A subexponential parameterized algorithm for Proper Interval Completion

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(Proper) interval graphs

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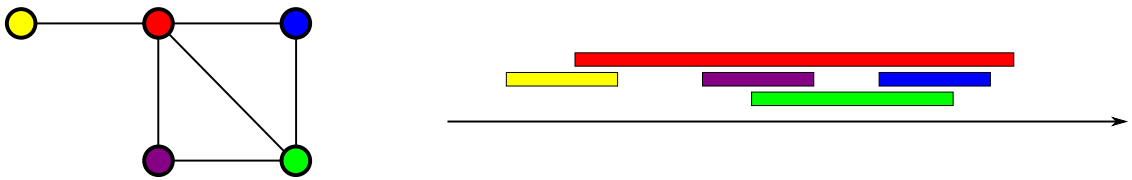

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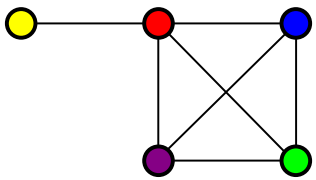

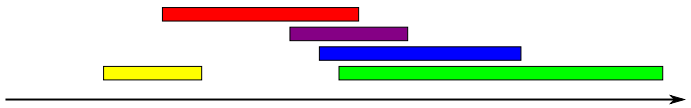

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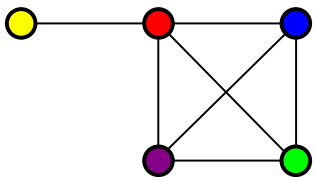

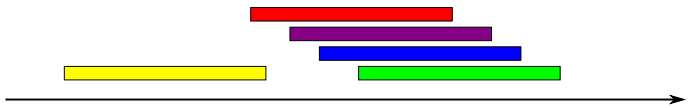

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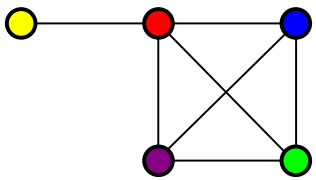

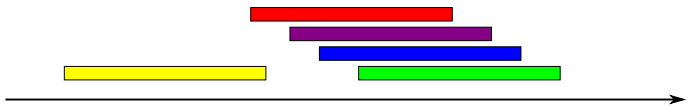

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- **Related paper**: the same about IC

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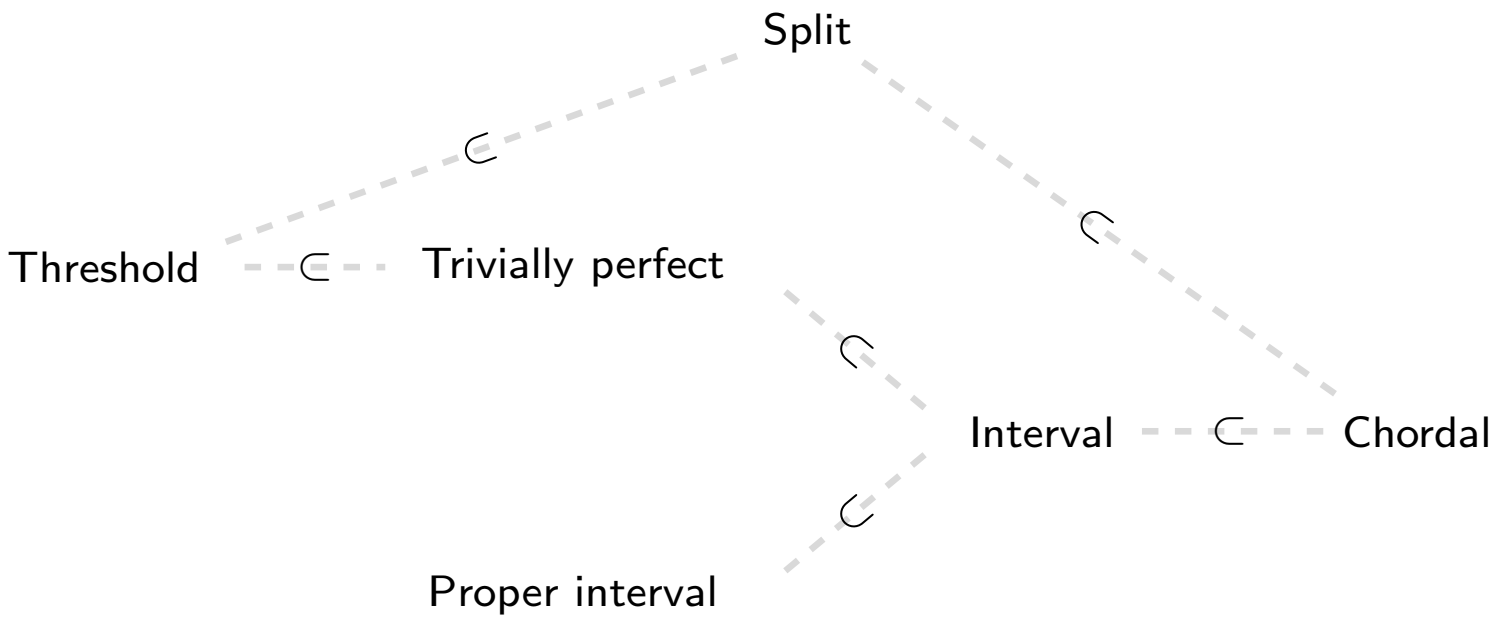
- Do other completion problems admit subexponential parameterized algorithms?

Subexponential algorithms



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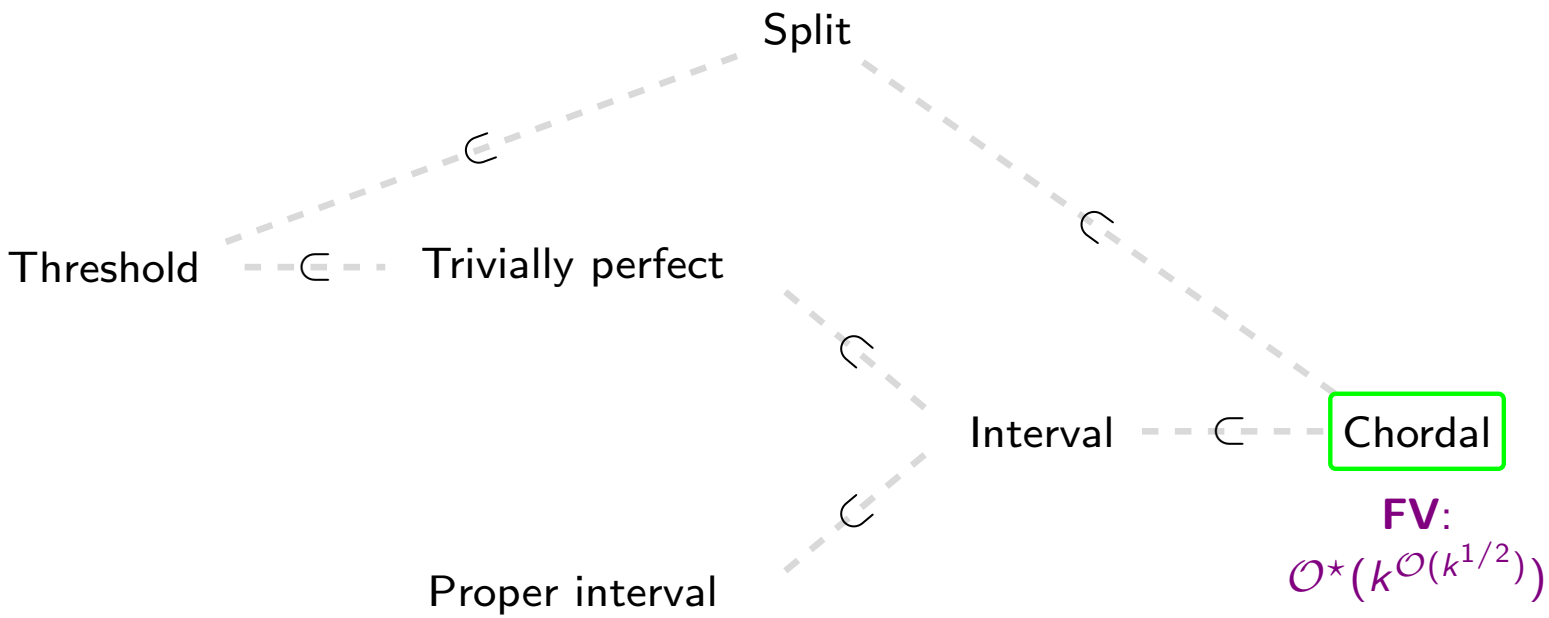

