial crystal has admissible shape.

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10 On the singular limit of some reaction-diffusion systems

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We consider an Allen-Cahn type equation of the form $u_t = \Delta u + \varepsilon^{-2} f^\varepsilon(x,t,u)$, where ε is a small parameter and f^ε a bistable nonlinearity associated with a double-well potential whose well-depths can be slightly unbalanced. Given a rather general initial data u_0 that is independent of ε, we perform a rigorous analysis of both the generation and the motion of interfaces. More precisely we show that the solution develops a steep transition layer within the time scale of order $\varepsilon^2|\ln\varepsilon|$, and that the layer obeys the law of motion that coincides with the formal asymptotic limit within an error margin of order ε. This is an optimal estimate that has not been known before for solutions with general initial data, even in the case where $g^\varepsilon \equiv 0$.

Next we consider systems of reaction-diffusion equations of the form

$$\begin{cases} u_t = \Delta u + \varepsilon^{-2} f^\varepsilon(u,v) \\ v_t = D\Delta v + h(u,v), \end{cases}$$

which include the FitzHugh-Nagumo system as a special case. Given a rather general initial data (u_0, v_0), we show that the component u develops a steep transition layer and that all the above-mentioned results remain true for the u-component of these systems.

This is joint work with Matthieu Alfaro and Hiroshi Matano.

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11 Periodic solutions of the Navier-Stokes equations around a rotating obstacle

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We study the motion of a viscous incompressible fluid filling the whole 3-dimensional space exterior to a rigid body, that is rotating with constant angular velocity ω, under the action of external force. By using a frame attached to the body, the equations are reduced to

$$\partial_t u + u \cdot \nabla u = \Delta u + (\omega \times x) \cdot \nabla u - \omega \times u - \nabla p + f, \qquad \text{div } u = 0$$

in a fixed exterior domain D with boundary condition $u|_{\partial D} = \omega \times x$. Given $f \in BUC(\mathbb{R}; \dot{W}^{-1}_{3/2,\infty}(D))$, we consider this problem in $D \times \mathbb{R}$ and prove that there exists a unique solution $u \in BUC(\mathbb{R}; L_{3,\infty}(D))$ when f and $|\omega|$ are sufficiently small. If, in particular, the external force for the original problem is independent of t, then f is periodic with period $2\pi/|\omega|$. In this situation, as a corollary of our result, we obtain a periodic solution with the same period.

12 Local existence and uniqueness for one-dimensional tumor invasion model

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In this talk, we consider the simplified and modified one-dimensional tumor invasion model which was originally proposed in Chaplain and Anderson. We show the local existence and uniqueness of solutions to our tumor invation system. By introducing a new function, we succeed in showing that our system can be rewritten into the (nonlinear) second-order PDE. Roughly speaking, our system is not a system no longer. And this fact is quite essential to show the local existence of solutions to our tumor invasion model.

13 Applications of the abstract theory on parabolic variational inequalities

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Recently, parabolic variational inequalities have been studied and an existence result was obtained in an abstract form. Here we give its application to obstacle problem in which the constraint functions depend on the unknown in a non local way.

For example, let Ω be a bounded domain in $\mathbb{R}^N$, $1 \le N < \infty$, with smooth boundary $\Gamma := \partial\Omega$. Furthermore, let $k_c(\cdot)$ be a Lipschitz continuous function on $\mathbb{R}$ with bounded

Lipschitz continuous derivative $k_c'(\cdot)$ on $\mathbb{R}$. Let $\delta_0 > 0$, $f \in L^2(-\delta_0, T; L^2(\Omega))$, and $u_0 \in W^{1,2}(-\delta_0, 0; L^2(\Omega)) \cap L^\infty(-\delta_0, 0; L^2(\Omega))$.

Then our problem is to find such $u \in W^{1,2}(-\delta_0, T; L^2(\Omega))$ that

$$u \geq k_c \left(\int_0^T \int_\Omega \rho(s, x, u(x,s)) dx ds \right) \text{ a.e. on } (0,T),$$

$$\int_0^T \int_\Omega (u' + \Delta u - f)(u - w) dx dt \leq 0,$$

$$\forall w \in W^{1,2}(-\delta_0, T; L^2(\Omega)),\ w \geq k_c \left(\int_0^T \int_\Omega \rho(s, x, u(x,s)) dx ds \right) \text{ a.e. on } (0,T).$$

$$u = u_0 \text{ on } [-\delta_0, 0]$$

where ρ be a bounded smooth function $\bar{\Omega} \times [0,t] \times \mathbb{R}$.

