Tutorial 9 4.12

Population protocols - example

Deaf Black Ninjas in the Dark (Javier Esparza et al.)

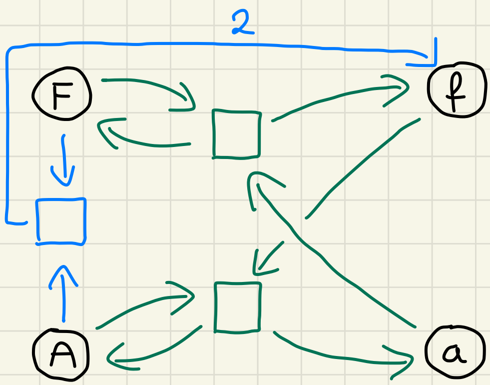
* deaf black ninjas meet at a Zen garden in the dark - they must decide by majority to attack or not (no attack if tie)
* how can they conduct to vote?

* ninjas wander randomly, interacting when they bump into each other
* they store their current estimation of the final outcome: attack (for) / don't attack (against)
* additionaly they are rested or tired (easy to get convinced to other's point of view)
* initially all ninjas are rested (estimation = own vote)
* goal of voting protocol: eventually all ninjas reach the same estimation (corresponding to the majority)

* we need to decide the outcome of each possible meeting (for any two types of ninjas)
* let us model this situation using a binary Petri net with the set of places

rested { For, Against }   tired { for, against }

* first, consider the following meeting outcomes



Fa $\rightsquigarrow$ Ff

Af $\rightsquigarrow$ Aa

FA $\rightsquigarrow$ ff

in other words, each rested ninja convinces tired ninjas they meet to their point of view, while two rested ninjas: one for, one against, become tired and for the attack

* however, there is a problem ...

Some theory recap - binary Petri nets

$I \subseteq P$ $\qquad\qquad$ $I = \{A, F\}$

set of places used in the initial marking

set of all places

Final outcome function

$O : P \to \{0, 1\}$

$F, f \rightsquigarrow 1 \qquad A, a \rightsquigarrow 0$

Configurations

$\mathbb{N}^P$ - the number of tokens at each place

$\mathbb{N}^I = \{ M \in \mathbb{N}^P : \forall p \in P \setminus I \ \ M(p) = 0 \}$

set of initial configurations

Runs

$M_0 \to M_1 \to M_2 \to \dots$ infinite

$$O(M) = \begin{cases} b & \text{if } \forall p \in P \ \ M(p) > 0 \Rightarrow O(p) = b \\ \bot & \text{otherwise} \end{cases}$$

final outcome of a net with configuration M

Fair runs

$\forall M \to M'.\ \{i : M_i = M\}$ is infinite

$\Rightarrow \{i : M_i = M,\ M_{i+1} = M'\}$ is infinite

$\Updownarrow$

$\forall M'.\ \{i : M_i \to M'\}$ is infinite

$\Rightarrow \{i : M_i = M'\}$ is infinite

Run stabilizes to b if $\exists k\ \forall i > k\ O(M_i) = b$.

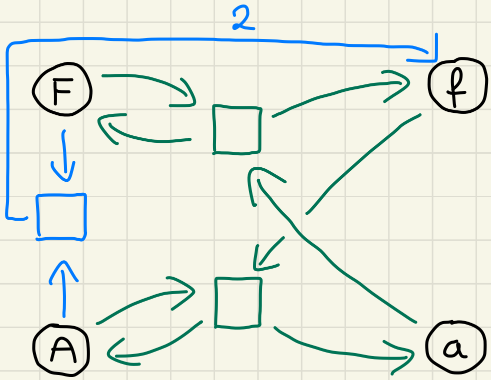
Protocol computes a predicate $\varphi : \mathbb{N}^I \to \{0, 1\}$ if every fair run stabilizes to $\varphi(M_0)$.

Protocol is well-specified if it computes some predicate.

Alternative approach – probabilistic semantics

Protocol computes φ if $\forall M_0 \in \mathbb{N}^I$ a run starting at M_0 stabilizes to $\varphi(M_0)$ with probability 1

* now, back to the example



$Fa \rightsquigarrow Ff$

$Af \rightsquigarrow Aa$

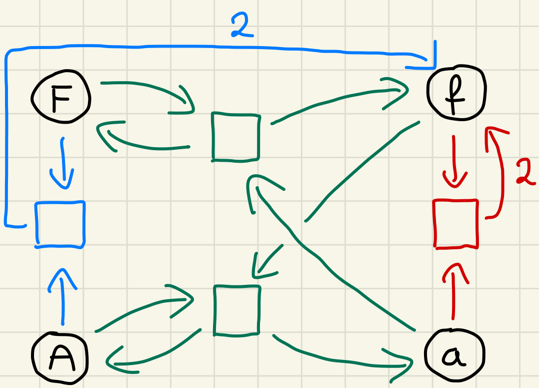
$FA \rightsquigarrow ff$

* the problem here occurs as we look at the following run

$$FFAA \to ffFA \to faFA \to faff$$

it doesn't stabilize, so we cannot use this protocol to decide whether to attack or not

* a quick fix



$Fa \rightsquigarrow Ff$

$Af \rightsquigarrow Aa$

$FA \rightsquigarrow ff$

$fa \rightsquigarrow ff$

1. One can check that the protocol on the previous page works, however, it still has some disadvantages. For instance, the simulations show that

- if the number of ninjas is chosen uniformly at random between 10 and 15, and we allow the simulation to run for at most 500 steps, then the protocol does not reach a consensus in about 25% of the runs on average,
- such a problem happens only for the case in which the initial number of A-ninjas is slightly above the number of F-ninjas (then we reach a situation were we have one A-ninja and all the other ninjas are f-ninja: A turns f to a, but f reverse it...),

- for the initial configuration of 8 A-ninjas and 7 F-ninjas reaching the consensus takes on average 9 072 494 steps, which means that dawn arrives long before consensus.

Thus, the question arises of how to improve this protocol?

* since ties are the source of all problems, we should deal with them explicitly
* consider having a third option for each ninja's outcome expectation being a tie T
* now, when two active ninjas meet, only one of them becomes passive, and both change their expectations in the natural way
* still, passive ninjas adopt the expectation of active ninjas

* formally, we consider the following meeting outcomes

$$FA \to Tt \qquad \forall \alpha \in \{f, a, t\}$$

$$FT \to Ff \qquad F\alpha \to Ff$$

$$AT \to Aa \qquad A\alpha \to Aa$$

$$TT \to Tt \qquad T\alpha \to Tt$$

and set $O(T) = O(t) = 0$ (tie = no attack)

* notice that the last transition: $T\alpha \to Tt$ is useful only in the case of a tie, for instance, we need to be able to go through the following run

$$FAfff \to TTfff \to \ldots \to Ttttt$$

* for such a protocol, in simulations with up to 15 ninjas, consensus never takes more than 170 steps

2. Prove that all semi-linear predicates can be computed by population protocols.