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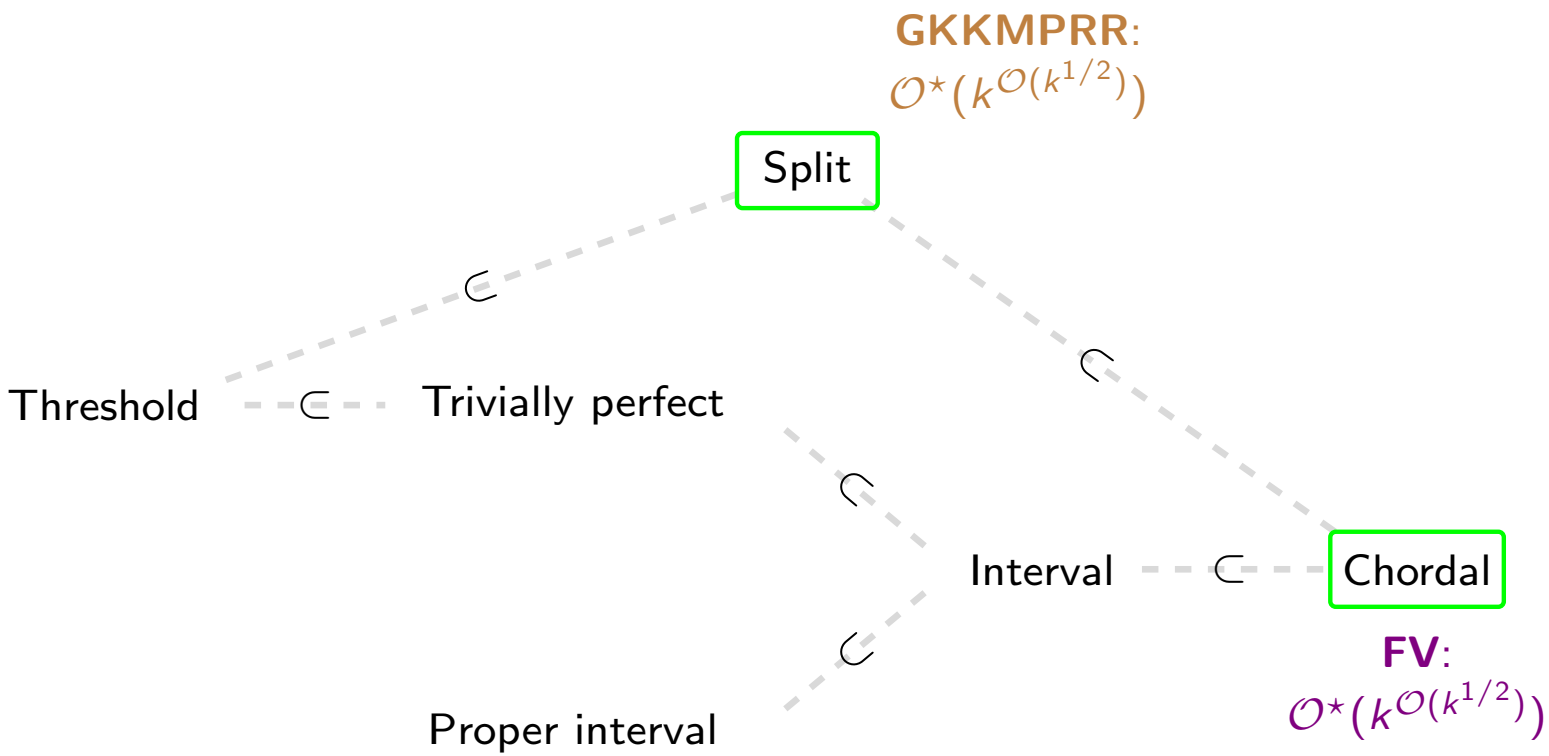

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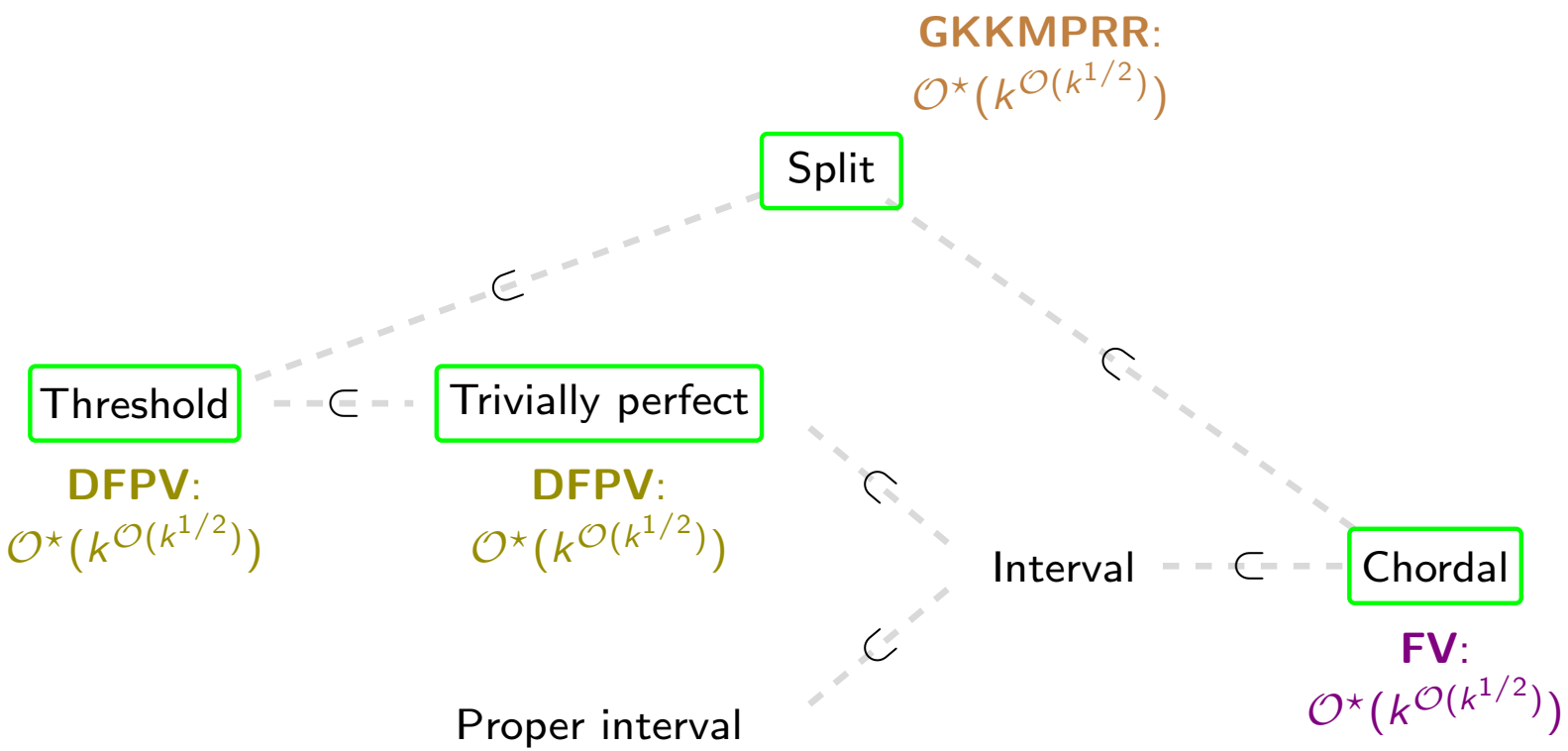

