Unambiguous languages exhaust the index hierarchy

Michał Skrzypczak

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Foundation for Polish Science

UNIVERSITY OF WARSAW

NATIONAL SCIENCE CENTRE POLAND

Regular languages of **infinite trees**

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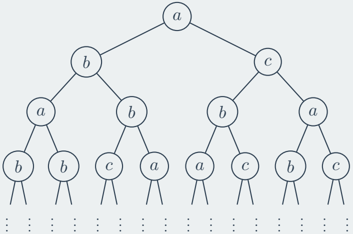

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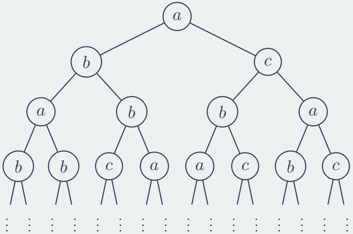

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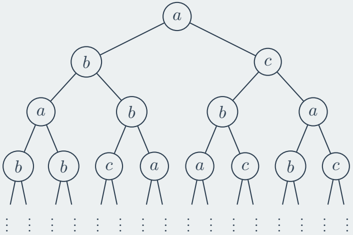

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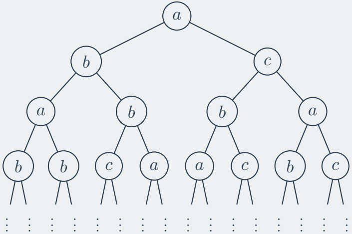

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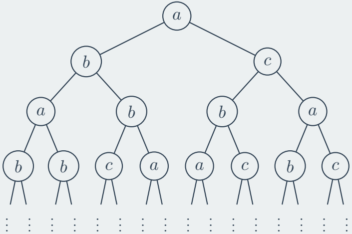

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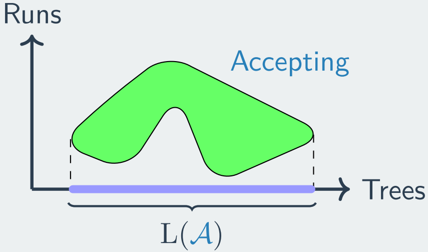

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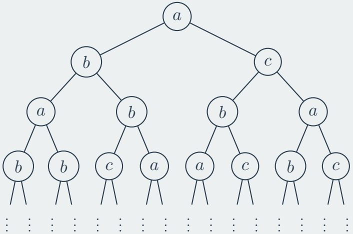

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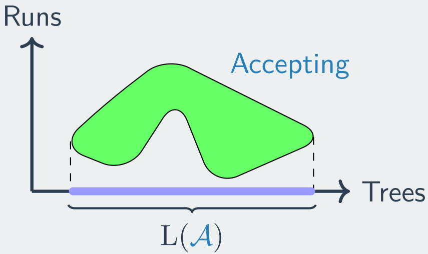

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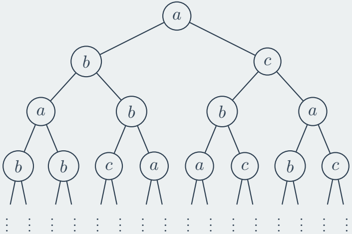

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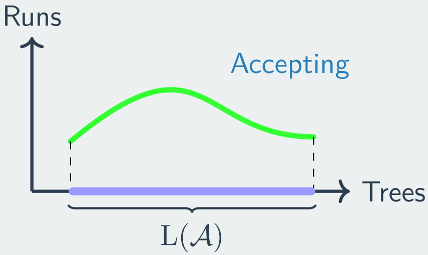

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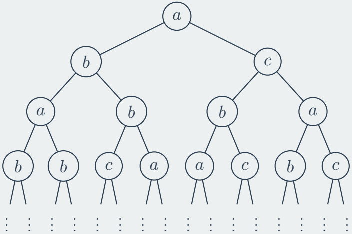

