On the strength of non-determinism for Büchi VASS

Olivier Finkel, Michał Skrzypczak

Highlights 2019

Warsaw 19.09.2019

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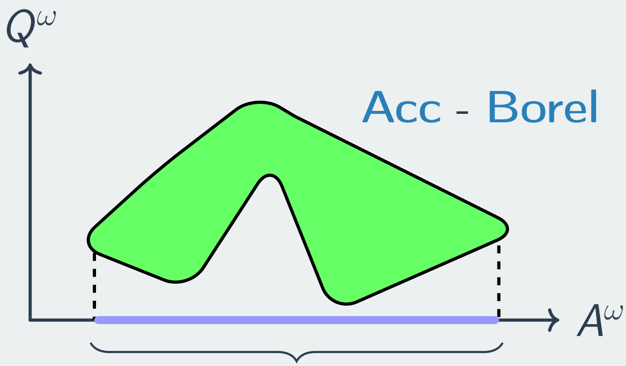

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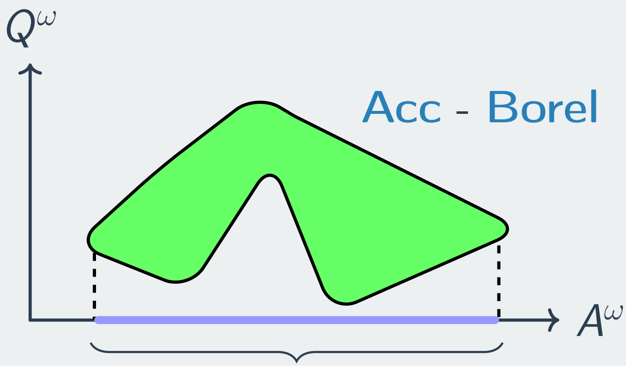

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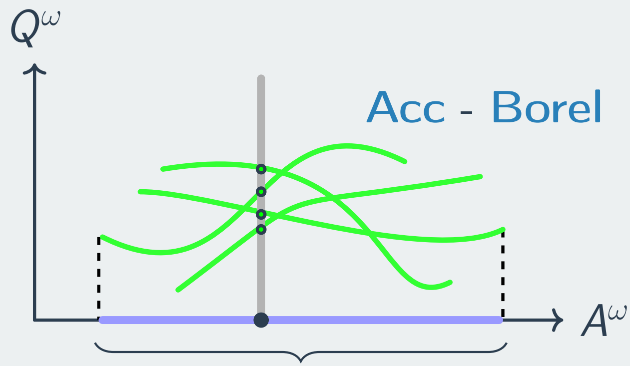

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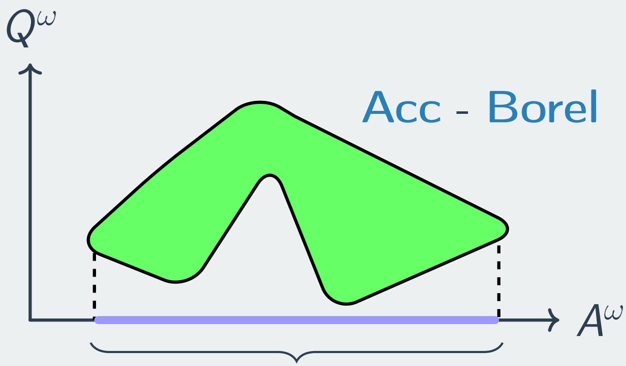

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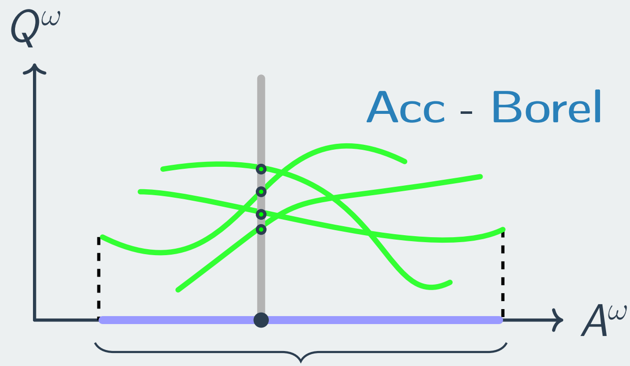

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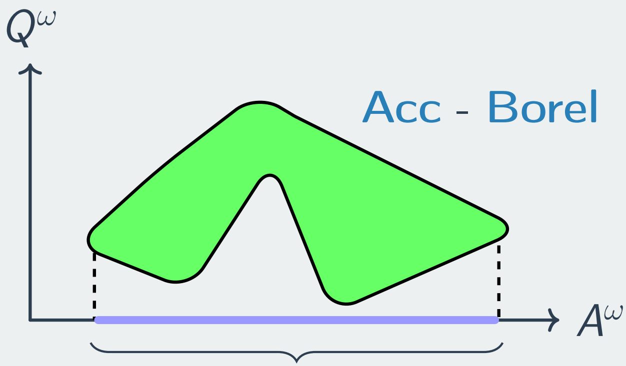

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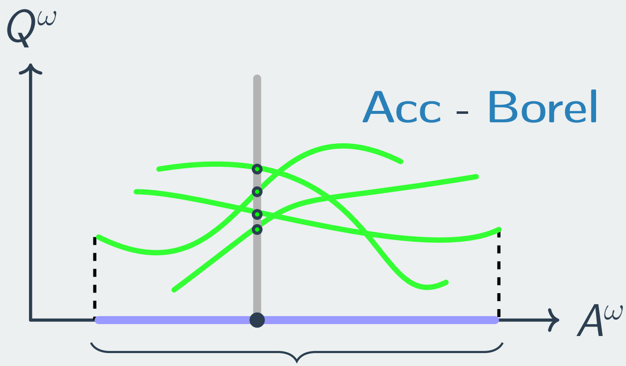