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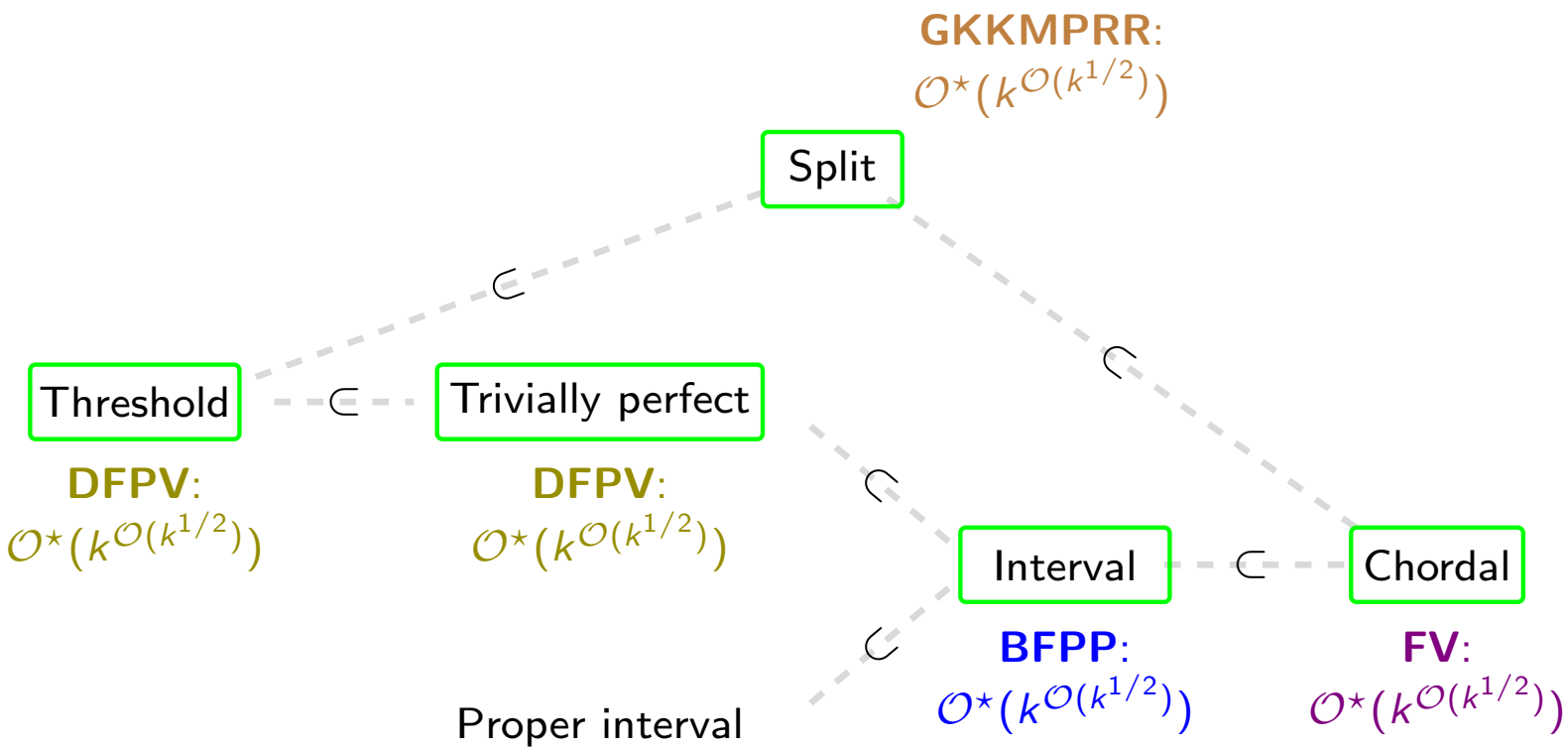

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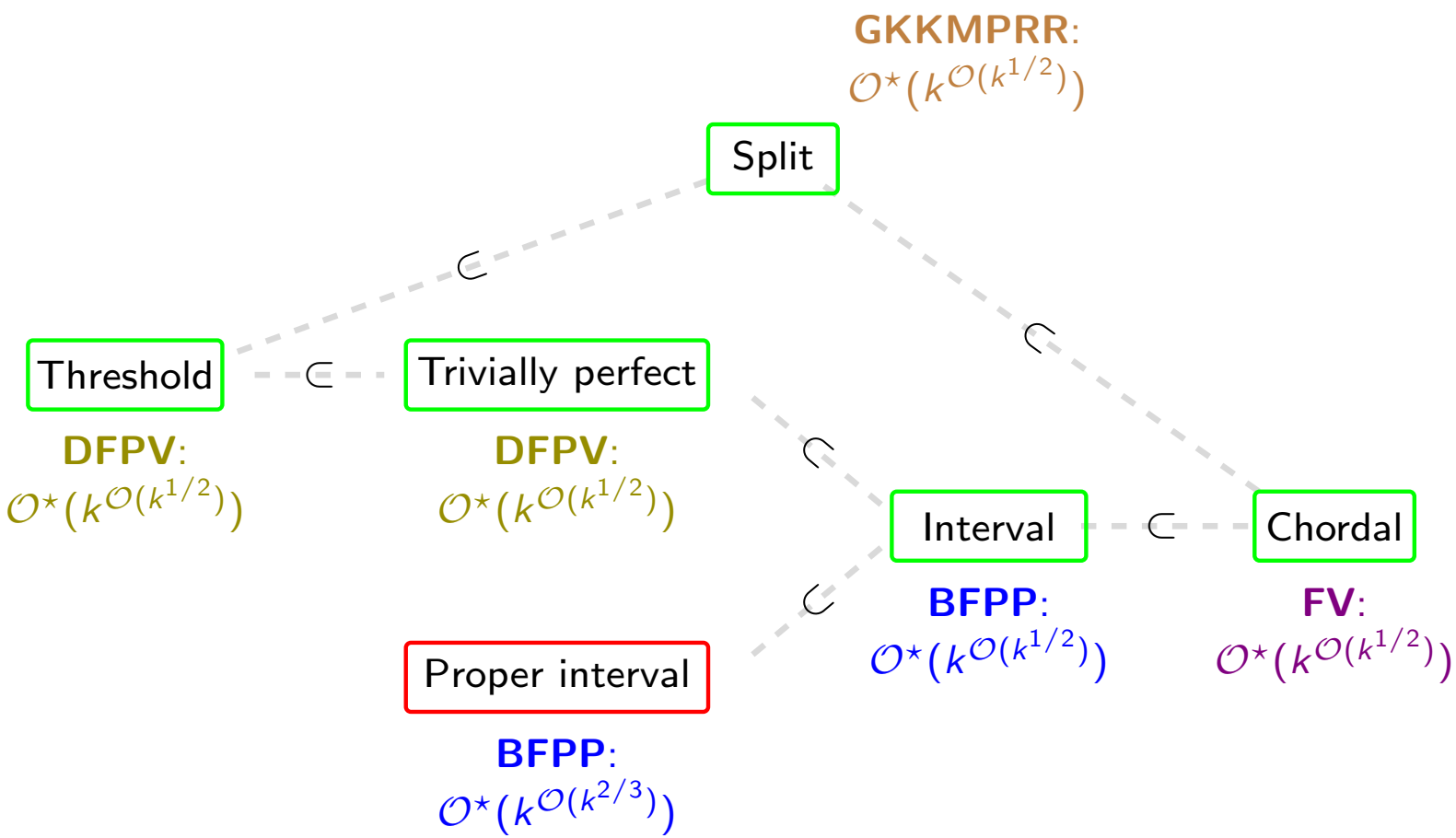