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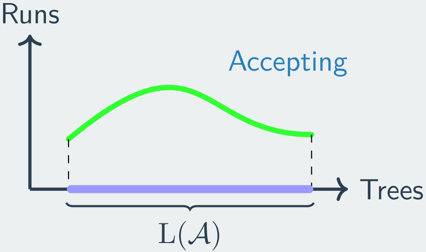

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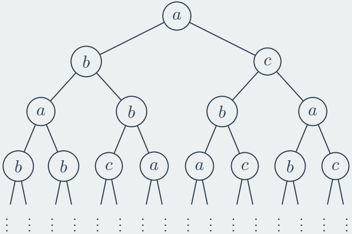

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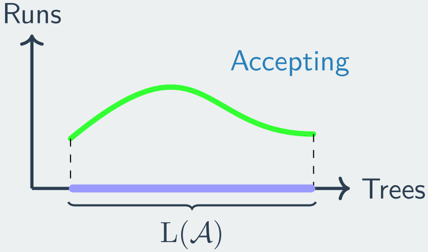

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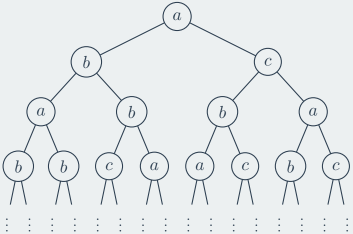

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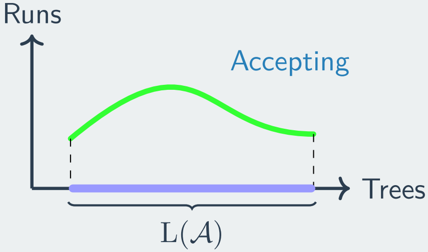

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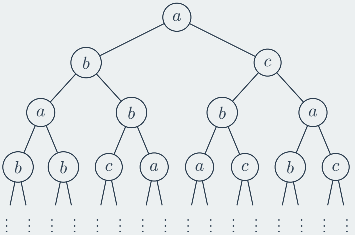

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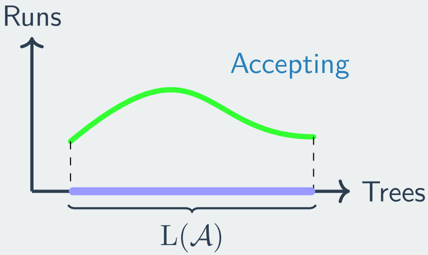

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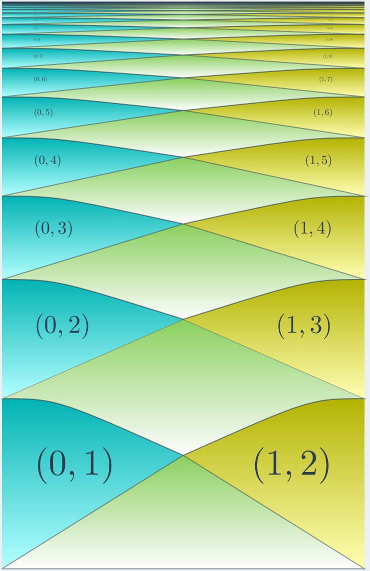

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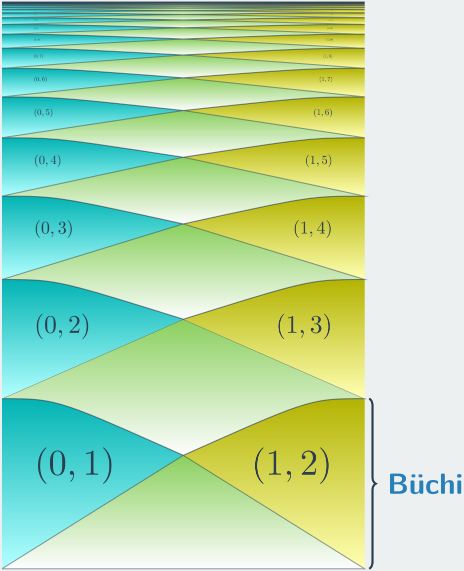

