Bayesian Networks for Multimodal Mode Choice Behavior Modelling: A Case Study for the Cross Border Workers of Luxembourg

Article *in* Transportation Research Procedia · December 2015

DOI: 10.1016/j.trpro.2015.09.040

Tai-yu Ma
Luxembourg Institute of Socio-Economic Research

Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 10 (2015) 870 – 880

Transportation Research Procedia

www.elsevier.com/locate/procedia

18th Euro Working Group on Transportation, EWGT 2015, 14-16 July 2015, Delft, The Netherlands

Bayesian networks for multimodal mode choice behavior modelling: a case study for the cross border workers of Luxembourg

Tai-Yu Ma*

Luxembourg Institute of Socio-Economic Research (LISER), 3, avenue de la Fonte, L-4364 Esch-sur-Alzette, Grand–Duché de Luxembourg

Abstract

Reducing car use and promoting public transport in the cross border area of Luxembourg has become a priority for sustainable development of the Greater region. In this study, we analyze daily mobility mode choice behavior of these cross border workers, in particular, focusing on their multimodal mode choices (e.g. park and ride mode choice) and on their trip chaining behavior. A rule-based approach based on Bayesian networks is proposed to capture the non-linear effects of related determinants/constraints on individuals' mode choice behavior. The result shows the propose Bayesian network has a competitive performance compared with classical discrete choice models with reasonable good corrected prediction rates.

Peer-review under responsibility of Delft University of Technology

Keywords: mode choice; bayesian network; multimodal; Luxembourg; uncertainty

1. Introduction

Reducing car use and promoting public transport in the cross border area of Luxembourg has become a priority for sustainable development of the Greater region. According to STATEC (STATEC 2015), the number of cross border workers, i.e. individuals working in Luxembourg but living in Germany, France or Belgium, has increased to 161 300 in 2013. In spite of good impression of public transportation service, car is still the main transport mode for their daily commuting trips (Schmitz et al. 2012). Facing to increasing daily mobility demand and a high car-use dependency, better understanding travelers' mode choice behavior provides useful insight for the stakeholders to

* Corresponding author. Tel.: +352 585855308; Fax: +352 585855700.
E-mail address: tai-yu.ma@liser.lu

Peer-review under responsibility of Delft University of Technology
doi:10.1016/j.trpro.2015.09.040

promote public transport and soft modes. Although the cross-border worker mobility survey of Luxembourg (Schmitz et al. 2012) has collected detailed daily travel-activity data, the issues in the causal structure and decision process of their mode choice behavior are still less studied. By distinguishing various multimodal combinations as alternatives (instead of grouping them by a main mode with longest travel time), this study aims to model complex mode choice behavior in relationship with relevant determinants and causal structure of their mode choice behavior.

Mode choice models are usually based on random utility theory, which assumes individuals aim at maximizing the utility of their choice. The utility of an alternative of choice is expressed as a function of its attributes. Unobserved preference of decision makers is modeled as an error term following some probability distributions. The error term can represents bounded/non-complete rational behavior. More flexible models have also been developed such as nested logit or probit models for modelling the correlations between choice alternatives (Bhat, 1997) and mixed logit models for incorporating preference heterogeneity (Greene and Hensher, 2003; Train 2009). In spite of the popularity of the discrete choice models, past studies have revealed the limits of utility-maximization framework (Ben-Akiva and Lerman, 1985; Train 2009) for modeling discrete choice behavior, resulting in recent developments in rule-based reasoning/modeling system for inferring travelers' choice behavior (Vause, 1997; Arentze and Timmermans, 2004, 2007; Janssens et al. 2004, 2006). The rational choice theory assumes individuals make their choice decisions based on the comparison of different alternatives in their choice set and select one with maximum utility. The theory assumes that a decision maker has complete information about his choice alternatives and has full cognitive capacity and time to compare the alternatives. These disadvantages lead to its model extensions based on bounded rationality of human decision making (Newell and Simon 1972, Rubinstein, 1998) and to the development of causal modeling of decision making under uncertainty (Sloman 2005). In this study, we focus on the latter one by applying Bayesian networks (BN, or Bayesian belief networks) for modeling mode choice behavior. We analyze their daily mobility mode choice behavior, in particular, focusing on the causal relationship by taking into account multimodal mode choice (e.g. park and ride mode choice) and trip chaining.