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- Examples: C_4-free Deletion, C_4-free Completion, Trivially Perfect Deletion, Cograph Completion... (DFPV).
- Essentially, the presented completion problems are singular cases for which subexponential parameterized algorithms are possible.

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- A chordal graph has $\leq n + 1$ maximal cliques.
- **Idea**: A graph that lacks k edges to being a chordal graph, has $\leq k^{\mathcal{O}(k^{1/2})} \cdot n^{\mathcal{O}(1)}$ sets that can become a clique after completion (*potential maximal cliques*).

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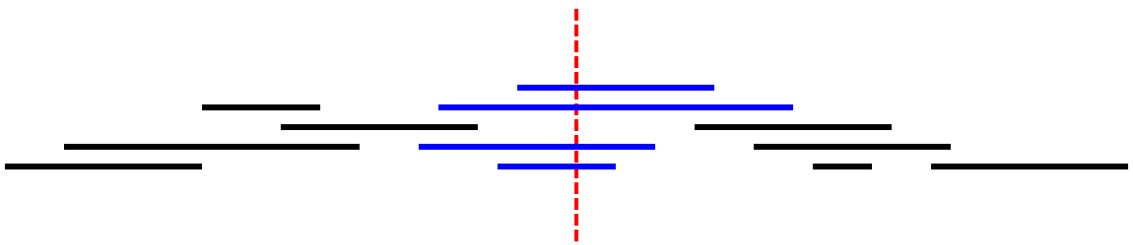
First try for Interval Completion

- **Section**: a set of intervals pinpointed by a vertical line.



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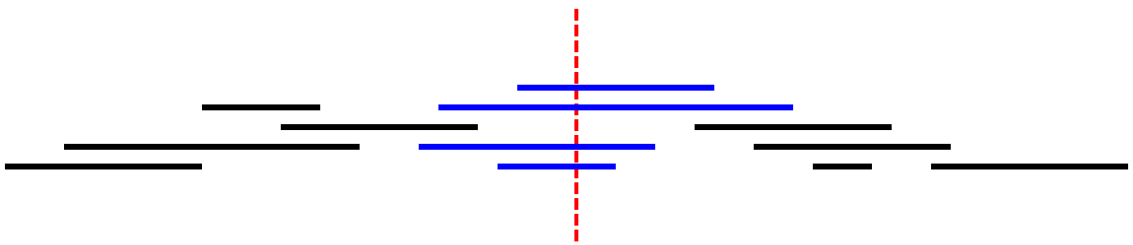

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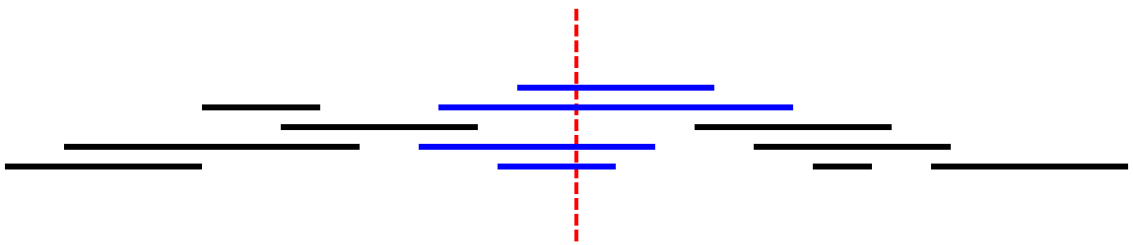

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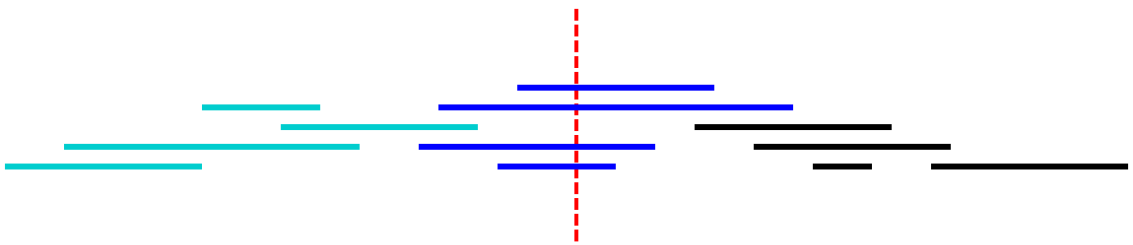

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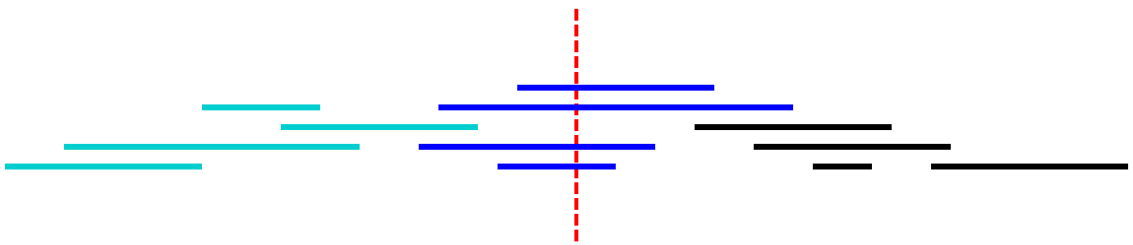

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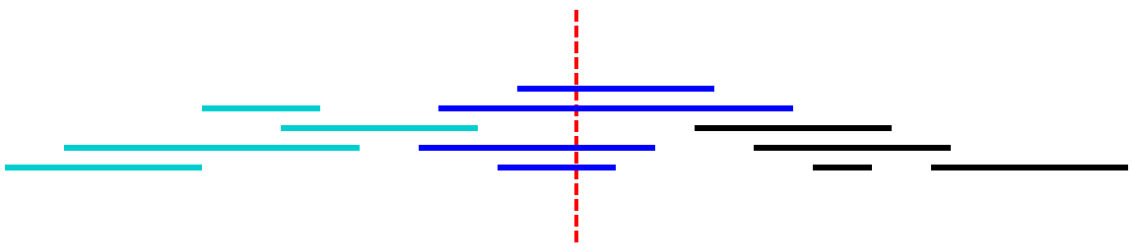
- If we had a family $\mathcal{N}$ capturing all the relevant states, then the DP computes the optimum solution in time $\mathrm{poly}(|\mathcal{N}|)$.