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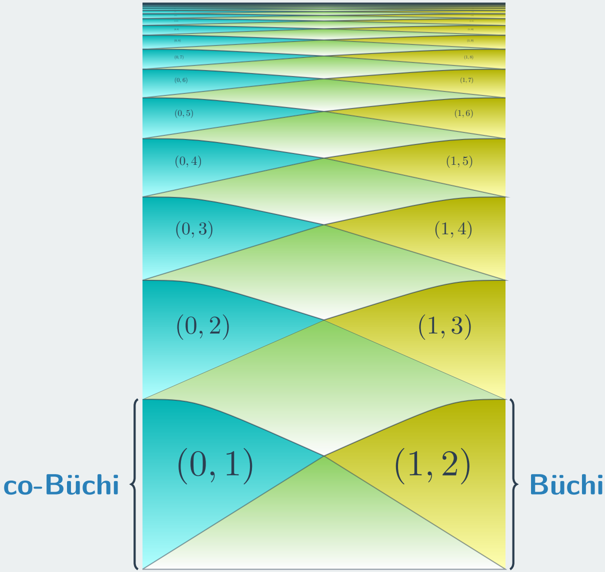

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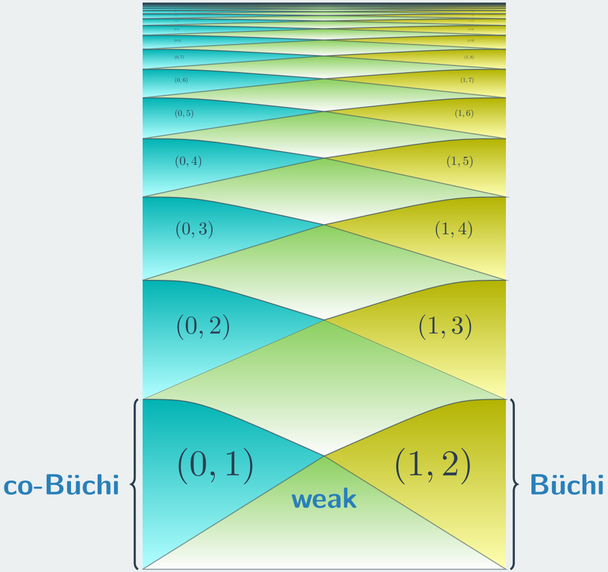

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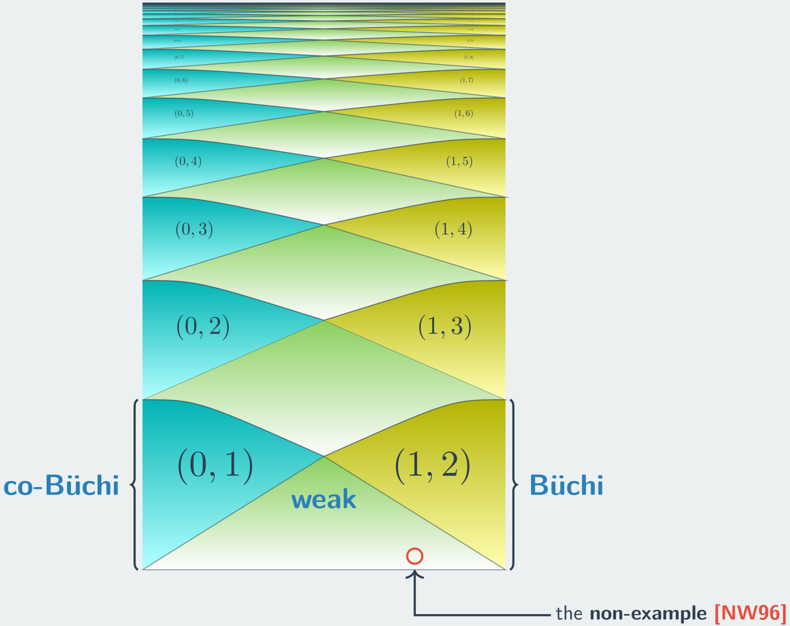