14 On convergence of solutions of fractal Burgers equation toward rarefaction waves

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In the talk, I will present my recent work, joint with Changxing Miao and Xiaojing Xu from Beijing, on the large time behavior of solutions of the Cauchy problem for the one dimensional fractal Burgers equation $u_t + (-\partial_x^2)^{\alpha/2} u + u u_x = 0$ with $\alpha \in (1,2)$. We showed that if the nondecreasing initial datum approaches the constant states $u_\pm$ ($u_- < u_+$) as $x \to \pm\infty$, respectively, then the corresponding solution converges toward the rarefaction wave, *i.e.* the unique entropy solution of the Riemann problem for the nonviscous Burgers equation.

15 Recent Development Parabolic Quasi-Variational Inequalities

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In this talk we discuss a class of parabolic quasi-variational inequalities of the form

$$u'(t) + \partial\varphi^t(u; u(t)) \ni f(t), \quad 0 < t < T, \quad \text{in } X^*,$$
$$u(t) = u_0(t) \quad \text{for } -\delta_0 \le t \le 0, \quad \text{in} X$$

under the following setting: X is a real reflexive Banach space compactly embedded in a real Hilbert space H, so $X \subset H \subset X^*$, where X^* is the dual of X and δ_0 and T are fixed positive numbers; $\varphi^s(v; z)$, $-\delta_0 \le s \le t$, is a proper l.s.c. convex function in z on X for v belonging to a suitable class of functions from $[-\delta_0, t]$ into X in which $\varphi^s(v; \cdot)$ depends continuously on v in a non-local way; u_0 is a prescribed initial function given on $[-\delta_0, 0]$.

As a simple example shows, the existence question for the above type of quasi-variational inequalities are quite delicate. We shall give sufficient conditions for the existence of a local in time solution of our problem. Moreover, in order to illustrate our conditions we apply our abstract result to a concrete obstacle problem having the obstacle depending on the unknown function.

16 A quasisteady Stefan problem with surface tension and kinetic undercooling

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The Stefan problem is a model of phase transition in solid-liquid systems. This accounts for heat diffusion in each phase and exchange of latent heat at the solid-liquid interface. Its strong formulation is a free boundary problem, since the interface evolution is a priori unknown. We study a one-phase quasisteady Stefan problem with surface tension and kinetic undercooling. We show that classical solutions exist globally and tend to spheres exponentially fast, provided that they are close to a sphere initially. Our analysis is based on center manifold theory and on maximal regularity.
AMS Subject Classification: 35R35, 35B35.
Keywords: Stefan problem, free boundary problem, center manifold.

17 Stability analysis of phase boundary motion by surface diffusion with triple junction

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The motion of the phase boundary by the geometrical evolution law in a bounded domain $\Omega \subset \mathbb{R}^2$ is studied in this talk. We consider the surface diffusion flow equation which is denoted by $V = -m\,\gamma\,\kappa_{ss}$. Here V is the normal velocity of the interface, κ is the curvature of the interface, s is the arc-length parameter along the interface, $m > 0$ is the mobility constant, and $\gamma > 0$ is the constant concerning the surface energy of the interface. The basic properties of this geometrical evolution law are H^{-1}-gradient flow structure for the total length of the interface and the area-preserving property for the domain surrounded by the interfaces. Also, it is known that this geometrical evolution law is derived as the sharp interface limit of a Cahn-Hilliard equation with degenerate mobility.

Our goal in this talk is to derive criteria of the linearized stability of stationary solutions for the three-phase boundary motion by surface diffusion with triple junction. It will be shown by investigating the sign of eigenvalues for the eigenvalue problem corresponding to the linearized problem around the stationary solutions.

18 Navier-Stokes equations in 2D exterior domains

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I would like to present some results on existence of solutions to the Navier-Stokes equations considered in exterior domain $\Omega = \mathbb{R}^2 \setminus B$, where $B \subset \mathbb{R}^2$ is a simply-connected bounded domain. On the boundary $\partial\Omega$ we impose slip boundary conditions, i.e.

$$\vec{n} \cdot \mathbb{T}(v,p) \cdot \vec{\tau} + f(v \cdot \vec{\tau}) = 0, \tag{3}$$

where $\mathbb{T}(v,p)$ is the Cauchy stress tensor, f is a positive friction coefficient, v is the velocity of the fluid, p - corresponding pressure and $\vec{n}, \vec{\tau}$ are respectively the normal and tangential vector to boundary $\partial\Omega$ and we also assume that the velocity tends to data at infinity.