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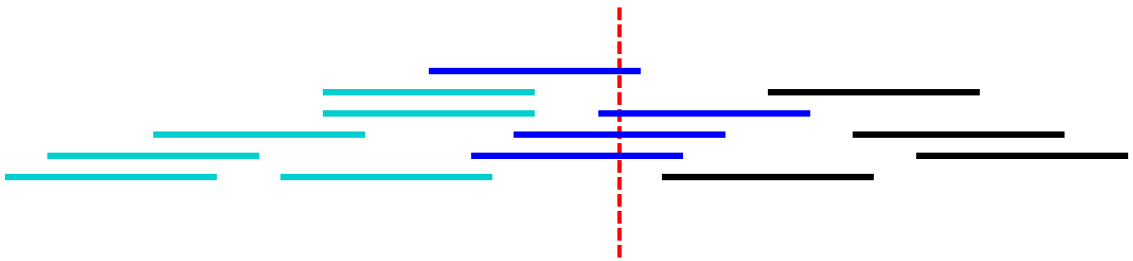

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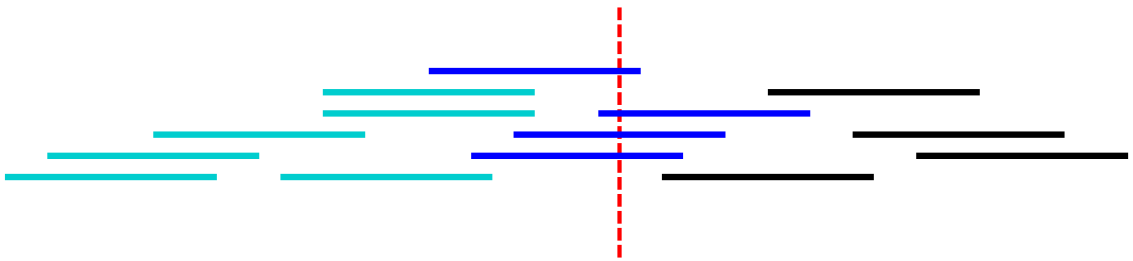

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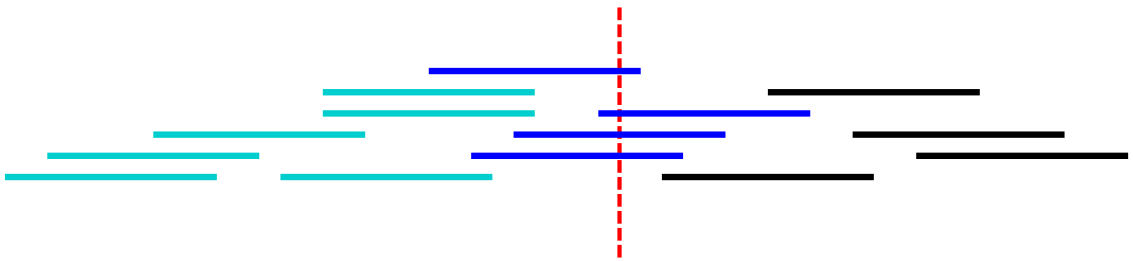

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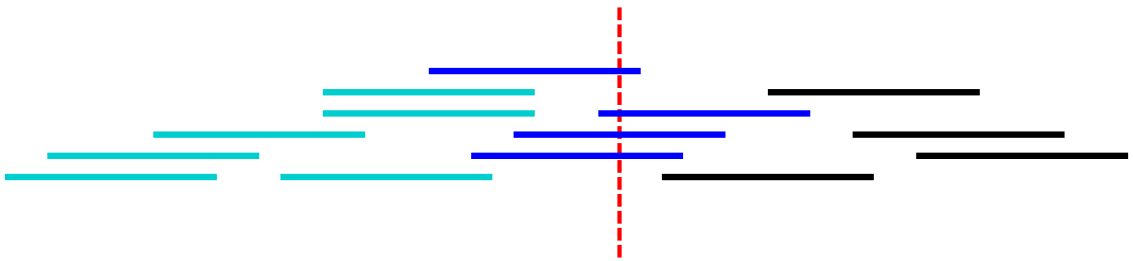

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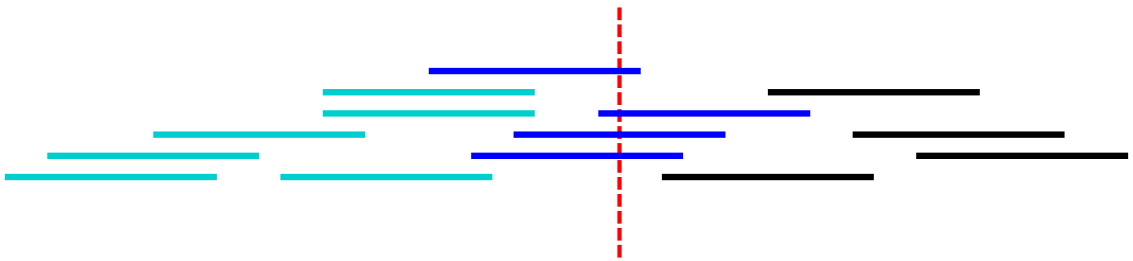
- Having all possible orderings is too expensive.



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  - We are not able to enumerate $2^{o(k)}$ candidates for the ordering.

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- Provided that expensive vertices are guessed, there is $k^{\mathcal{O}(\tau)} = k^{\mathcal{O}(k^{1/3})}$ candidates for sections and left/right.