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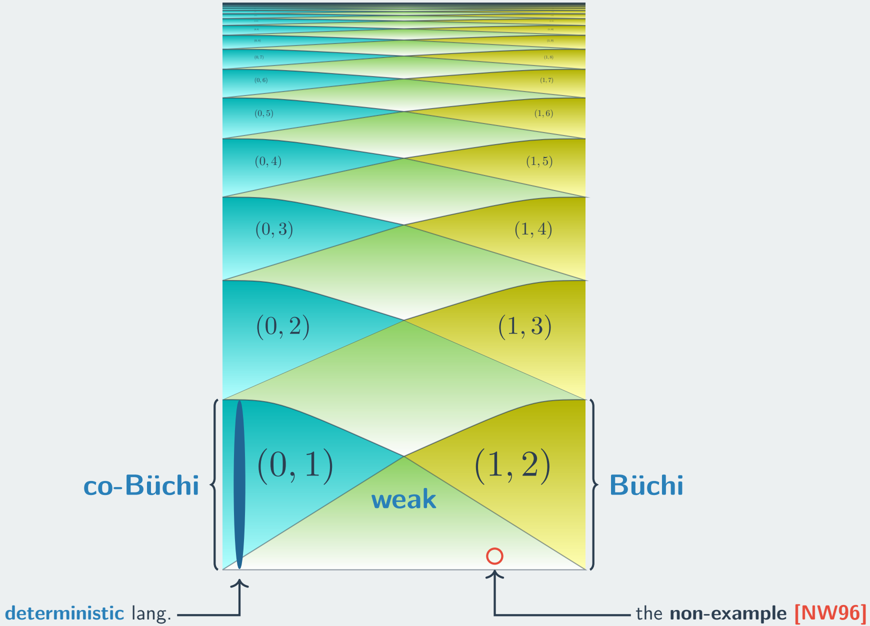

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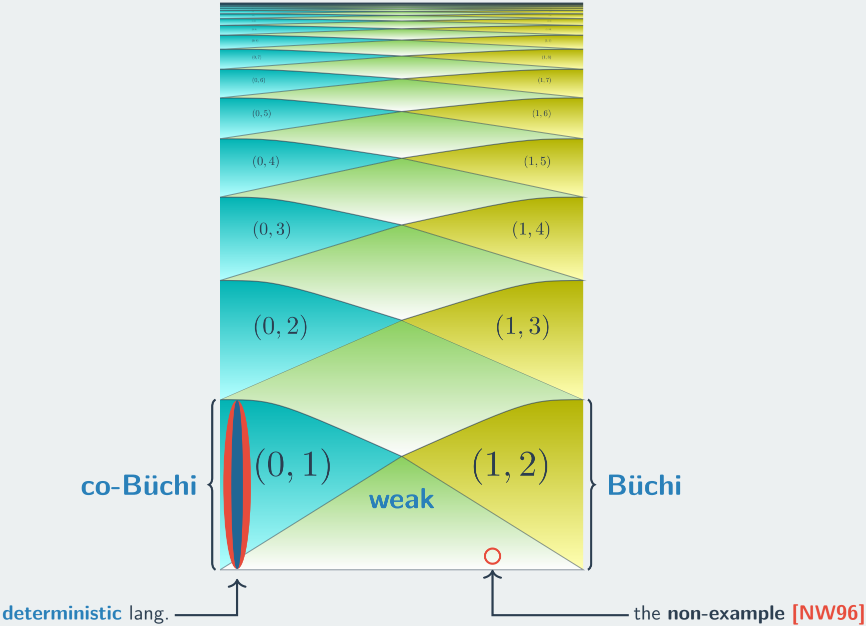