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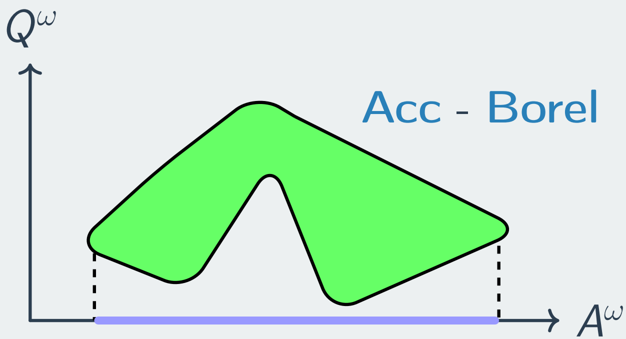

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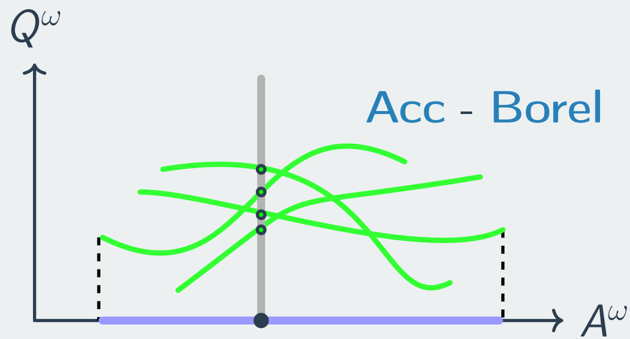

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Theorem (S. ['18])

There exists a **non** Borel ω-language

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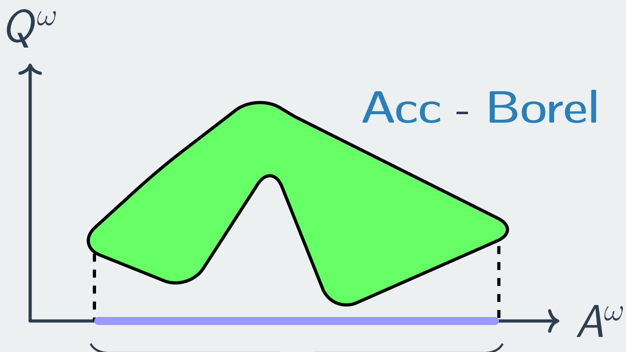

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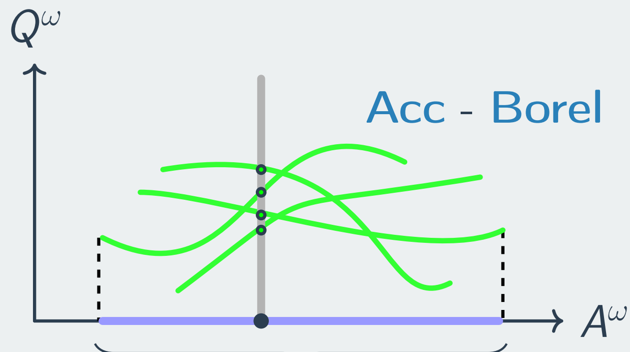

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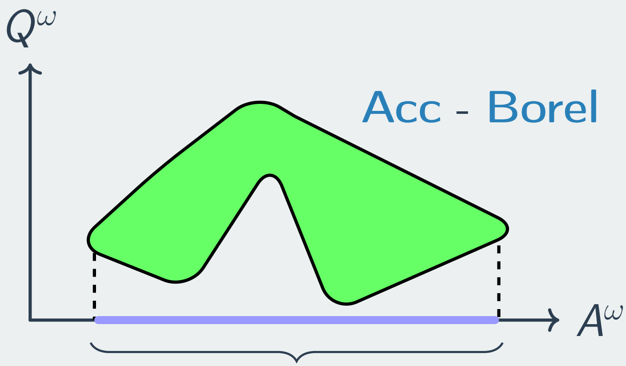

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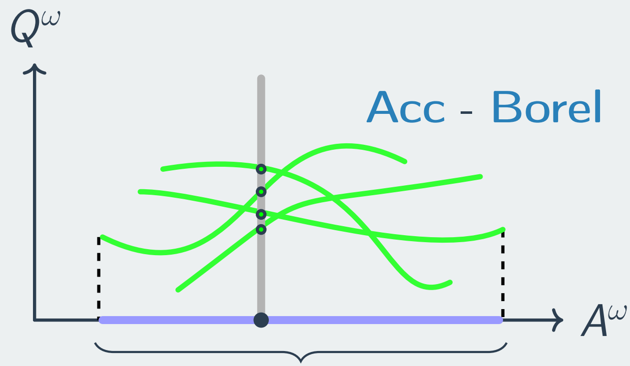

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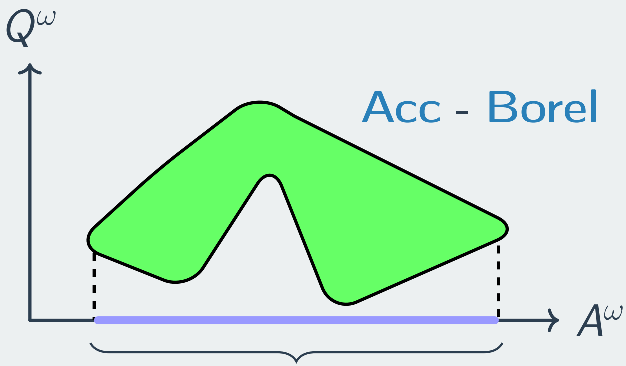