The slip boundary conditions do not give us full information about velocity on the boundary, like it is the case in Dirichlet boundary conditions. However our problem admits an existence of weak solutions without restriction on smallness of the data. A different approach is needed. We reformulate our problem in terms of the vorticity $\alpha = \text{rot } v$, where the slip boundary conditions become the Dirichlet relations on α. Then we are able to show the existence of solutions to our problem.

Moreover, I would like to present a problem one encounters when trying to recover the whole information about the velocity from α. This problem does not occur in higher dimensions.

19 Poisson equation in weighted space

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We consider Poisson equation in weighted space $H^2_\mu(R^3)$, where the weight is a power of the distance from a distinguish axis. We demonstrate for which real exponent μ there are a priori estimates for the solutions of Poisson equation. We also calculate the dimension of kernel an co-kernel of the Laplace operator for non integer value of μ.

20 Non-diffusive large time behaviour for a degenerate parabolic equation with gradient absorption

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We investigate the large time behaviour of non-negative weak solutions u to the Cauchy problem

$$\partial_t u - \text{ div } \left(|\nabla u|^{p-2} \nabla u\right) + |\nabla u|^q = 0 \ , \quad (t,x) \in (0,\infty) \times \mathbb{R}^N \, , \tag{4}$$

$$u(0) = u_0 \, , \quad x \in \mathbb{R}^N \, , \tag{5}$$

the initial condition being bounded, continuous, and compactly supported, and the parameters p and q ranging in $(2,\infty)$ and $(1,p-1]$, respectively. For $p \in (1,p-1)$, the positivity set $\mathcal{P}(t) := \{x \in \mathbb{R}^N \, , \ u(t,x) > 0\}$ of $u(t)$ increases to a bounded subset $\mathcal{P}_\infty$ of $\mathbb{R}^N$. The solution to (4), (5) then converges towards a self-similar solution to the Hamilton-Jacobi equation $\partial_t z + |\nabla z|^q = 0$ which is uniquely determined by $\mathcal{P}_\infty$. For $q = p-1$, the positivity set expands and eventually converges to $\mathbb{R}^N$. The large time behaviour is self-similar and still given by a self-similar solution to the Hamilton-Jacobi equation $\partial_t z + |\nabla z|^q = 0$, but with a different time scale (joint works with Juan Luis Vázquez, Universidad Autónoma de Madrid, Spain)

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21 Mechanisms of pattern formation in degenerated reaction-diffusion systems

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Dynamics of multicellular systems is controled by the dynamics of intracellular signalling pathways and cell-to-cell communication. The description of biological processes often leads to nonlinear dynamical models with multiple steady states, which differ from the usual reaction-diffusion systems. Also, processes containing switching between different pathways or states lead to new types of mathematical models, which consist of nonlinear partial differential equations of diffusion, transport and reactions, coupled with dynamical systems controlling the transitions. In this talk I consider a systems of reaction-diffusion equations coupled with ordinary differential equations and approach the question of possible mechanisms of the formation of spatially heterogeneous structures. I study two mechanisms of pattern formation, diffusion-driven instability and hysteresis-driven mechanism, and demonstrate their possibilities and constraints in explanation of different aspects of structure formation in cell systems. Depending on the type of nonlinearities I show the existence of spatially inhomogeneous stationary solution of periodic type, the maxima and minima of which may be of the spike or plateau type, and the existence of transition layer solutions. These concepts are basic for the explanation of the morphogenesis of the fresh water polyp, hydra as well as growth of early lung cancers.

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22 Numerical simulation of differential inclusions, application to the modelling of crowd motion

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23 A caricature of a singular curvature flow in the plane

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We study a simplifying version of the singular weighted mean curvature flow in the plane. The equation preserves the singular character of the original system and behave roughly like

$$u_t - \delta_0(u_x)u_{xx} = 0.$$

We show existence and uniqueness of weak solutions. Our main results are on precise regularity of a class of solutions which we call 'almost classical'. We study formation of flat facets, their interaction and asymptotic behavior.