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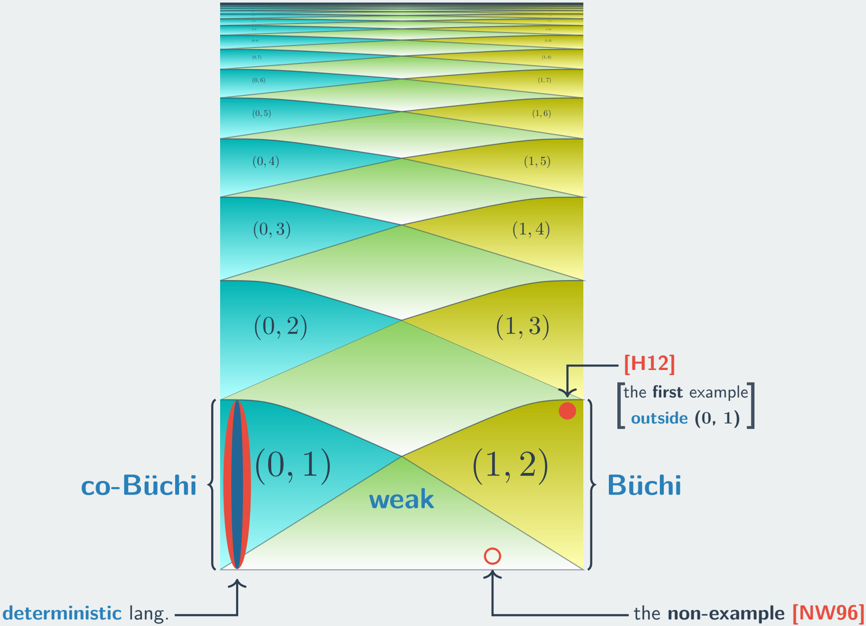

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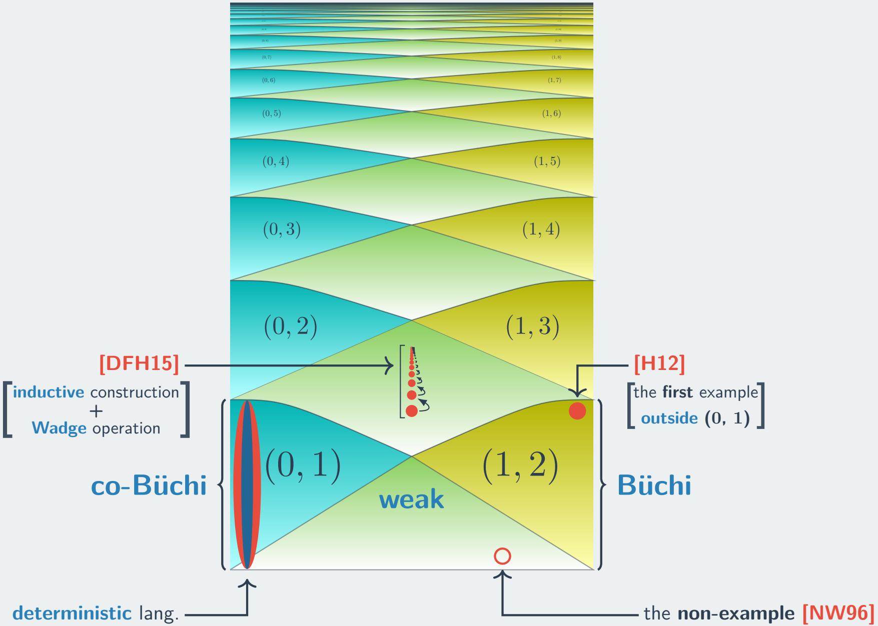