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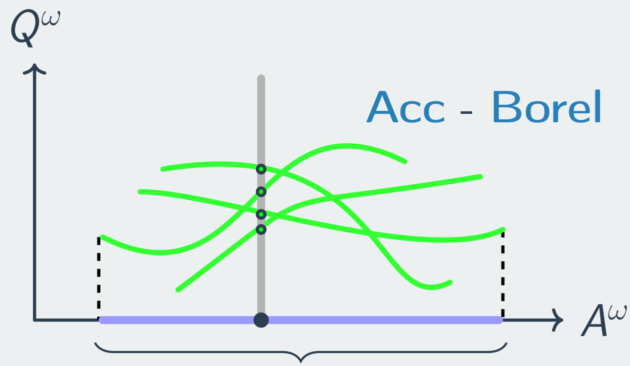

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⟶ Similar result (+Wadge) with four counters in (Finkel ['18]) ⟵

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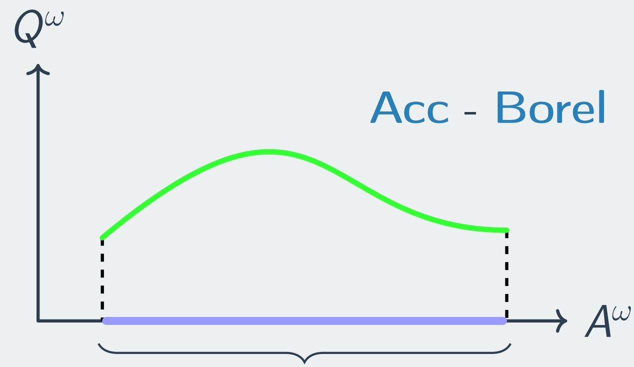

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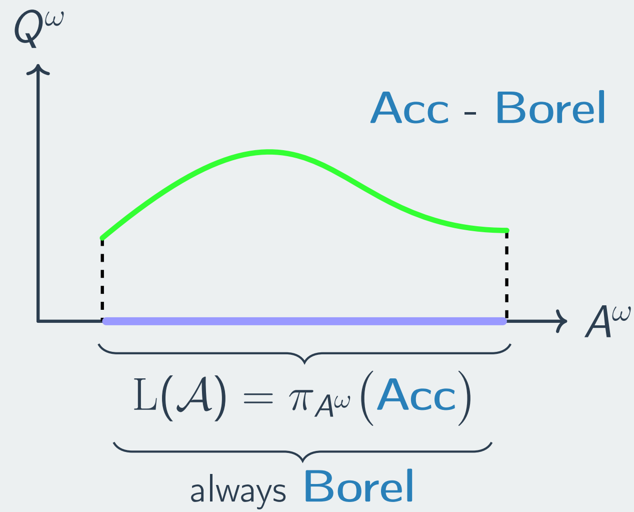
How to make this effective?



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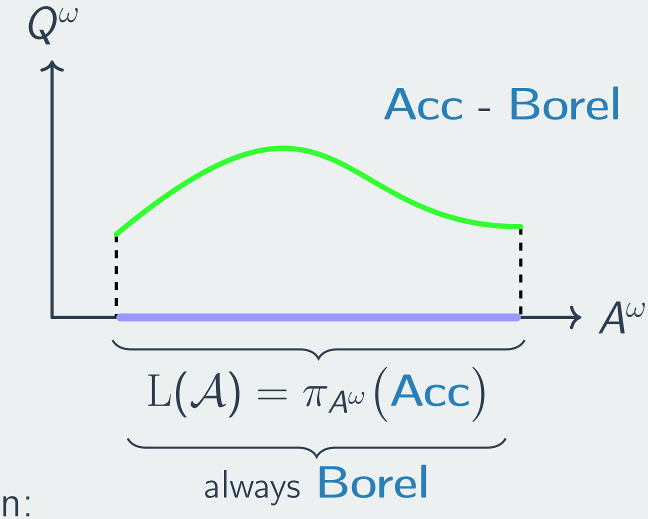

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There exists an effective determinisation:

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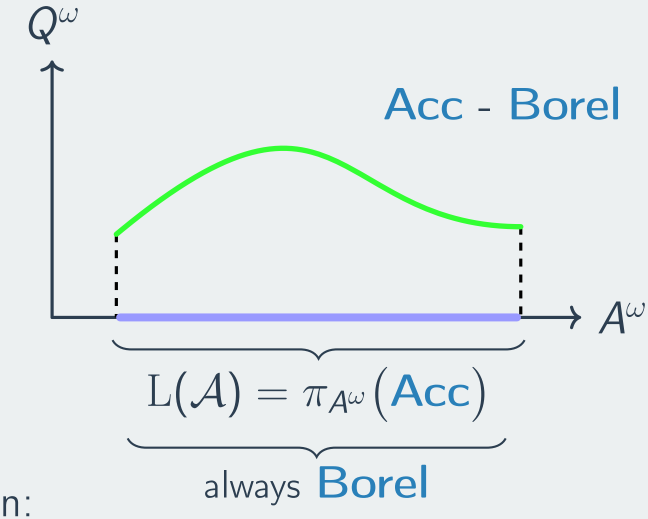

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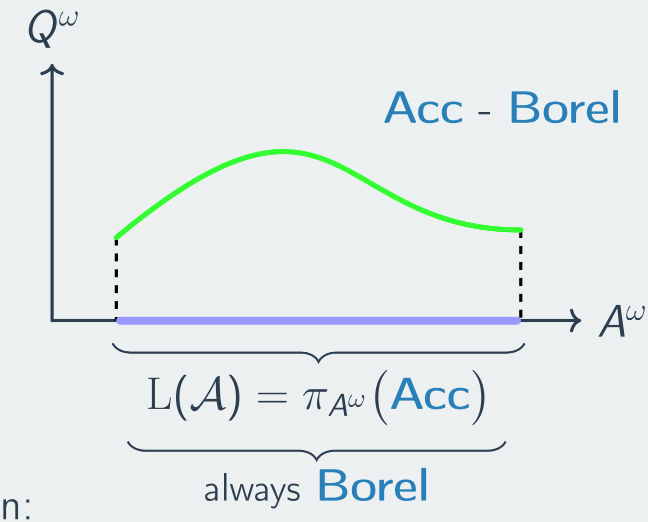