24 Elliptic quasi-variational inequalities and applications

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Let X be a reflexive real Banach space, X^* be its dual space. We denote by $< \cdot, \cdot >$ the duality pairing between X^* and X. Let $K(u)$ be a bounded closed convex subset of X, which depend on $u \in X$ and $g^* \in X^*$, and A be a mapping from X into X^*. We consider the following problem $P(g^*)$ which is called "Quasi-variational inequality"

$$u \in K(u),\ u^* \in A(u);$$

$$< u^* - g^*, u - v > \leq \forall v \in K(u)$$

If $\tilde{A} : X \times X \to X$ satisfies (SM1) and (SM2) then $\tilde{A}$ is called semimonotone.

(SM1) $\forall v \in X,\ \tilde{A}(v, \cdot) : D(\tilde{A}(v, \cdot)) \to X^*$ is maximal monotone

(SM2) $\{v_n\} \subset X, v_n \to v$ weakly in $X \Rightarrow \forall u^* \in \tilde{A}(u,v),\ \exists \{u_n^*\} \in X^*$ such that

$$\begin{cases} \{u_n^*\} \in \tilde{A}(u_n, v) \ \ \forall n = 1, 2, ... \\ u_n^* \to u^* \text{ in } X^* \ (n \to \infty), \end{cases} \tag{6}$$

Theorem Let $\tilde{A} : X \times X \to X$ be semimonotone and bounded, K_0 be a bounded closed convex subset of X, $\{K(v)|v \in X\}$ be a family of non-empty bounded closed convex subsets of X and $K(v) \subset K_0$ for $\forall v \in K_0$. Let $A(u) = \tilde{A}(u,u)$ for any $u \in X$, and assume that

$(K1)$ $\{v_n\} \subset K_0, v_n \to v$ weakly in X

$$\Rightarrow \forall w \in K(v) \exists \{w_n\} \in X \text{ such that } w_n \in K(v_n),\ w_n \to w \text{ in } X$$

$$(K2) \qquad v_n \to v \text{ weakly in } X, w_n \in K(v_n),\ w_n \to w \text{ weakly in } X$$
$$\Rightarrow w \in K(v)$$

Then for $\forall g^* \in X$, problem $P(g^*)$ has at least one solution.

Applications (Gradient Obstacle Problem)

Let $\Omega \in \mathbb{R}^N$ be a bounded domain with smooth boundary, and $1 < p < \infty$. We put $X = W_0^{1,p}(\Omega)$. Let $a_i : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}(i = 0, 1, 2, ..., N)$, and $k_c(\cdot) : \mathbb{R} \to \mathbb{R}_+$. Then the following Quasi-variational inequality has at least one solution u.

$$\left\{ \begin{array}{c} |\nabla u| \leq k_c(u) \text{ a.e. on } \Omega \\ \sum_{i=1}^{N} \int_\Omega a_i(x, u, \nabla u)(\frac{\partial u}{\partial x_i} - \frac{\partial v}{\partial x_i})dx + \int_\Omega a_0(x, u, \nabla u)(u - v)dx \leq \int_\Omega f(u - v)dx \\ \text{for } \forall v \in X,\ |\nabla v| \leq k_c(u) \text{ a.e. on } \Omega \end{array} \right. \tag{7}$$

under some assumptions on $a_i(\cdot, \cdot, \cdot)$ and $k_c(\cdot)$.

This is a mathematical model for elastic-plastic torsion problem for visco-elastic materials in $\mathbb{R}^3$, and $k_c(\cdot)$ represents threshold value for $|\nabla u|$ of displacement u.

25 On the Mathematical Problems Arising from the Motion of a Viscous Fluid Around a Rotating body

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The motion of one or several rigid bodies in a viscous incompressible fluid has been a topic of numerical theoretical and numerical studies. Over last 40 years the study of the motion of small particles in a viscous liquid has become one of the main focuses of the applied research. The presence of the particles affects the flow of the liquid, and in term, affects the motion of the particles, so that the problem of determining the flow characteristics is highly coupled. It is just the latter feature that makes any fundamental mathematical problem related to the liquid-particle interaction a particularly challenging. We would like to discuss the mathematical analysis of certain aspects of particle sedimentation. We assume that the liquid fills the whole space, in accordance with the fact that, as established by experiments "wall effects" play no role on the preferred orientation of the particles. The mathematical analysis of the particle sedimentation is based on the concept of free fall of the body $\mathcal{B}$ in a liquid $\mathcal{L}$. To investigate the asymptotic behaviour of weak or strong solutions, the knowledge of the asymptotical structure of steady solutions is of the fundamental importance, and we will consider some properties of the linearized operators arising in this problem.