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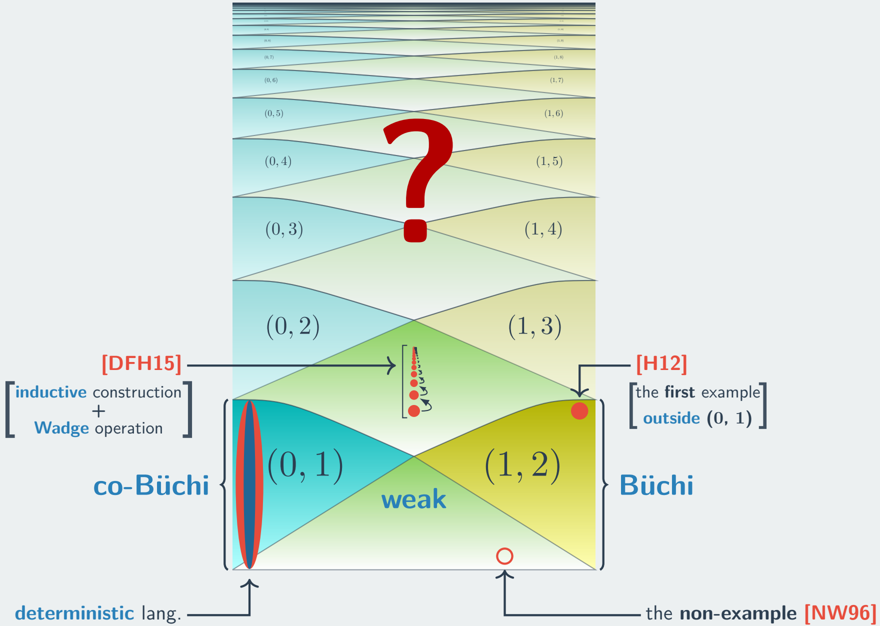

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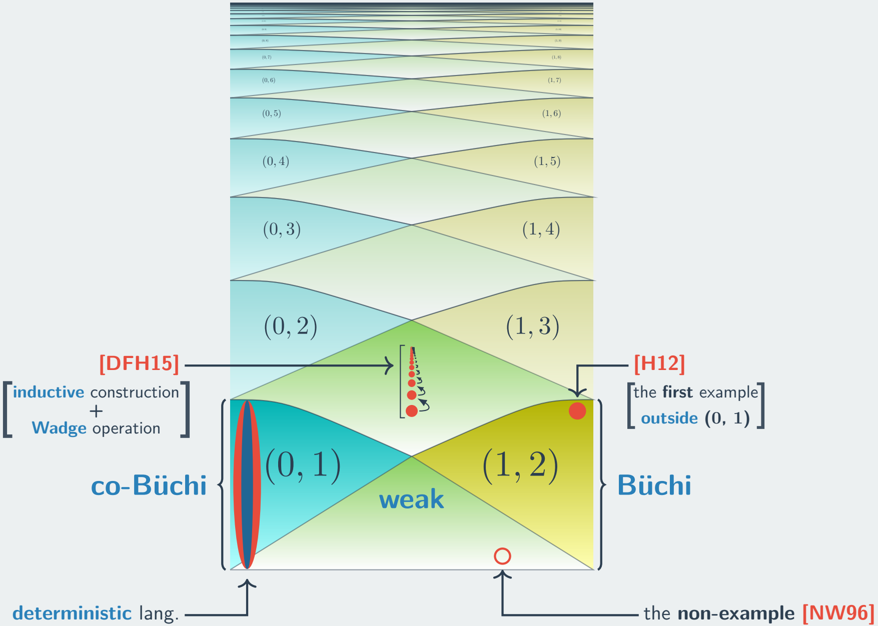

