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[ a nice class, closed under **Boolean** operations ]

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$$\forall t' \in \text{Tr}_A. t' \upharpoonright_\tau = t \upharpoonright_\tau \Rightarrow (t \in L \Leftrightarrow t' \in L)$$