Keywords: Stokes problem, Osseen problem, Maximal operator, Lizorkin multiplier, strong solution, weak solution, Paley-Littlewood theory.

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26 Variational approximation of anisotropic geometric evolution equations

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We present a variational formulation of fully anisotropic motion by surface diffusion and mean curvature flow, as well as related flows. The proposed scheme covers both the closed curve case, and the case of curves that are connected via triple junction points. On introducing a parametric finite element approximation, we prove stability bounds and report on numerical experiments, including regularized crystalline mean curvature flow and regularized crystalline surface diffusion. The presented scheme has very good properties with respect to the distribution of mesh points and, if applicable, area conservation.

27 Large Mean field models for self interacting particles

Existence and asymptotics of solutions of the Debye-Nernst-Planck system in $\mathbb{R}^2$

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We analyse the large time behavior of solutions of a system of partial differential equations describing the time evolution of a cloud of electrically charged particles. This problem, which was formulated in the XIXth century by Nernst and Planck as a drift-diffusion system, was investigated thoroughly and a large bibliography around this topic is available. However, this long history and the number of different approaches did not answer all the questions and there are still some open problems of fundamental nature.
Using methods of functional analysis, ordinary differential equations and fixed point

theorems in various function spaces, we prove the existence of global in time solutions. Moreover, we describe the large time asymptotic behavior of such solutions as t tends to infinity. We are particularly interested in the two-dimensional case, when the system is considered in the whole space $\mathbb{R}^2$. We show that in the special case of small initial conditions the large time behavior of the solutions much differs from the higher-dimensional case.
The analysis of the problem which we present here does not close all the scientific challenges around this issue. For example, one of the still open questions is to describe the asymptotic behavior of the solutions without the mass constraint in the two-dimensional system.

28 Nonlocal behaviour of crystalline curvature flow

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The well-known mean curvature flow for an interface is governed by an evolution law which is parabolic in type and share many common features with the heat equation. In an anisotropic setting we can still define a corresponding curvature flow, consistent with the anisotropic version of surface energy. In this context, anisotropy is defined by a norm with a unit ball which is required to be regular and strictly convex.

Crystalline anisotropy corresponds to a faceted unit ball, and in this case appropriate definition of "crystalline" flow is not completely obvious.

One property that we definitely want to preserve in the crystalline case is a comparison principle which states that a set contained into a bigger set, both with boundary evolving by curvature flow, will remain inside the evolving larger set for all times.

The crystalline evolution law is nonlocal in the sense that the local velocity will depend on the shape of the surface far away. A quick example is given by an evolving (admissible) faceted surface, in which case the velocity of motion of an internal point in a facet will depend on the shape of the whole facet.

Another somehow unespected feature, specific of the 3D evolution, is the possibility of bending of facets, which can lead to nontrivial evolution even starting from convex admissible sets.

The special case of a cylindric anisotropy is also discussed.

29 Calcium waves with mechano-chemical couplings

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Calcium waves in cells and tissues play crucial role in communication between different regions of a cell or between cells. We are mainly interested in the influence of coupling between chemical and mechanical effects on the existence and properties of such waves. It is known that mechanical stresses activate the release of calcium from internal stores. Mathematically, we have to deal with a system of reaction-diffusion equations for the calcium and various buffer proteins concentrations, coupled with the equations of the balance of mechanical forces in the tissue. We have shown that this system has solutions in the form of travelling waves, provided the stresses are not too large. We study also the existence of more complex solutions representing the merging of two travelling waves.

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30 Stationary compressible Navier-Stokes system with slip boundary conditions in a square

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We consider a stationary compressible Navier-Stokes system with slip boundary conditions in a square domain $Q \subset \mathbf{R}^2$. We show the existence of a solution in a class $(W^2_p(Q))^2 \times W^1_p(Q)$ that can be regarded as a small perturbation of a constant flow. The proof is divided into three steps. Fistly we use standard energy methods to obtain apriori bounds on H^1-norm of the velocity and L^2-norm of the pressure. Next we introduce the Helmholtz decomposition to derive estimates on W^2_p-norm of the velocity and W^1_p-norm of the pressure. Finally we use these apriori estimates and the Friedrichs' lemma about commutators to show the existence.

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31 Global and local existence of strong solutions to a viscoelastic fluid model

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We consider a model for viscoelastic fluid, recently studied in the papers [1] and [2]. The studied model is a suitable reformulation of the Oldroyd model. In this talk we show existence of a strong solution (local in time and global in time, provided the data are sufficiently small) under less restrictive conditions on the initial velocity than presented in the papers cited above.