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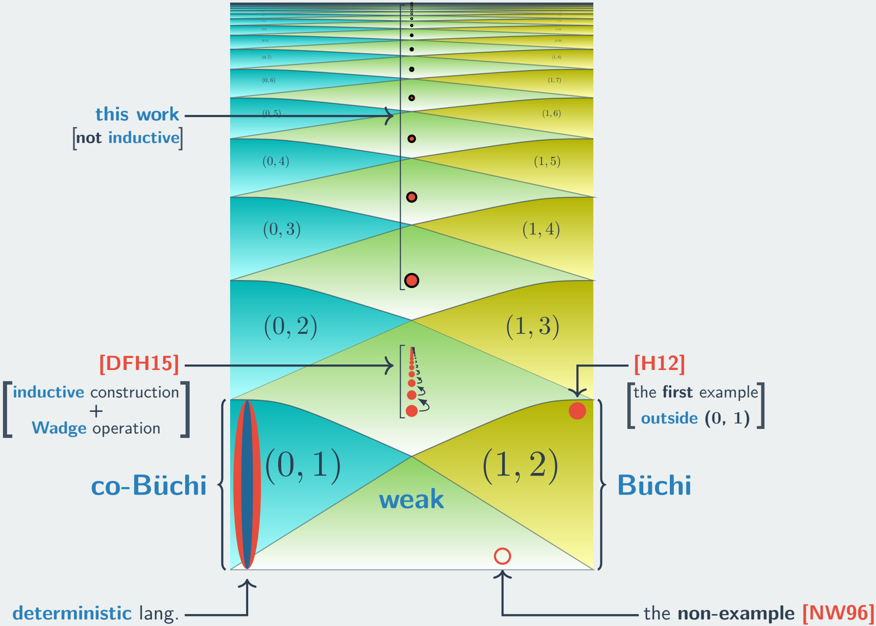

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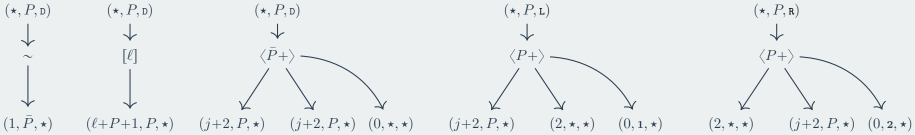

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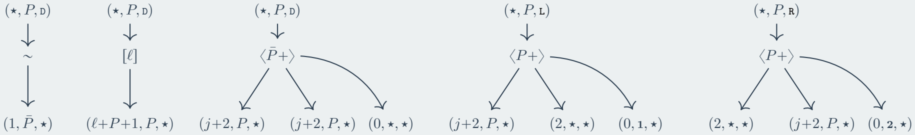

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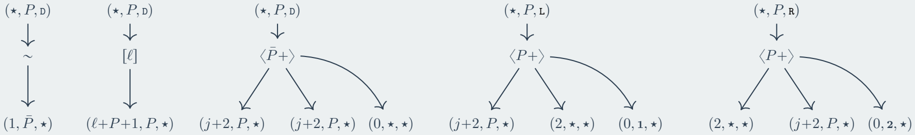

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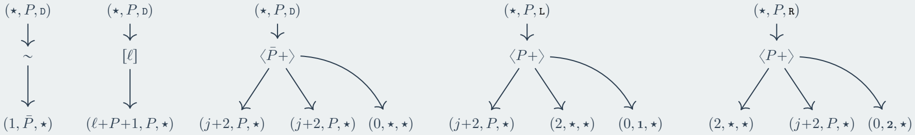

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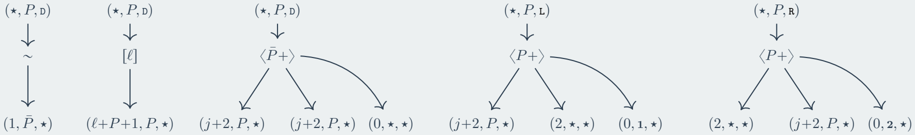

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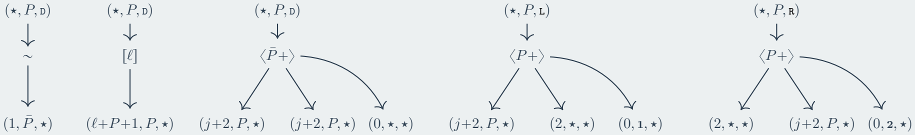