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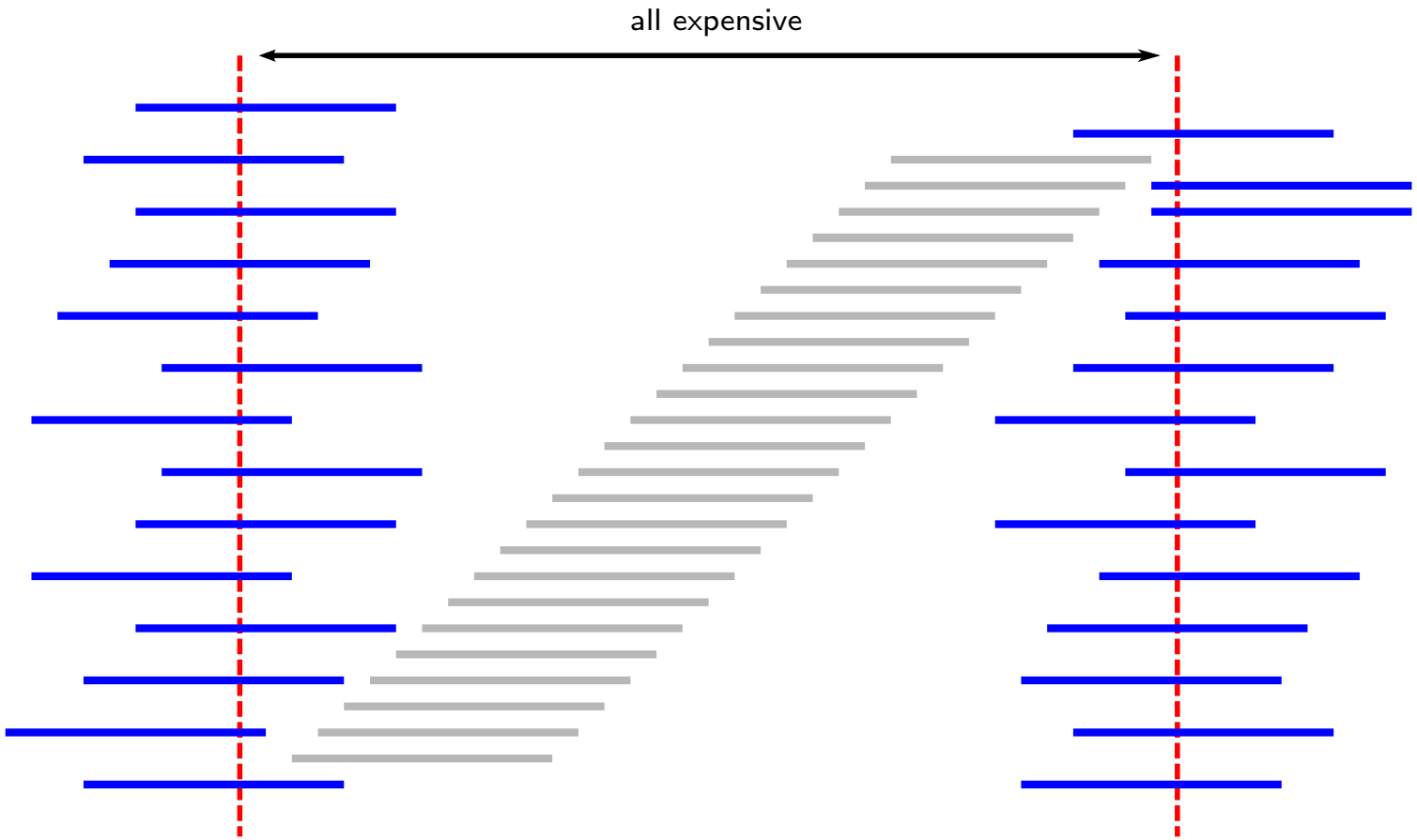

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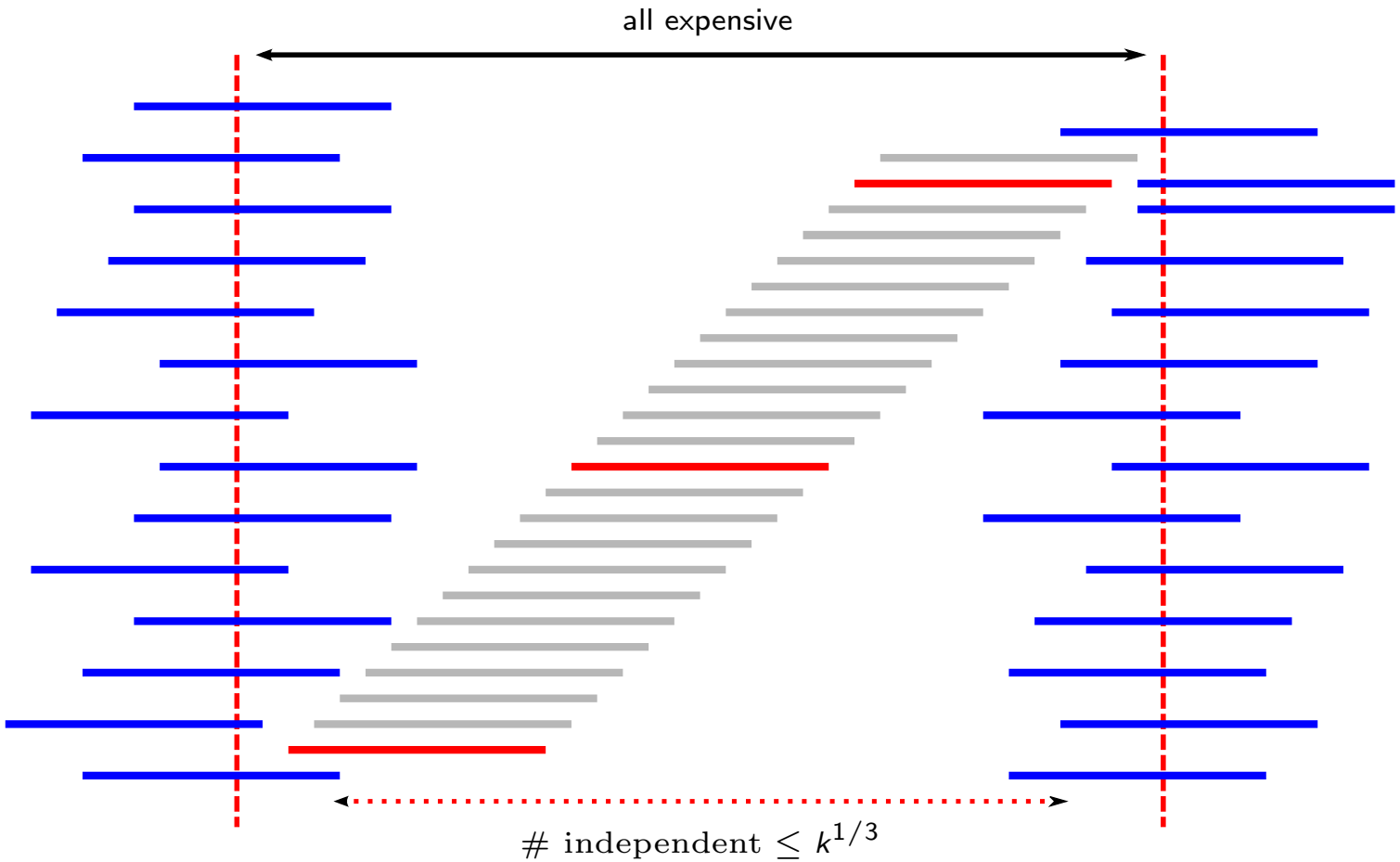