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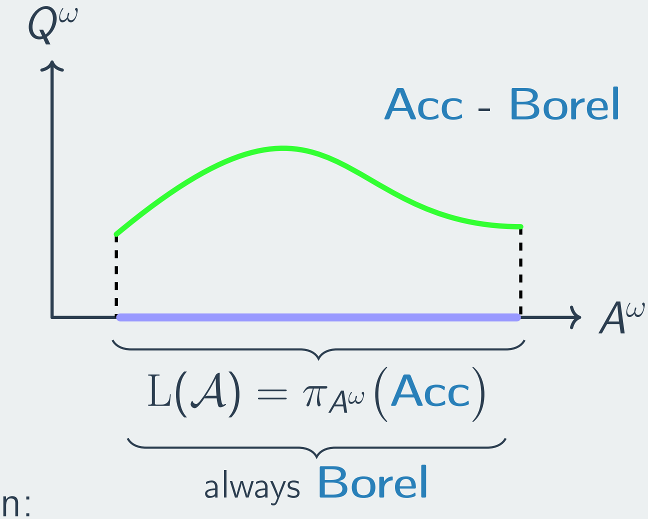

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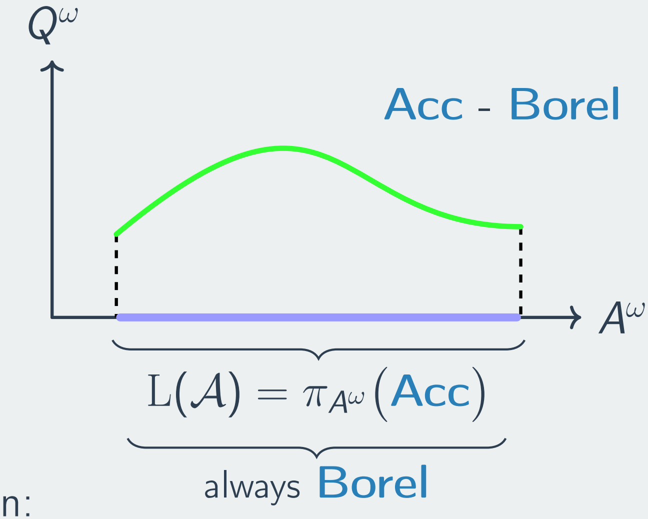

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Unambiguous VASS, when reading $w \in A^*$,

can reach **at most one** configuration per sector in $Q \times \mathbb{N}^C$.

Sectors / clubs of configurations

$$Q \times \mathbb{N}^C$$

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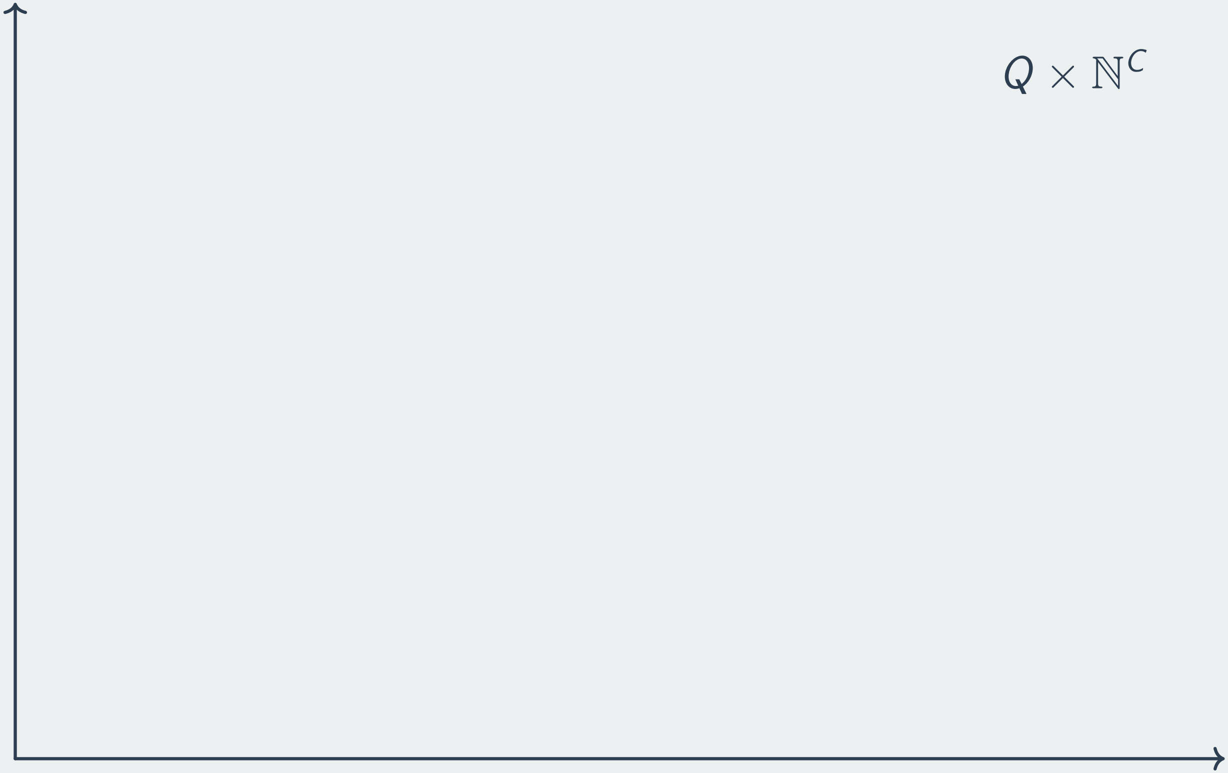