The organization of this paper is as follows. In Section 2, we review firstly main determinants of mode choice decision making and then present Bayesian network modeling approach for reasoning travelers' mode choice decision. Section 3 reports descriptive statistics of the survey data and discusses possible causal relationships between the determinants. Section 4 presents the proposed Bayesian networks for causal structure modeling of travelers' mode choice. Model calibration and validation are tested and compared with the traditional multinomial logit model. Finally the conclusion is drawn and future extensions are discussed.

2. Determinants of mode choice behavior and Bayesian network modeling approach

2.1. Determinants of mode choice behavior

Modeling mode choice behavior needs to understand the relevant determinants and their causal relationships to explain travelers' decisions. Past studies show mode choice may be influenced by different factors (Ye et al. 2007; Krygsman et al. 2007; Carpentier and Gerber 2009; De Witte et al. 2013; Enaux and Gerber 2014; Ma et al. 2015):

- Journey characteristics: trip purpose, trip chaining, weather condition, departure time, travel time, travel distance, travel cost etc.
- Socio-demographic characteristics: driving license, car availability, season ticket ownership, presence of children, gender, age, household income, household composition, occupation etc.
- Spatial characteristics: population density of residential location, proximity to infrastructure and services, parking, frequency of public transport etc.
- Socio-psychological factors: habits, lifestyle, transport mode perception, past positive/negative experience etc.

In general, these factors are correlated, and their relationships are usually implicit and sometimes unobservable. To investigate the causal structure of choice outcomes, some causal structure modeling technique, e.g. structural equation modeling approach, has been applied to identify these complex relationships (Simma and Axhausen, 2001). The identified causal structures can be served as references for further adjustments to find better causal models based on the empirical data.

2.2. Bayesian network modeling approach

A Bayesian network is a mathematical modelling of causal relationships between a set of variables for a problem domain. Such a probabilistic network is a good modeling tool for representing graphically causal relations associated with uncertainty for related inference problems or decision making problems. Bayesian network modelling approach have been successfully applied to medical diagnostics, decision support systems, semantic search, bioinformatics etc. (Abramson and Finizza, 1991; Anderson et al.1991; Bacon et al. 2002). In Bayesian networks, causal relations between variables are represented by conditional probabilities, indicating a kind of if-then rule under uncertainty. A Bayesian network consists of two components: a structural model (causal relations among variables) and parameters (conditional probabilities). The structural model assumes dependent/independent relations among variables and characterizes the conditional probability of variables. Bayesian networks allow us to identify the causal relations and inference travellers' mode choice according to each traveller's profile. Moreover, the precision of the model can be improved over time when new coming evidences further updating the beliefs associated with the network.

Let a Bayesian network be described by a directed acyclic graph (DAG) $G=(V, E)$ where V is a set of nodes and E is a set of directed arcs. Let us define a set of domain variables X, relevant to characterize the problem variable Y, i.e. mode choice decision over a finite set of transport mode alternatives. X_v denotes the variable associated to node v. A directed arc (u, v) represents a dependent relationship from u (cause) to v (effect). Node u is called parent of node v and node v is called child of node u. A DAG is a graphical representation of dependent relationships of a set of variables. The type of variables can be binary, discrete or continuous, representing possible outcomes of variables. In Bayesian networks, if there is no connection between two nodes, they are assumed conditional independent (Markov property, Lauritzen et al. (1990)). Given each node is conditional independent of its non-descendants, the joint probability distribution of domain variables defined by a BN can be calculated by applying chain rule as (Pearl, 2009)

$$P(X_v) = \prod_{v\in V} P(X_v \middle| X_{pa(v)}) \tag{1}$$

, where $pa(v)$ is the parent nodes of v. $P(X_v | X_{pa(v)})$ is the conditional probability distribution of the child node v, given the values of its parent nodes. The chain rule allows to compute the joint probability efficiently. The reader is referred to Pearl (2009) and Kjærulff and Madsen (2013) for more detailed description.