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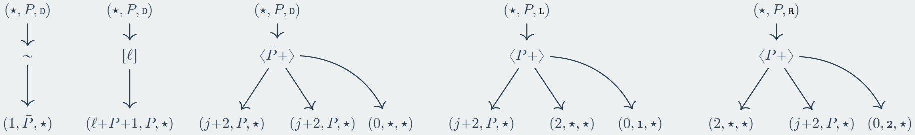

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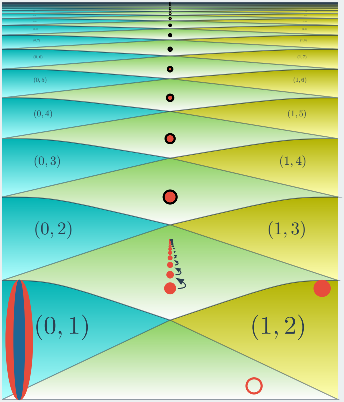

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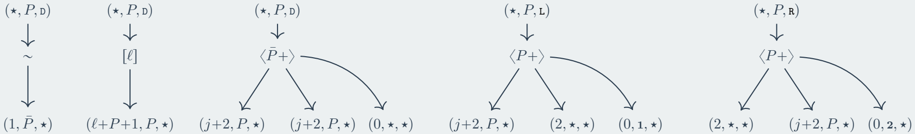

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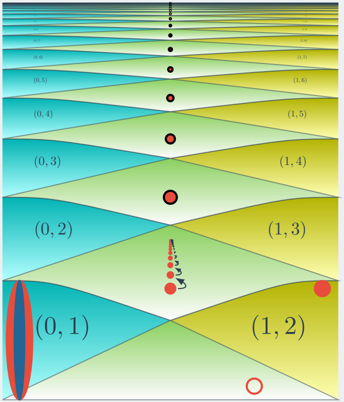

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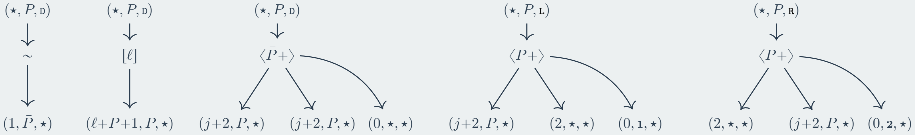

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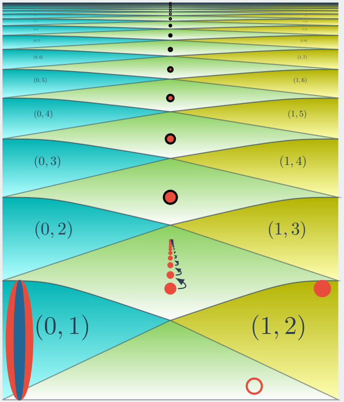

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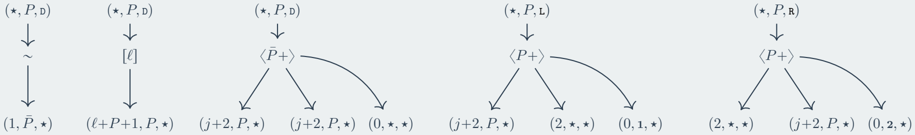

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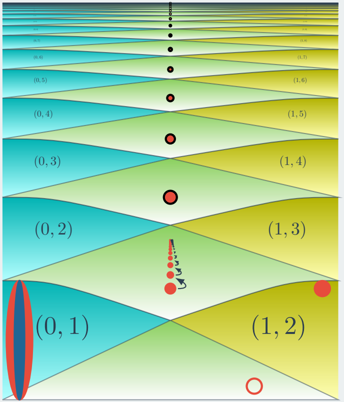

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