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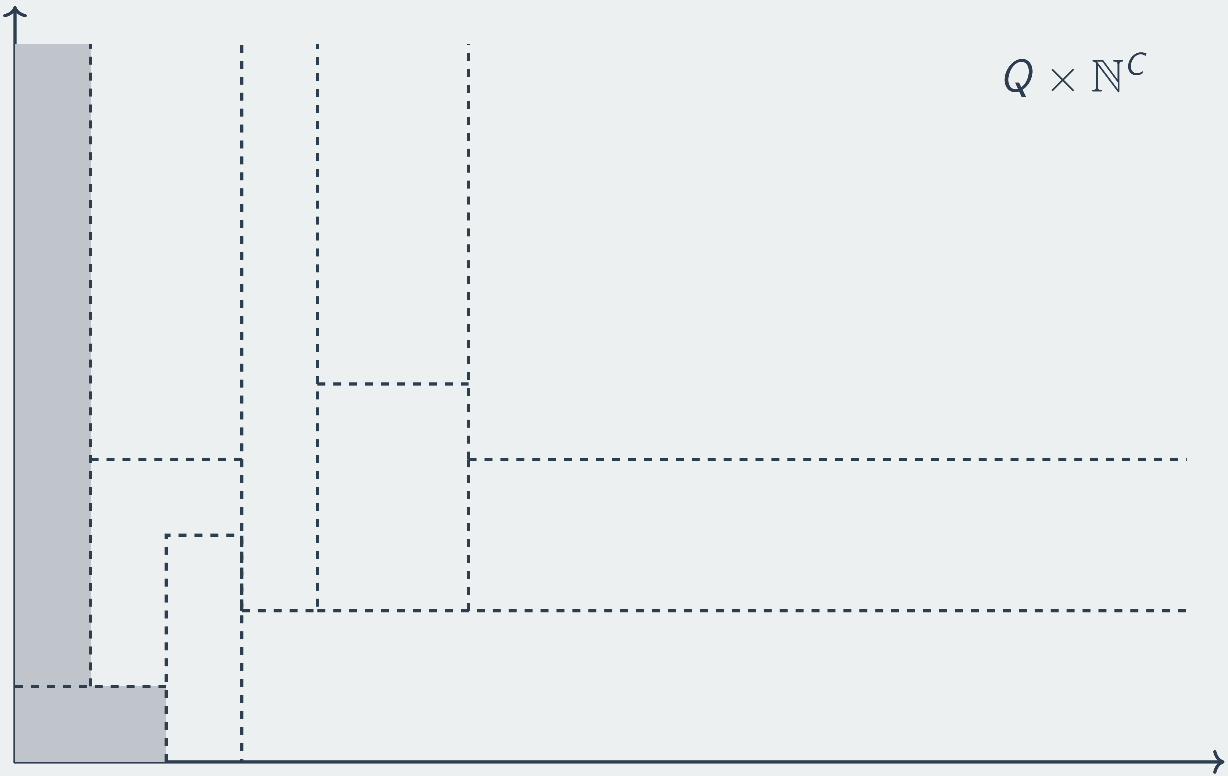

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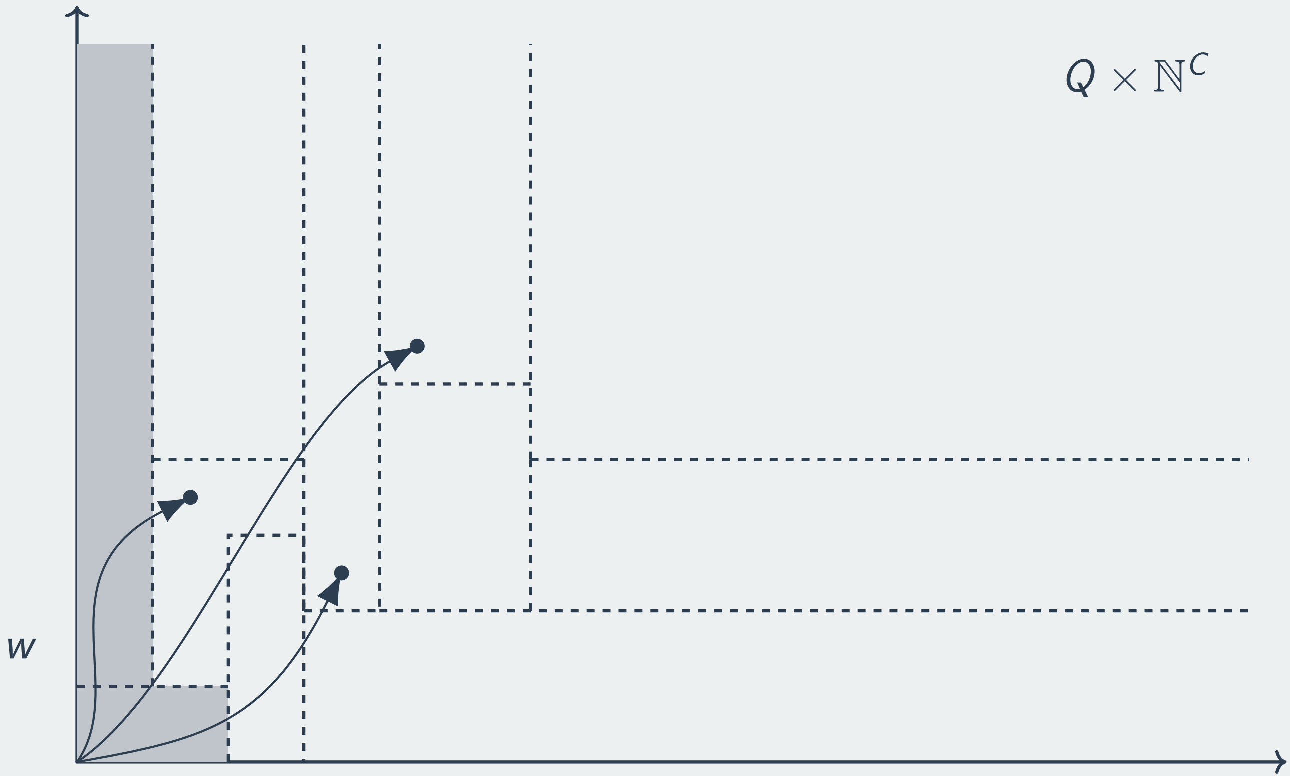