The learning of Bayesian networks consists of: (1) structure learning, and (2) parameter learning. Structure learning determines conditional dependency/independency between nodes (variables). Parameter learning determines the conditional probability tables (CPT), given network structure and evidence from the data. For example, a simple Bayesian network is shown in Fig. 1. Mode choice outcome is conditioned by three parent nodes: presence of children in the household (child), difficulty of parking (Parking) and driving license ownership (D_license). The probability distributions for Child, Parking and D_license are unconditional probability distributions because there is no parent nodes. The interesting applications of Bayesian networks is when entering new evidences, i.e. the fact of knowing the value of some variables, we can predict the probabilities of our interested outcome variable. The computation of the updated CPTs can be based on the maximum likelihood estimation or Bayesian estimation (Jensen and Nielsen, 2007; Pearl, 2009)

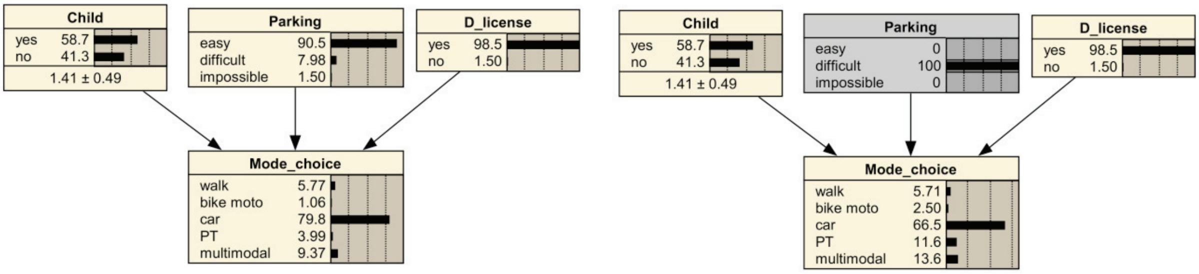

Fig.1 Example of a small Bayesian network with its conditional probability tables (left). After entering new evidence (i.e. given the value of some variables, in this case, parking is difficult), mode choice probability changes (probability of choice for car reduces from 0.798 to 0.665 if parking is difficult for a respondent). Note that for "child" node, 1.41±0.49 indicates its mean (1.41) and standard deviation (0.49).

As for the structure learning, we adopt a greedy search heuristic by firstly setting up an initial legal (i.e. acyclic and logically sound) network structure and then iteratively adjust its structures until its performance score cannot be improved. The performance score can be measured by score functions based on Euclidean distance between the probability distributions or on Bayesian Information Criterion. The greedy search heuristic is defined as (Jensen and Nielsen, 2007):

Greedy search heuristic

Step 0: Given initial structure S_0, set initial best performance score $u^*=u(S_0)$, i=0.

Step 1: Do legal arc operations $o(S_i)$ and compute the difference of performance scores $\Delta(o(S_i))$.

Step 2: if $\Delta(o(S_i))>0$ then update best obtained structure $S^*=S(S_i, o(S_i))$, i=: i+1.

Step 3: Repeat Step1 and Step 2 until S* cannot be further improved.

3. Data

The data set is based on the mobility survey of cross border workers in the Greater Region of Luxembourg (Schmitz et al. 2012). The greater region contains the cross border area between Luxembourg and its three neighbors, namely France, Germany and Belgium. In 2010, there was totally 146 000 cross border workers who used car as main commuting mode, resulting in recurrent road network congestion in this area. Thus there was an important need to collect data for better understanding their daily mobility practices in order to reduce car use and promote public transport uses. A total of 7235 valid samples was collected. The respondents were asked to report the travel diary of their precedent working day before the day of the survey. The survey data contains one-day travel diary and related spatial and socio-demographic information. Table 1 shows the descriptive statistics of the sample. Surveyed questions about trip chaining information contain: departure and arrival times and destinations, principal trip purposes, waiting time and searching time for parking. For each trip, transport mode, travel time and estimated distance are collected. The descriptive statistics is shown in Table 1. Most cross border workers are man (60%), living in couple (72%) with presence of children (56%). The average number of cars in the household is relatively high (3.24) and most respondents have driving license (96%). The household income distribution is relatively high. As for transport mode choice, it is classified into five categories: walk, bike, car, public transport, and multimodal (car + public transport). It is shown car is the dominant mode for the cross border workers, accounting for 79.7% for all trips. The average number of trips is 2.65 and travel time is 39.2 minutes.