[1] Lin, F., Liu, C. and Zhang, P.: *On hydrodynamics of viscoelastic fluids*, Comm. Pure Appl. Math. **58** (2005) 1437–1471.
[2] Chen, Y. and Zhang, P.: *The global existence of small solutions to the incompressible viscoelastic fluid system in 2 and 3 space dimensions*, Comm. Partial Diff. Equations **31** (2006) 1793–1810.

32 Qualitative behaviour of Stefan problems with surface tension

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We show strong well-posedness of Stefan problems with surface tension, locally in time, and discuss the induced semiflow on a natural phase manifold. In contrast to the classical Stefan problem, in presence of surface tension there are nontrivial equilibra which consist of families of spheres. We investigate stability of such equilibria and prove convergence of solutions to Stefan problems with surface tension as time approaches infinity. This is based on a new Ljapunov functional for the problem.

33 On two models of chemotaxis

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In the talk we want to present the existence of the solutions of two models of chemotaxis:
parabolic-elliptic

$$u_t = \Delta u - \nabla(u\nabla\phi)$$
$$\Delta\phi + u = 0$$

and parabolic-parabolic with parameter ε

$$u_t = \Delta u - \nabla(u\nabla\phi)$$
$$\varepsilon\phi_t = \Delta\phi + u.$$

We prove the existence of solutions in the space of pseudomeasures for two-dimensional case. Additionally we prove the convergence (with $\varepsilon \to 0$) the solutions of parabolic-parabolic problem to solutions of parabolic-elliptic one.

34 The Laplace equation in weighted spaces - existence of solutions

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We study the existence of solutions in weighted Sobolev spaces. It is assumed that the weight is the distance to the plane $x_3 = 0$ in a positive power μ.

35 Zygmund spaces, inviscid limit and uniqueness of Euler flows

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The result is an improvement of the classical uniqueness results for the incompressible Euler system in the n dimensional case assuming $\nabla u^E \in L_1(0, T; BMO(\Omega))$, only.

Moreover, the rate of convergence for the inviscid limit of solutions to the Navier-Stokes equations is obtained, under same regularity of the limit Eulerian flow. A key element of the proof is a logarithmic inequality between the Hardy $\mathcal{H}^1$ and L_1 spaces which is a consequence of the basic properties of the Zygmund space $\mathbf{L}\ln\mathbf{L}$.

36 A simple case of the driven singular mean curvature flow in the plane

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In many free boundary problems (FBPs) the interface evolves according to the Gibbs-Thomson law with kinetic undercooling. We have in mind growth of ice from vapor or growth of crystals from solution. Mathematically, the Gibbs-Thomson relation is a driven mean curvature flow for the interface. In the problems mentioned above the underlying surface energy density function γ is of poor regularity, hence we are lead to consider singular mean curvature. The driving term is the coupling to the diffusion field (e.g. supersaturation or temperature). However, in the present problem, we assume that it is given and enjoys some monotonicity property, which is valid in the full FBP.

We study the dynamics of the driven singular mean curvature flow in the plane if function γ is piecewise linear and its Wulff shape of a rectangle. We are mostly interested in the evolution of closed Lipschitz curves which are slight perturbations of rectangles. We show existence of solutions for a generic driving satisfying the mentioned above monotonicity property. However, our focus is a caricature of a singular curvature flow in the plane Piotr B. Mucha and Piotr Rybka

We study a simplifying version of the singular weighted mean curvature flow in the plane. The equation preserves the singular character of the original system and behave roughly like

$$u_t - \delta_0(u_x)u_{xx} = 0.$$

We show existence and uniqueness of weak solutions. Our main results are on precise regularity of a class of solutions which we call 'almost classical'. We study formation of flat facets, their interaction and asymptotic behavior. the process of bending of initially flat facets.

The motivation for the present choice of the Wulff shape comes from the fact that a rectangle is a cross-section of a circular cylinder (which is an approximation to a hexagonal prism being the equilibrium shape of ice) with a plane containing the symmetry axis.