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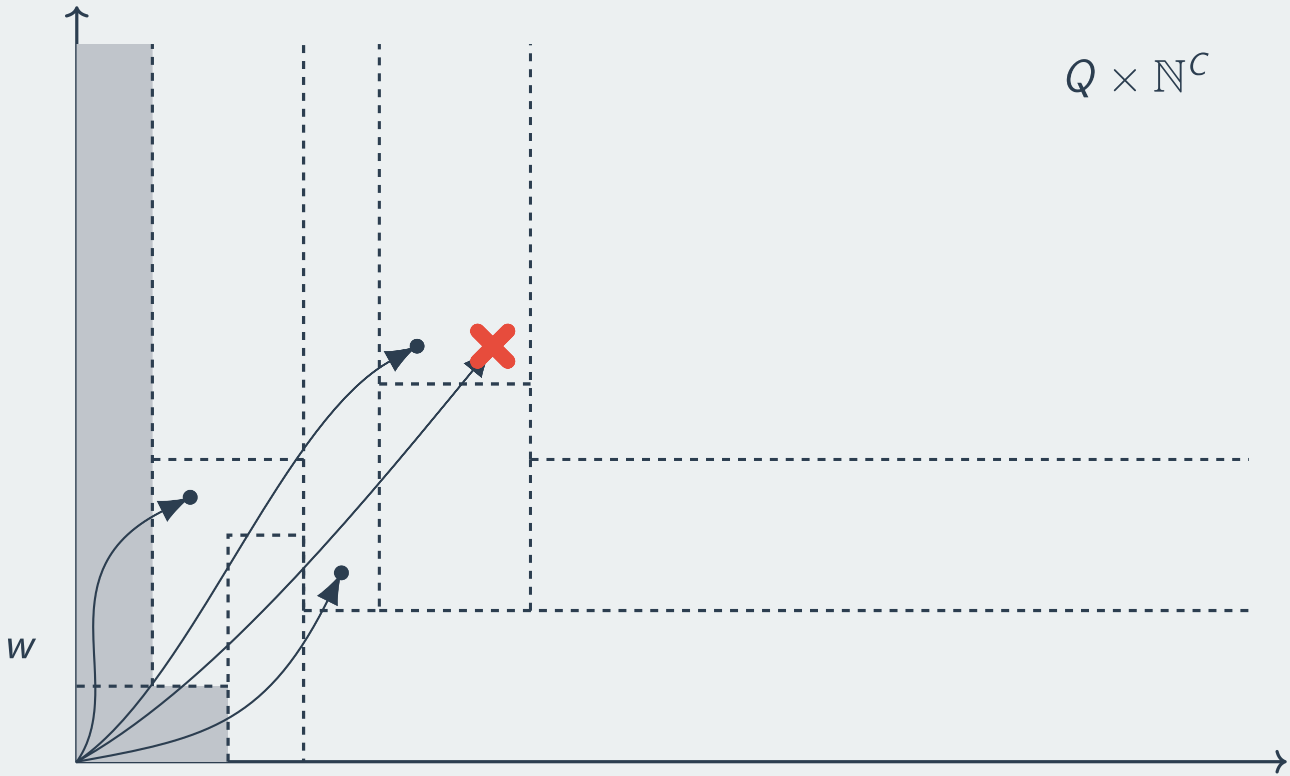

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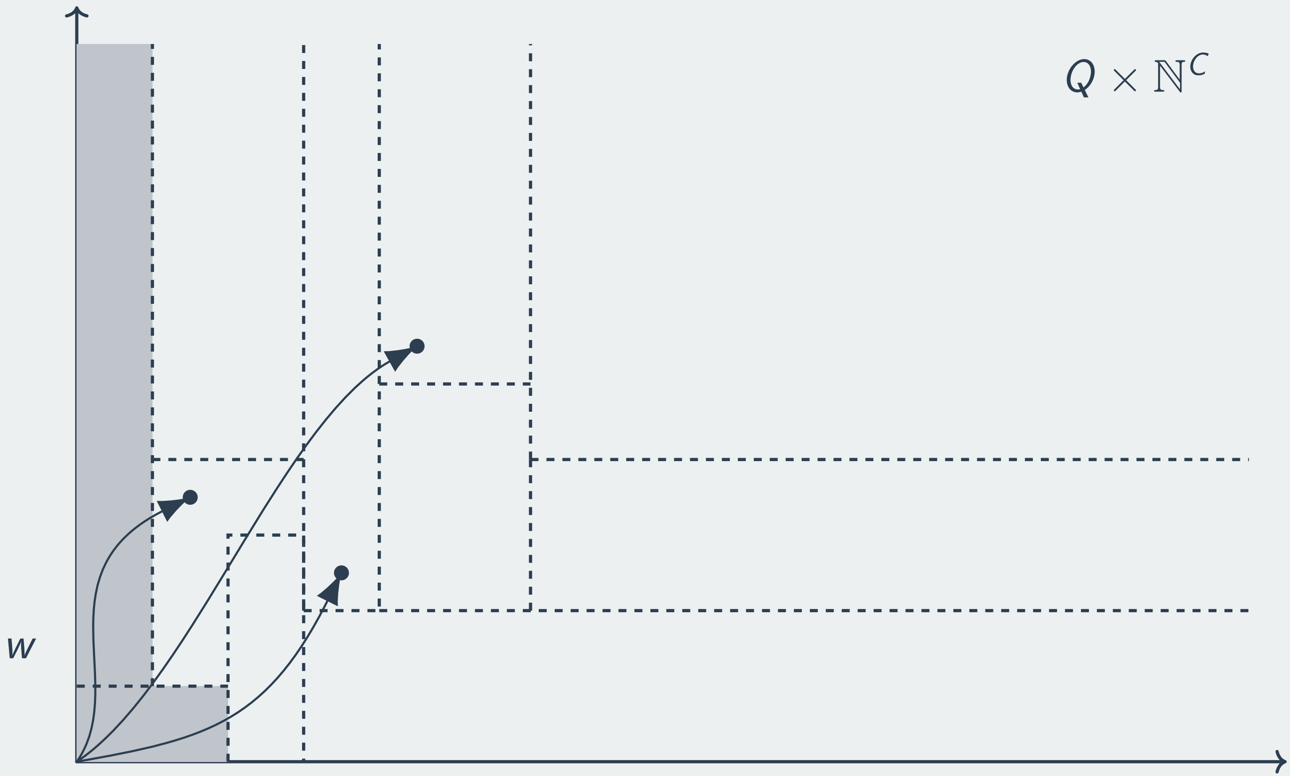