Table 1. Descriptive statistics of the sample (sample size =7235 individuals)

Variable	Definition	Mean
Socio-demographic and spatial characteristics		
Male	1 if male, 0 female (% of 1)	0.60
Couple	1 if couple, else 0 (% of 1)	0.72

Children	1 if presence of children in the household, and 0 otherwise	0.56
N_car	Number of cars in the household	3.24
License	1 if the individual has a driving license, and 0 otherwise (% of 1)	0.96
Household income after tax	0-2000 euros/month	6.08
	2000-4000 euros/month	42.30
	4000-6000 euros/month	30.67
	6000-8000 euros/month	11.61
	8000 or more euros/month	4.31
Trip characteristics		
Mode_work	Walk	5.74%
	Bike and motorcycle	0.18%
	Car	79.7%
	Public transport	4.1%
	Car + Public transport	7.5%
N_trip	Number of trips during individuals' last working days	2.65
Parking_easy	1 if finding a parking at work place is easy, and 0 otherwise	0.45
WP_fixed	1 if the individual's work place is fixed, and 0 otherwise	0.92
Dest_act	Trip destination is in Luxembourg	37%
	Trip destination is in the country of residence	61%
Travel time	Travel time of trip (minute)	39.2

Remarks: Soft mode is related to walk, bicycle and moto-bike, public transport is related to bus and train.

4. Estimation results

In this section, Bayesian network modeling approach is applied to model the causal structure of mode choice behavior. The performance of the Bayesian network is compared to a traditional multinomial logit model. The data set is divided into a training dataset (random 75% of total trips) and a test dataset (the rest of total trips) for model validation. Based on the literature review in Section 2.1, the variables considered are listed in Table 2. For socio-demographic variables, gender, household composition may influence the probability of choosing car for conducting certain activities (e.g. pick up or drop off children to school). Household income, number of cars in the household and driving license may influence car availability for an individual. Full-time or part-time job may influence departure time for work trip and the scheduling of other activities, resulting in different mode choice outcomes. For journey characteristics, trip purposes influence destination choices of trip, resulting in different mode choice decision with least travel time/cost. Other relevant decision variables are travel time, departure time of trip and travel cost. Moreover, trip chain complexity may also influence an individual's mode choice decision to optimize his/her trip chaining. As for spatial factor, easy/difficult to find a place of parking is related to trip destination. The outcome variable is an individual's mode choice which is classified into: walk, bike/moto, car, public transport and multimodal (car+ public transport). The continuous variables (travel time of trip and travel distance of trip) are further discretized based on its frequency. Note one can use other methods to handle continuous variables (Kjærulff, and Madsen 2013) which may influence the performance of Bayesian networks.

The obtained structure of Bayesian network is shown in Fig. 2. Mode choice decision is directly influenced by journey characteristics: parking difficulty, travel time of trip, travel distance of trip, driving license ownership and number of car in the household. Trip purposes influence departure time choice and activity destination which may determine travel time, travel distance and parking availability at destination and then influence individuals' mode choice. Household characteristics (gender, couple, presence of children and household income) determine household members' activity needs and derives travel demand on space and transportation services. We can find Bayesian networks provide intrinsic possible causal relationships between the determinants, which can be easily understood for explaining individuals' choice decision. The probability conditional tables of the Bayesian network are shown in Fig 3. For the nodes without parents (e.g. gender), the probability tables represent related marginal probability distributions. For child nodes, its conditional probability tables are computed based on the probability distributions of its parents' nodes.

Table 2. Node definition and values

Node	Definition	Value
Socio-demographic characteristics		
Gender	Gender of the individual	1 male, 2 female
T_job	Full-time or part-time job	1 full-time job, 2 part-time job
Couple	The individual living in couple	1 yes, 2 no
Child	Presence of children in the household	1 yes, 2 no
HH_income	Household net monthly income	1 < 2000€, 2 2000€-4000€, 3, 4000€-6000€, 4 6000€-8000€, 5 >8000€
N_car	Number of car in the household	1 no car, 2 1 car, 3 two or more cars
D_license	Having driving license	1 yes, 2 no
Journey characteristics		
T_purpose	Trip purpose	1 pickup/drop off someone, 2 go home, 3 work/school, 4 having a meal / shopping, 5 personal business, 6 social-recreation, 7 others
T_tour	Type of tour in which the trip is located	1 1-stop (one out-of-home activity), 2 2 or 3 stops? 3 4 or 5 stops, 4 more than 5 stops
D_time	Departure time of trip	1 within peak hours (7:00-9:00 or 17:00-19:00) 2 within non-peak hours
T_time[1]	Travel time of trip	1 <=5 min., 2 5-10 min., 3 10-20 min., 4 20-40 min., 5 40-60min., 6 60-80 min., 7 80-90 8 >=90 min.
T_distance[2]	Travel distance of trip	1 <=1km, 2 1-3km, 3 3-11.4km, 4 11.4-34 km, 5 34-50km, 6 50-70km, 7 70-85km, 8 >85km
Spatial characteristics		
Dest_act	Destination	1 Luxembourg, 2 Country of residence, 3 others
Parking	Facility/difficulty to find a parking at work place	1 easy, 2 difficult, 3 impossible
Dependent variable		
Mode_choice[3]	Transport mode of trip	1 walk, 2 bike or motorcycle, 3 car, 4 bus/train, 5 car+ bus/train