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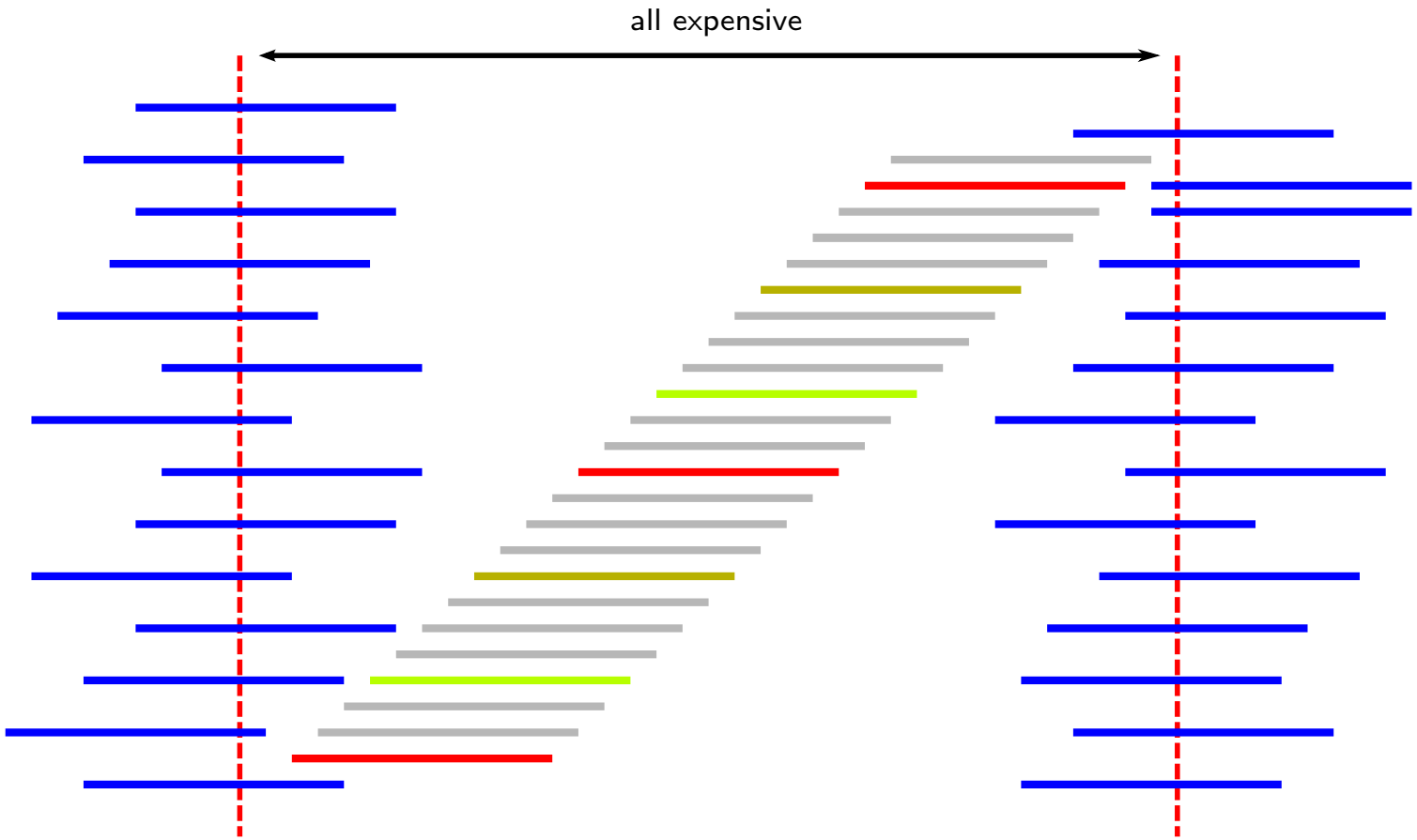

chains are DP states

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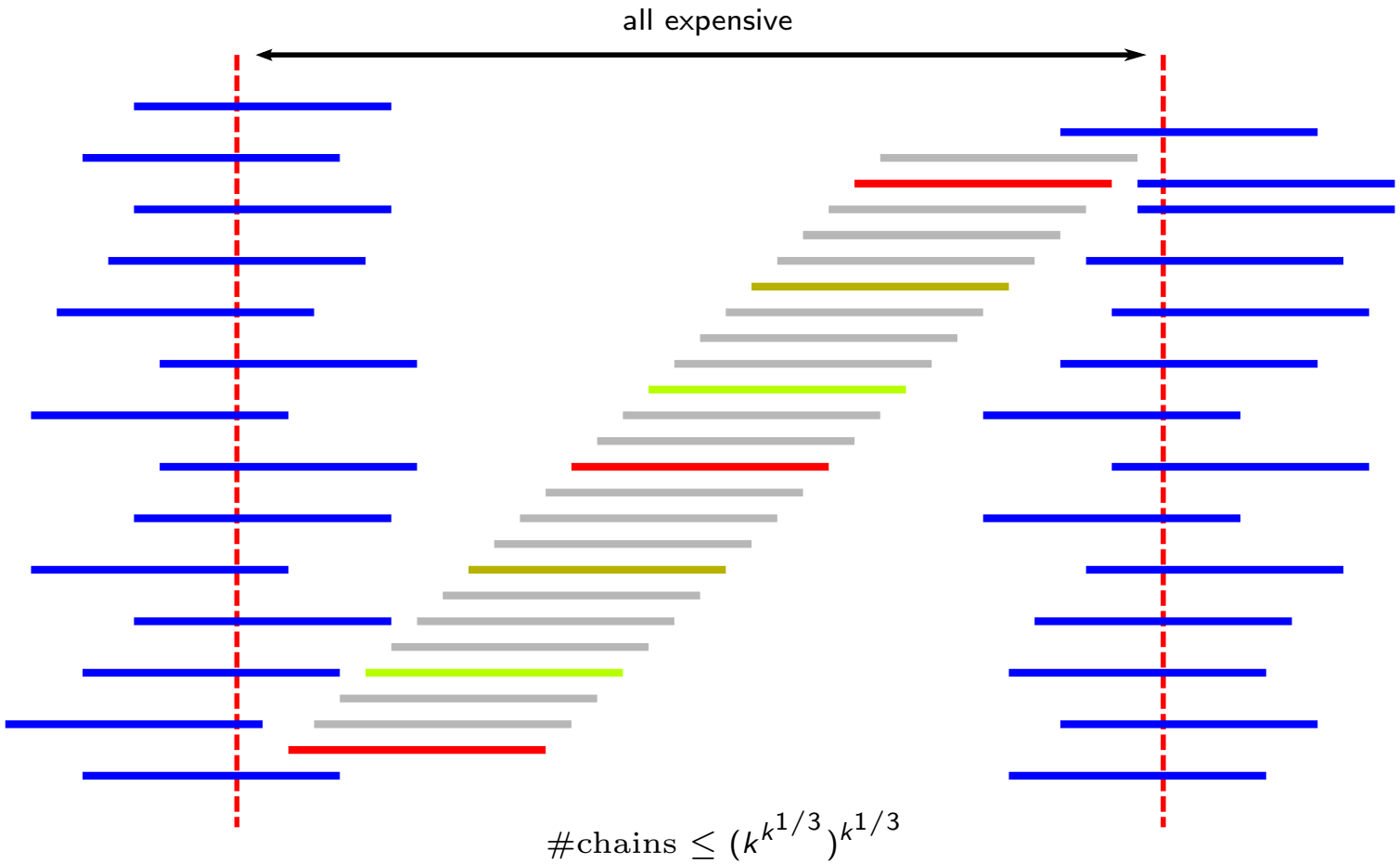

$$\#\text{chains} \leq (k^{k^{1/3}})^{k^{1/3}}$$

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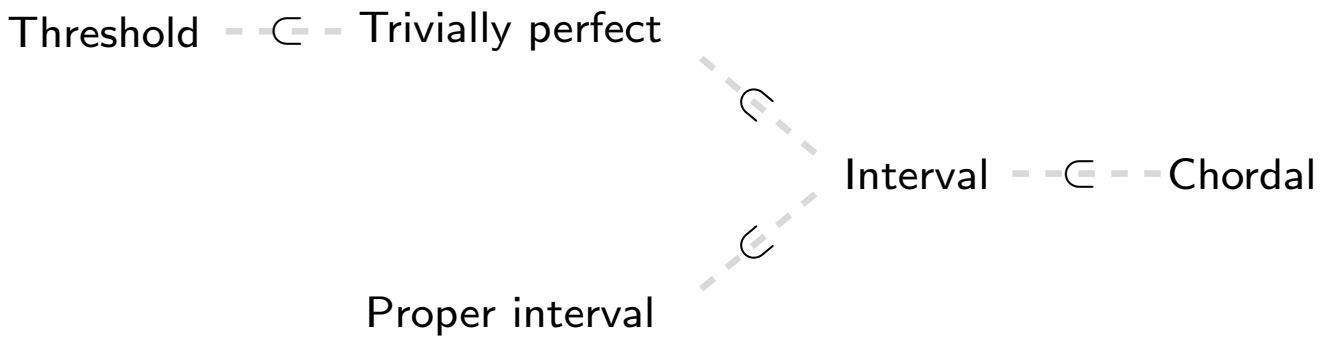
- Many more details in the second-layer DP.