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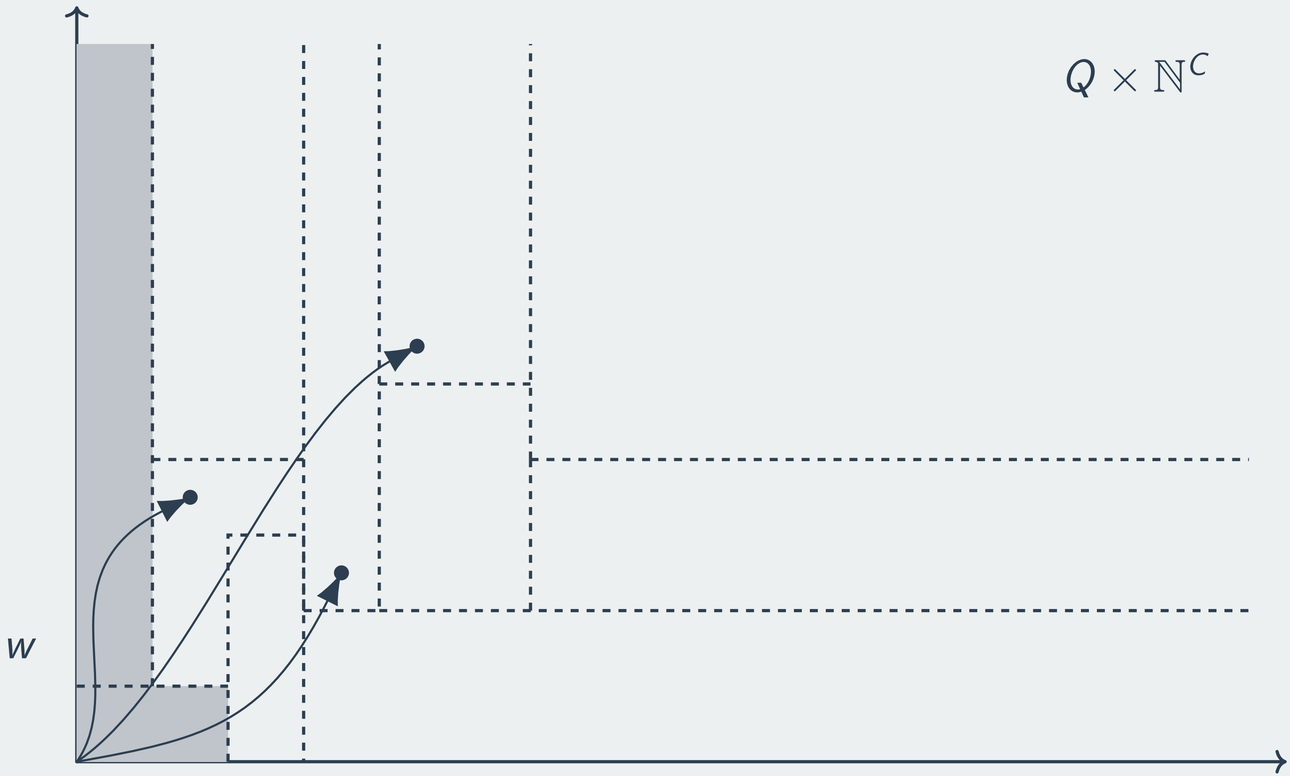

⇝ **finitary** representation of all runs

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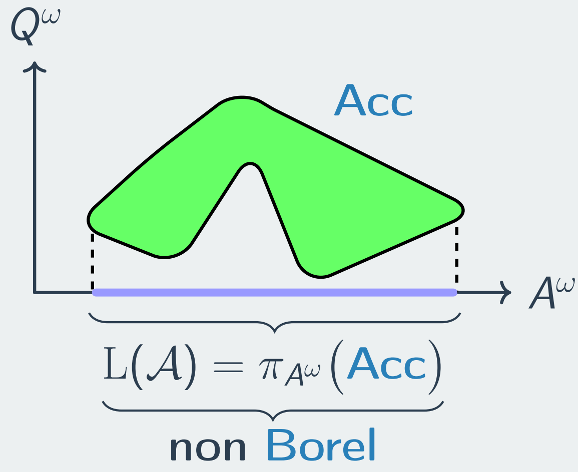
Full non-determinism



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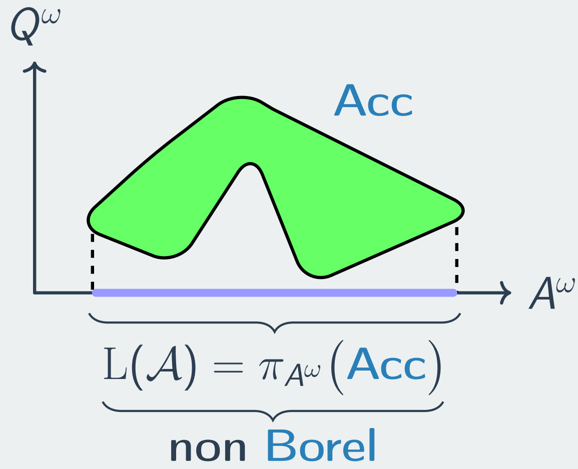