Remark: 1. travel time of a trip is computed as the difference between its arrival time and departure time; 2. travel distance of a trip is computed based on the shortest routes of network; 3 car: including car driver and car passenger.

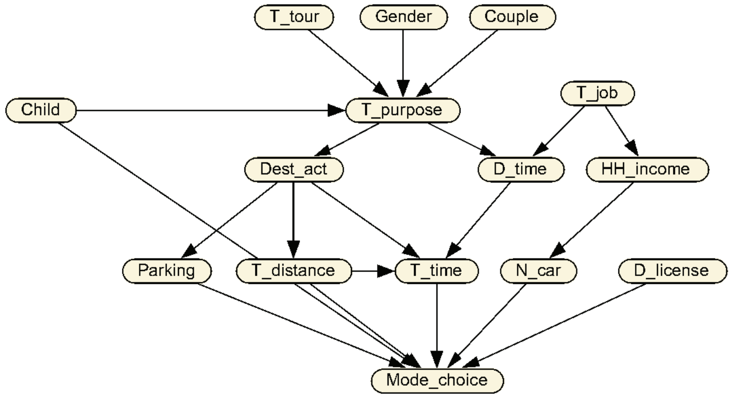

Fig.2 Bayesian network of mode choice decision of cross border workers of Luxembourg

