Parameterized algorithms — tutorial 8

Algebraic techniques 2

Problem 1. Assume that we are given a graph G together with its nice tree decomposition of width t and linear number of bags. Prove that we can solve:

- Maximum Matching in $O(2^t n)$.
- Dominating Set in $O(3^t n)$.

Problem 2. Given a graph G and a coloring $c : V(G) \to [k]$ determine in $O(2^k \cdot poly(|G|))$ and polynomial space whether there exists a k-path that has vertices of all k colors.

Problem 3. In Weighted Longest Path problem we are given a directed weighted graph G with weighting function $E(G) \to \{0, 1, \ldots, W\}$ and an integer k and our goal is to find k-path of smallest total weight. Prove that this problem can be solved by Monte Carlo algorithm in $O(2^k \cdot W \cdot poly(|G|))$ time and $O(W \cdot poly(|G|))$ space.

Problem 4. In Triangle Packing problem we are given undirected graph G and an integer k and we are asked whether G contains k disjoint triangles. Prove that this problem can be solved in $O(2^{3k} \cdot poly(|G|))$ time and polynomial space.

Problem 5. Let f be a function that takes a set of nonnegative integers L and outputs an integer as follows.

- First, all integers in L are padded with leading zeros so they are all the same length as the maximum length number in L.
- We will construct a string where the $i-$th character is the minimum of the i-th character in padded input numbers.
- The output is the number representing the string interpreted in base 10.

For example $f(10, 9) = 0, f(123, 321) = 121, f(530, 932, 81) = 30$.

Define a function

$$G(x, T) = \left(\sum_{S \subseteq T, S \neq \emptyset, f(S) = x} \left(\sum_{y \in S} y \right)^2 \right) \mod (10^9 + 7)$$

where T is a set of integers. Assume that elements of T are smaller than 10^n.

Compute $G(0, T), G(1, T), \ldots, G(10^n - 1, T)$ in time $O(|T| + 10^n \cdot n)$.