37 Singular Limits for the Two-Phase Stefan Problem

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The Stefan problem is a model for phase transitions in liquid-solid systems (e.g. water and ice). Here we are especially interested in the justification of the classical model (classical Stefan problem) as an approximation of the Stefan problem with surface tension and/or kinetic undercooling.
More precisely, we examine the behavior of the solution (u, Γ) (u temperatur, Γ free interface) of the two-phase Stefan problem, if surface tension σ and kinetic undercooling γ (separately or simultaneously) tend to zero. It is a well known fact that both of them have a regularizing effect on the free interface. In other words the regularity class of the solution (u, Γ) depends on the presence of surface tension and/or kinetic undercooling, hence the limits $\sigma \to 0$ and $\gamma \to zero$ are singular.
In my talk I am going to present how strong convergence of the solution in the corresponding topology of the limit solution for singular limits of the above type can be obtained. This, for instance, justifies the classical Stefan problem to be a reasonable approximation of the Stefan problem with Gibbs-Thomson correction, if surface tension is small. This is a joint project with Jan Prüß (Halle) and Gieri Simonett (Nashville).

38 Continuous dependence for solution classes of Euler-Lagrange equations generated by convex energies with linear growth

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In this session, let us fix constants $0 < \kappa << L < +\infty$ to consider the following inclusions, denoted by $(P)_\varepsilon$ $(0 < \varepsilon < 1)$ and $(P)_0$

$$(P)_\varepsilon \quad (w_\varepsilon) \in -\kappa \left(\frac{Dw_\varepsilon}{\sqrt{\varepsilon^2 + |Dw_\varepsilon|^2}} \right)_x + \partial I_{[-1,1]}(w_\varepsilon) \text{ in } L^2(0, L),$$

$$(P)_0 \qquad\qquad (w) \in -\kappa \left(\frac{Dw}{|Dw|} \right)_x + \partial I_{[-1,1]}(w) \text{ in } L^2(0, L),$$

subject to homogeneous Neumann boundary conditions, where $I_{[-1,1]}$ denotes the indicator function on the compact interval $[-1, 1]$, and $\partial I_{[-1,1]}$ denotes its subdifferential.

Inclusions $(P)_\varepsilon$ $(0 < \varepsilon < 1)$ and $(P)_0$ are respectively Euler-Lagrange equations of appropriate energy functions, including convex functions with linear growth:

$$V_\varepsilon(z) := \int_0^L \sqrt{\varepsilon^2 + |Dz|^2} := \sup\left\{ \int_0^L (\varepsilon\phi_0 + z\phi_x)dx | \phi_0, \phi \in C_c^1(0, L), |\phi_0^2 + \phi^2|_{C[0,L]} \leq 1 \right\},$$

$$(0 < \varepsilon < 1)$$

and $V_0(z) := \int_0^L |Dz|dx$, $\forall z \in L^2(0, L)$, where $\int_0^L |Dz|dx$ denotes the total variation of z.

Here the energy function for $(P)_0$ is supposed to be a possible free energy in solid-liquid phase transition, and then the inclusion $(P)_0$ just corresponds to the steady-state problem in the phenomena. On the other hand, the sequence $\{V_\varepsilon | 0 < \varepsilon < 1\}$ often appears as an approximation of total variation V_0, and actually V_ε converges to V_0 on $L^2(0, L)$ in the sense of Mosco, as $\varepsilon \searrow 0$. Hence we figure out that inclusion $(P)_0$ can be regarded as limiting problem of inclusions $(P)_\varepsilon$ as $\varepsilon \searrow 0$, and the represented situation by each of $(P)_\varepsilon$ is to be some regularized version of that by the limiting problem $(P)_0$.

Recently the structural analysis for solutions of $(P)_0$ has been developed, and the reported results have enabled us to see concrete steady-states in phase transitions, represented by the problem $(P)_0$. Therefore, applying similar approach to problems $(P)_\varepsilon$, we can expect a precise observation of the situation that solutions w of $(P)_0$ are approximated by solutions w_ε of $(P)_\varepsilon$ as $\varepsilon \searrow 0$.

The objective of this session is on the basis of the above expectation. Then, the main focus od the discussion will be concerned with:

(a) structural analysis for solutions of the approximating inclusions $(P)_\varepsilon$; $(0 < \varepsilon < 1)$

(b) continuous dependence between the solution class $(P)_0$, and the limit of the solution classes of $(P)_\varepsilon$ as $\varepsilon \searrow 0$.

Consequently, the approximating situation, pointed above, will be exactly demostrated by means of analytical methods in set valued analysis.