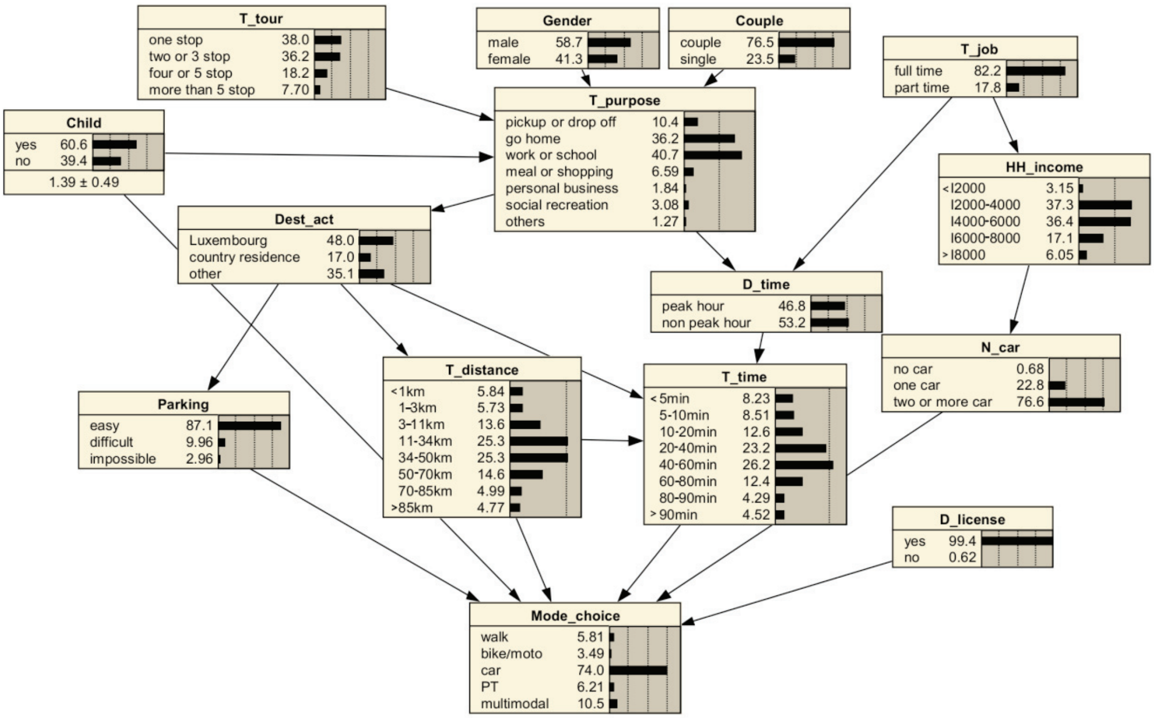

Fig3 Conditional probability tables of the Bayesian network for individuals' mode choice decision of the cross border workers of Luxembourg

The performance of Bayesian networks is shown in Table 3. The training set accounts for 75% of total sample and the test set for the rest 25% of total sample. The overall corrected prediction rates (i.e. percentage of the same observed and predicted choices) for BN is higher (86.7%) than multinomial logit model (MNL) (84.9%) in the training set. However, for the test data set, their performances are similar (83.4% v.s. 83%). When regarding in a more detail, car and walk are more successfully predicted compared to public transport and multimodal choices. The result could be explained by the fact that most relevant determinants for car and walk choices are included in the model, e.g. travel time, travel distance, easy/difficult to find a parking space etc. However, for public transport and multimodal choices, some relevant variables, e.g. number of transfer or waiting time at stops, might be missing in the model. As a result, the corrected prediction rates for the two modes are less convincing. For comparison purpose, we utilized the same explanatory variables as the BN in Fig. 3 for the MNL model estimation except for driving license (D_license, binary variable). This is because there is a very high driving license ownership for the respondents (96%) in the sample. Moreover, we designated PT as the reference alternative to investigate the influence of these determinants on respondents' mode choice probabilities. The estimated MNL model is shown in Table 4. The overall fitness of the model (p-value of Chi-Square test <0.0001) shows the determinants have statistically significant effects. The McFadden's R^2 is 0.3413.

For the pair comparison between car and PT, we found travel time, travel distance, easy/difficult to find a parking space, type of tour, number of cars in the household, household income, presence of children, household type (couple or not) and type of job (full-time or part-time job) have significant effects on relative choice probabilities between these two modes. The result is consistent with the BN in Fig.3, in which the six determinants (T_time, T_distance, Parking, N_car, D_license, Child) directly influence mode choice probability distribution. As regards the estimated coefficients, the positive sign of a coefficient means increasing the value of an attribute will increase the utility of that alternative, resulting in increasing the relative choice probability compared to the reference alternative. Note that the MNL model assumes the IIA property (Independence of Irrelevant Alternatives Property), which assumes the choice probability between any two alternatives is independent of the presence of the other alternatives. Under the IIA property, we found full-time job workers have higher probability in the use of car. Couple, number of car, complex tour and long-distance trip have positive effect on the use of car. On the contrary, non-presence of children, higher household income, longer travel time and difficult to find a parking space have higher probability to use public transport. Similar analyses can be conducted for choice preference of walk-PT and multimodal-PT pairs. We found that the MNL model may show contrary effects of some attributes compared to other empirical findings in the literature. This may be due to the unrealistic IIA assumption of the MNL model. Moreover, compared to the BNs the MNL model cannot capture the endogenous/causal relationships between the explanatory variables.

Table 3. Corrected prediction rates of Bayesian networks and multinomial logit model for training and test datasets

	Training set (n=8941)			Test set (n=3052)		
		Corrected prediction[1] (%)			Corrected prediction (%)	
Mode	N of trips	MNL model	BN	N of trips	MNL model	BN
Walk	320	40.6	64.1	111	31.5	63.1
Bike/motorbike	18	0.0	22.2	9	0.0	0.0
Car	7391	97.5	97.6	2539	95.3	95.2
Public transport	422	22.0	36.5	136	26.5	11.0
Multimodal	790	21.1	21.9	257	21.4	12.5
Total	8941	84.9	86.7	3052	83.4	83.0

Remark: 1. Correction prediction rates is calculated with respect to each transport mode

