

HOW TO USE ATTACHED *.IEQ AND *.POI FILES

The algorithm given by Craw, Maclagan and Thomas in "Moduli of McKay quiver representations. I The coherent component." in *Proc. Lond. Math. Soc. (3)*, 95(1):179–198, 2007 (see Algorithm 6.5, Example 6.6) shows, how to pass from a particular choice of stability parameter θ to the explicit description of the toric fan of normalization of the coherent component of the moduli space of θ -stable McKay quiver representations.

Recall that every (possibly unbounded) polyhedron can be written as the Minkowski sum of a compact polyhedron and a polyhedral cone.

Fix coprime r, a with $r > a$ and let $B \in \text{Mat}(r \times 3r)$ be an incidence matrix of the corresponding McKay quiver. We assume that columns of B correspond to arrows $x_0, y_0, z_0, \dots, x_{r-1}, y_{r-1}, z_{r-1}$, respectively. Note that we use convention which assigns -1 to tail of an arrow and 1 to the head (contrary to the convention used by Craw, Maclagan, Thomas). Let $D \in \text{Mat}(3 \times 3r)$ be a $3 \times 3r$ matrix, which has 1 at the entries $(i \bmod 3, i)$ and 0 elsewhere (assuming the indices of rows and columns start from 0). We set $\theta = [\theta_0, \dots, \theta_{r-1}]^T$.

Theorem (Craw, Maclagan, Thomas). *The following algorithm computes fan of the normalization of the coherent component of moduli space of θ -stable representations of McKay quiver:*

- i) using PORTA express the cone of non-negative solutions of the equation $B\mathbf{x} = \theta$ as a sum of a compact polyhedron K_1 and a polyhedral cone K_2 ,*
- ii) replace K_1 with a list of lattice points L_1 such that K_1 is the convex hull of L_1 ,*
- iii) replace K_2 with a list of its generators L_2 ,*
- iv) define P to be polyhedron which is the Minkowski sum of compact polyhedron equal to the convex hull of DL_1 and the polyhedral cone generated by DL_2 ,*
- v) using PORTA express polyhedron P as a set of inequalities,*
- vi) the inner normal fan to polyhedron P is a toric fan of the normalization of coherent component of θ -stable representations of McKay quiver.*

Remarks.

- 1) PORTA uses two descriptions of polyhedra: files with extension `.ieq` describe polyhedron by a set of inequalities, files with extension

.poi describe polyhedron as the Minkowski sum of convex polyhedron (`CONV_SECTION`) and a polyhedral cone (`CONE_SECTION`). Command lines `traf -v *.poi` and `traf *.ieq` translate one description to the other, adjoining strong validity table to the end of resulting *.ieq file (that is a table showing incidence of points from `CONV_SECTION` and facets given by inequalities).

- 2) Removing multiple vectors in *.poi file can be obtained by converting it to an *.ieq file and then back to a *.poi file.
- 3) To compute the product of matrix D with vectors in lists L_1, L_2 we use algebra computer system `Maxima`.
- 4) The inner normal fan of polyhedron P is given at each vertex of P by the negatives of normal vectors corresponding to facets meeting at the chosen vertex (i.e. by the negative of coefficients of inequalities in *.ieq file).