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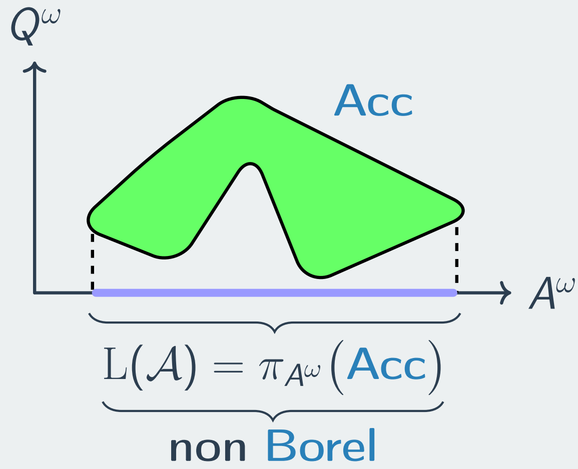

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Büchi VASS:

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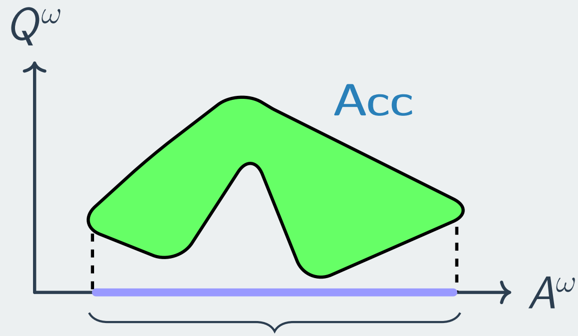

$L(\mathcal{A}) = \pi_{A^\omega}(\text{Acc})$

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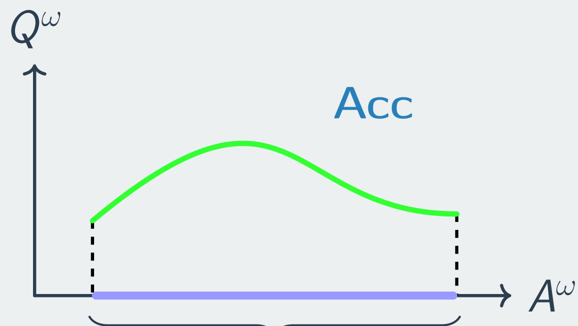

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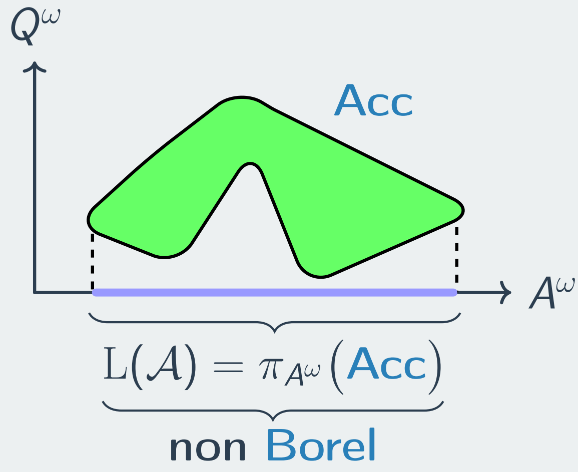

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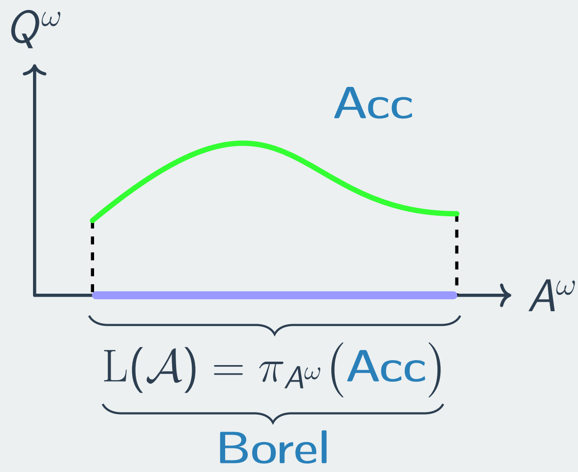

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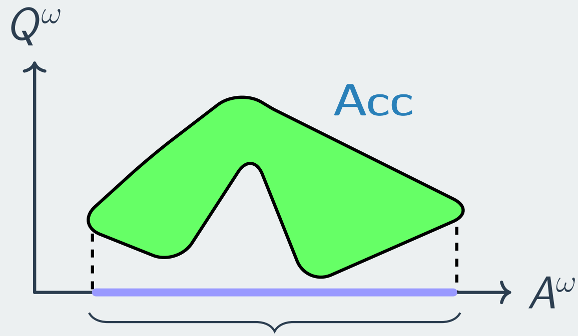

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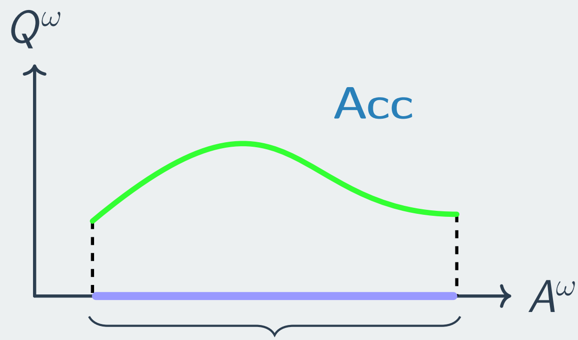

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[combinatorics of **sectors** of configurations]

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