Table 4 Estimation result of the MNL model

Mode	Variable	Coef.	Std.	Mode	Variable	Coef.	Std.
PT		*Reference mode*					
Walk	Gender	0.376*	0.199	Car	Gender	0.096	0.128
	T_job	-0.449*	0.264		T_job	-0.439**	0.175
	Couple	0.249	0.27		Couple	0.787***	0.159
	Child	-0.205	0.214		Child	-0.494***	0.136
	HH_income	-0.154	0.106		HH_income	-0.380***	0.069
	N_car	1.432***	0.219		N_car	2.178***	0.126
	T_purpose	0.338***	0.087		T_purpose	-0.114	0.075
	T_tour	0.385***	0.122		T_tour	0.406***	0.078
	D_time	0.771***	0.213		D_time	-0.008	0.118
	TTime	-0.028***	0.005		TTime	-0.069***	0.003
	TDistance	-0.352***	0.03		TDistance	0.055***	0.003
	Dest_act	-0.685***	0.155		Dest_act	-0.082	0.069
	Parking	-0.718***	0.178		Parking	-0.645***	0.096
	_cons	-0.971	1.165		_cons	0.349	0.693
Bike/Moto	Gender	-0.451	0.574	Multimodal	Gender	0.666***	0.143
	T_job	-0.512	0.863		T_job	-0.349*	0.194
	Couple	0.758	0.682		Couple	0.766***	0.184
	Child	-0.361	0.619		Child	-0.285*	0.153
	HH_income	-0.653*	0.341		HH_income	-0.250***	0.079
	N_car	0.549	0.546		N_car	2.092***	0.155
	T_purpose	0.359**	0.171		T_purpose	-0.163	0.103
	T_tour	0.171	0.31		T_tour	-0.108	0.092
	D_time	0.025	0.505		D_time	-0.029	0.133
	TTime	-0.053	0.019		TTime	0.002	0.002
	TDistance	0.002	0.022		TDistance	0.018***	0.003
	Dest_act	0.399	0.302		Dest_act	-0.078	0.083
	Parking	-1.178*	0.695		Parking	-0.580***	0.116
	_cons	-0.489	2.980		_cons	-4.589***	0.853
	N= 8941; Log likelihood = -3814.0013;Prob > chi2 < 0.0001;Pseudo R2 = 0.3413						

Remark: 1.TTime and TDistance is travel time of trip and travel distance of trip, respectively (continuous variables). 2. D_license is removed due to the extremely high driving license ownership. 3.* means 0.05 < p-value ≤ 0.1; ** means 0.01 < p-value ≤ 0.05 ;*** means p-value ≤ 0.01.

4. Conclusions

In this study, Bayesian networks are applied to model commuters' mode choice decision. We investigate possible causal structure of commuters' mode choice behavior and estimate the model parameters. A greedy search approach is applied to identify underlying causal structure of mode choice which allows to predict commuters' mode choice. The empirical data is based on the cross-border worker mobility survey of Luxembourg. We compared the performance of Bayesian network with that of multinomial logit model and found similar corrected prediction rates. Further studies are necessary to improve the performance of Bayesian networks. Possible directions include introducing other relevant variables, different discretization schemes for continuous variables or using data-driven causal structure learning approach. To improve the performance the present model, hierarchical Bayesian network models, in which the combination of continuous and discrete variables can be integrated by a link function, could be specified and compared to the present BN model. Moreover, different calibration methods of BNs could be compared and discussed in the future extensions. The test of the performance of the BNs on different mobility survey data would be helpful to confirm the validity of model comparisons between the BNs and utility-based discrete choice models.

Acknowledgements

We sincerely thank the Fonds National de la Recherche Luxembourg for their support of the CONNECTING project (Consequential Life Cycle Assessment of multi-modal mobility policies - the case of Luxembourg), FNR CORE programme (C14/SR/8330766).

References

Abramson, B., Finizza, A. 1991. Using belief networks to forecast oil prices. International Journal of Forecasting 7(3), 299–315.

Anderson, L.R., Krebs, J.H., Anderson, J.D., 1991. STENO: An Expert System for Medical Diagnosis Based on Graphical Models and Model Search. Journal of Applied Statistics 18, 139-153.

Arentze, T.A., Timmermans, H.J.P., 2004. A learning-based transportation oriented simulation system. Transportation Research Part B 38(7), 613-633.

Arentze, T.A., Timmermans, H., 2007. Parametric action decision trees: Incorporating continuous attribute variables into rule-based models of discrete choice. Transportation Research Part B 41, 772-783

Bacon, P., Cain, J., Howard, D., 2002. Belief network models of land manager decisions and land use change. Journal of Environmental Management 65(1), 1–23.