Conclusions



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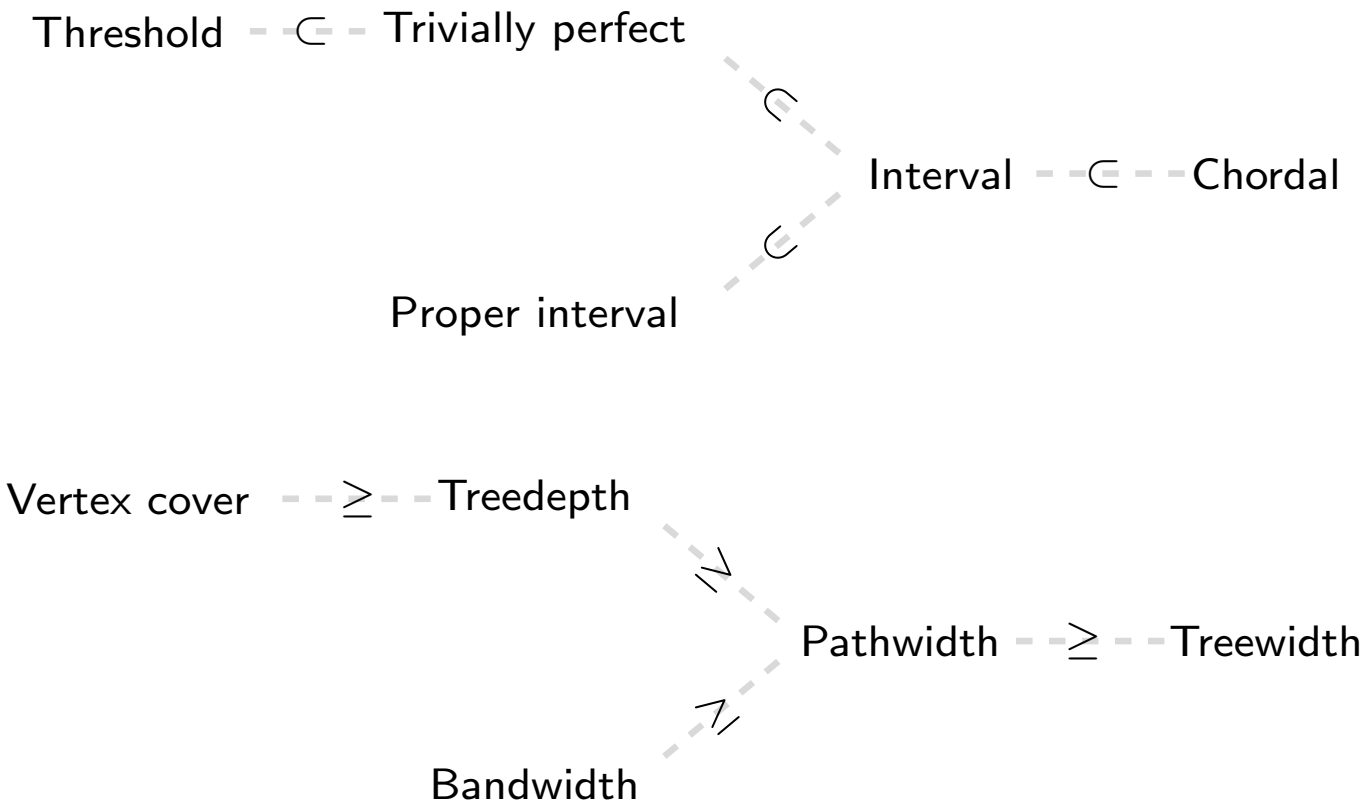

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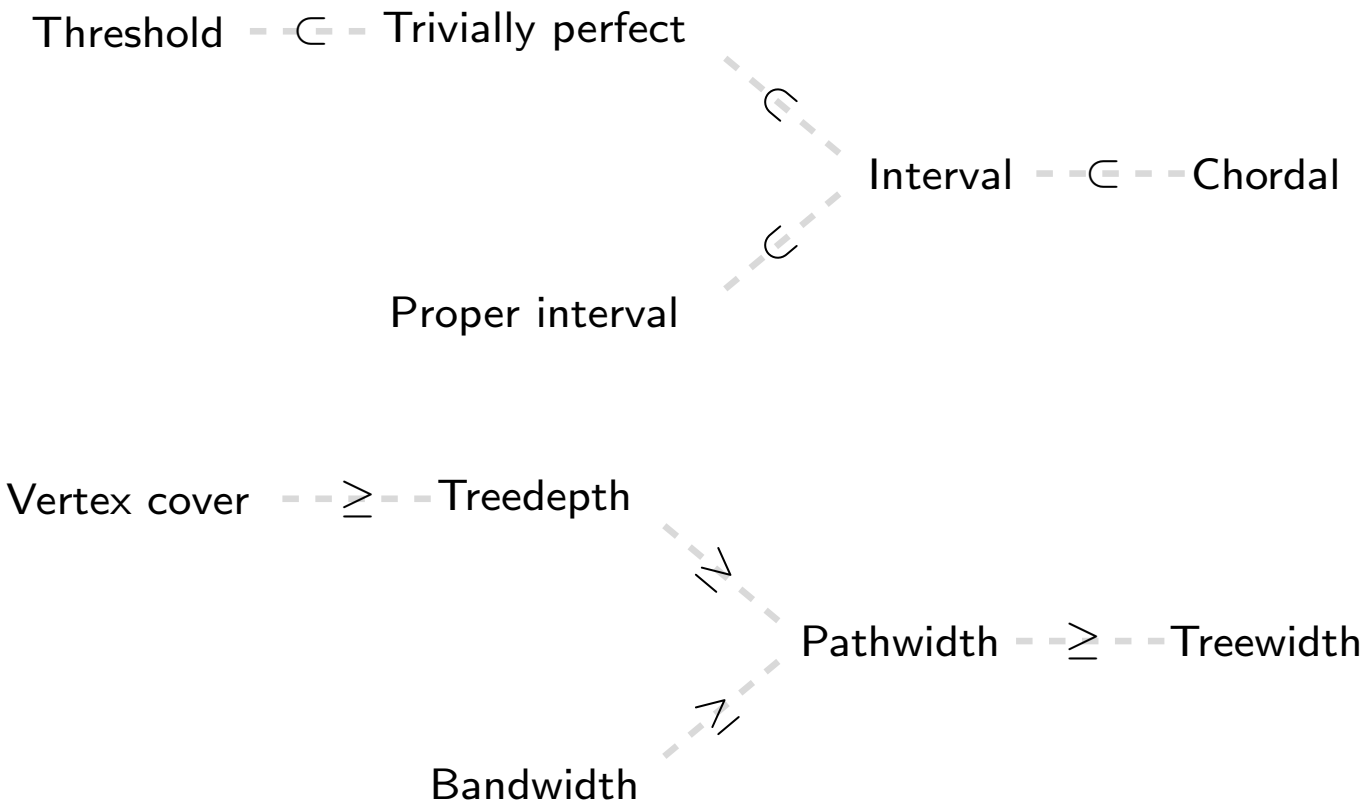

- **Correspondence**: parameter $p(\cdot)$ is the minimum possible maximum clique size in a completion to class Π (minus 1).

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- **Thank you for attention!**