39 Existence of solutions to non-Newtonian flow in generalized Orlicz spaces

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We will present existence results for non-Newtonian fluids with the stress tensor of very fast (faster than polynomial) and non-homogenous (in x) growth. Consequently L^p spaces framework is not suitable to formulate growth conditions of the stress tensor. Instead, the concept of the so-called *N-function* is introduced, which leads to the definition of the generalized Orlicz space (known also as *the Musielak–Orlicz space*). Example of the generalized Orlicz spaces are the spaces $L^{p(x)}$. They constitute the appropriate framework in analysis of situations when we deal with a problem of modeling strongly inhomogeneous physical behavior — like the so-called electrorheological fluids.

Part of the difficulties in mathematical analysis of such models are caused by the generalized Orlicz spaces themselves. Although they create a natural framework for such analysis, they exhibit new problems arising from the fast growth of an *N-function*, like the lack of reflexivity of the space or the lack of the density of smooth functions. The literature provides numerous results on the existence of solutions to abstract elliptic and parabolic problems in Orlicz spaces, but the framework of Musielak-Orlicz spaces for non-Newtonian flows is still a developing field.

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40 Quasilinear parabolic equation coupled with o.d.e. Model of morphogen gradient formation

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The talk is related to a joint work with Ph. Laurençot and P. Krzyżanowski. We construct a model describing the transport of morphogen - a chemical signaling positional information to cells in a developing tissue. The model couples nonlinear degenerate parabolic equation (with chemotaxis term) with o.d.e. The system is defined on an interval with mixed Dirichlet-Neumann conditions at the ends. Existence-uniqueness of solutions as well as their long time behaviour are studied. Then properties of numerical solutions are analyzed.

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41 Subifferential Operator Approach to Optimal Problems of Allen-Chan Equation with Constraint

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We consider a one dimensional Allen-Cahn equation associated with the total variation energy:

$$(P)\begin{cases} w_t - \kappa\left(\frac{w_x}{|w_x|}\right)_x + \partial I_{[-1,1]}(w) \ni w + u \quad \text{in} \quad Q_T := (0,T)\times(0,1), \\ (B.C.) + (I.C.), \end{cases}$$

where T and κ are positive constants, $u(t,x)$ is a given function on Q_T, and the constraint $\partial I_{[-1,1]}(.)$ is the subdifferential of an indicator function $I_{[-1,1]}(.)$ defined by

$$I_{[-1,1]}(z) := \begin{cases} 0 & \text{if} \quad z \in [-1,1], \\ +\infty & \text{otherwise.} \end{cases}$$

The main object to this talk is to consider an optimal control problem of (P) as follows:
Problem (OP): Find the optimal control $u_* \in L^2(Q_T)$ such that

$$J(u_*) = \inf_{u \in L^2(Q_T)} J(u).$$

Here, $J(u)$ is the cost functional defined by

$$J(u) := \frac{1}{2} \int_{Q_T} |(w - w_g)(t,x)|^2 dxdt + \frac{1}{2} \int_{Q_T} |u(t,x)|^2 dxdt$$

where $u \in L^2(Q_T)$ is the control, w is a unique solution to the state problem (P) with the source control function u, and w_g is a given target profile in $L^2(Q_T)$.

In this talk, we shall show the following:

(i) The existence of an optimal control to Problem (OP).

(ii) The relationship between the original problem (OP) and its approximating problem.

(iii) The optimality condition for the control problem (OP).

[1] I. Pawłow, Optimal control of two-phase Stefan problems -numerical solutions, pp. 179-206, In *Optimal control of partial differential equations, II (Oberwolfach, 1986),* Internat. Schriftenreihe Numer. Math,. Vol. 78, Birkhäuser, Basel, 1987.

[2] K. Shirakawa, Asymptotic convergence of p-Laplace equations with constraint as p tends to 1, Math. Methods Appl. Sci., **25**(2002), 771-793.

42 Global Regularity of the Supercritical 2D Quasi-Geostrophic Equation for Small Initial Data

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The 2D Quasi-Geostrophic equation describes the motion of fluid on a rotating surface, and has been found to be mathematically related to the 3D incompressible Navier-Stokes and Euler equations. In this talk, we discuss some global regularity result when the initial data is small for the supercritical Quasi-Geostrophic equation whose dissipation term is weaker than the convection term.

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43 Long time regular solutions to the Navier-Stokes equations in an axially symmetric domain

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We consider the Navier-Stokes motion in a bounded axially symmetric domain and with slip boundary conditions. We prove the existence of long time regular solutions under assumptions that azymutal components and azymutal derivatives of initial velocity and the external force are sufficiently small in some norms. The existence is proved by the Leray-Schauder fixed point theorem.