Ben-Akiva, M., Lerman, S.R. 1985. Discrete Choice Analysis Theory and Application to Travel Demand. The MIT press.

Bhat C.R., 1997. A nested logit model with covariance heterogeneity. Transportation Research part B 31, 11–21.

Carpentier, S., Gerber, P., 2009. Les déplacements domicile-travail : en voiture, en train ou à pied ? Vivre au Luxembourg, 53.

De Witte A, Hollevoet, J, Dobruszkes, F, Hubert, M, Macharis, C., 2013. Linking modal choice to motility: a comprehensive review, Transportation Research Part A 49, 329-341.

Enaux C, Gerber P., 2014. Beliefs about energy, a factor in daily ecological mobility? Journal of Transport Geography 41, 154–162

Greene, W.H. and Hensher, D.A., 2003. A latent class model for discrete choice analysis: contrasts with mixed Logit. Transportation Research Part B 37, 681–698.

Jensen, FV, Nielsen TD. 2007 Bayesian Networks and Decision Graphs, 2nd Edition, Springer, New York.

Janssens, D., Wets, G., Brijs, T., Vanhoof, K., Arentze, T.A., Timmermans, H.J.P., 2004. Improving performance of multiagent rule-based model for activity pattern decisions with Bayesian networks. Transportation Research Record 1894, 75-83.

Janssens, D., Wets, G., Brijs, T., Vanhoof, K., Arentze, T.A., Timmermans, H.J.P. 2006. Integrating Bayesian networks and decision trees in a sequential rule-based transportation model. European Journal of operational research 175, 16-34.

Kjærulff, U.B., Madsen, A.L., 2013. Bayesian Networks and Influence Diagrams: A Guide to Construction and Analysis. Springer, 2nd Edition.

Krygsman, S., Arentze, T., Timmermans, H. 2007. Capturing tour mode and activity choice interdependencies: A co-evolutionary logit modelling approach. Transportation Research Part A 41, 913–933.

Lauritzen, S. L., Dawid, A. P., Larsen, B. N., Leimer, H.-G. (1990). Independence properties of directed Markov fields, Networks 20(5) 491–505.

Ma T-Y, Gerber, P., Carpentier, S., Klein, S., 2015. Mode choice with latent preference heterogeneity: a case study for employees of the EU Institutions in Luxembourg. Transportmetrica A: Transport Science. doi: 10.1080/23249935.2015.1007175

Newell, A., Simon, H. A., 1972. Human Problem Solving. Englewood Cliffs, NJ: Prentice Hall.

Pearl, J., 2009.Causality: Models, Reasoning, and Inference, 2nd edition, Cambridge University Press, Cambridge, UK.

Rubinstein A (1998) Modeling Bounded Rationality. MIT press

Schmitz F., Drevon G., Gerber P. (Eds.), 2012, La mobilité des frontaliers du Luxembourg : dynamiques et perspectives, Hors-série, Les cahiers du CEPS/INSTEAD, 40 p.

Simma, A, Axhausen, K.W. 2001. Structures of commitment in mode use: a comparison of Switzerland, Germany and Great Britain. Transport Policy 8(4), 279–288.

Sloman, S. A. 2005. Causal Models: How People Think about the World and Its Alternatives, Oxford University Press, NY, USA.

STATEC, 2015 http://www.statistiques.public.lu/stat/TableViewer/tableView.aspx?sCS_ChosenLang=fr&ReportId=494

Train, K. 2009. Discrete Choice Methods with Simulation. Cambridge University Press, 2nd edition.

Tringides C.A, Ye, X., Pendyala R.M., 2007. Departure-Time Choice and Mode Choice for Non-work Trips: Alternative Formulations of Joint Model Systems. Transportation Research Record 1898, 1-9.

Vause, M., 1997. A rule-based model of activity scheduling behavior. In: Ettema, D.F., Timmermans, H.J.P. (Eds.), Activity-based Approaches to Activity Analysis. Pergamon Press, Oxford, pp. 73–88.

Ye, X., Pendyala, R.M., Gottardi, G., 2007. An exploration of the relationship between mode choice and complexity of trip chaining patterns. Transportation Research Part B 41, 96–113.