Symmetric Choice and the Quest for a Logic Capturing Polynomial Time

Moritz Lichter

LoGAlg 2023
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European Research Council
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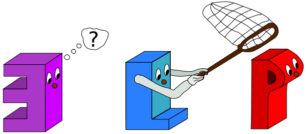
The Quest for a Logic Capturing Polynomial Time



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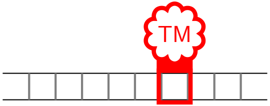
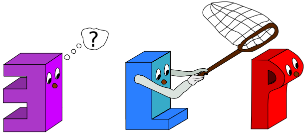


polynomial-time
Turing machine



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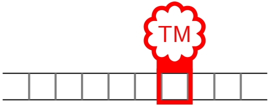

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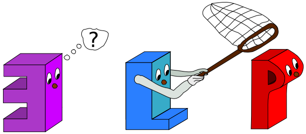

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formula of a logic

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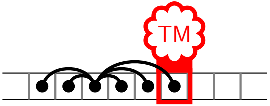

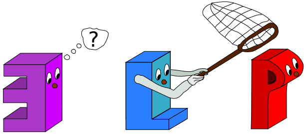

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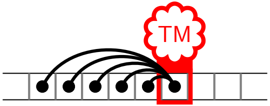

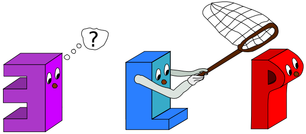

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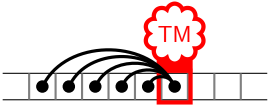

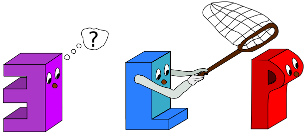

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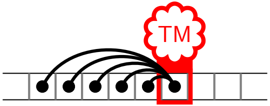

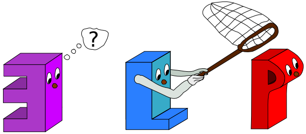

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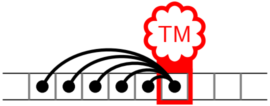

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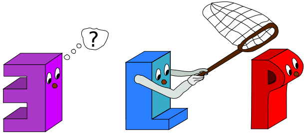

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Choiceless
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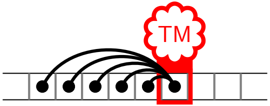

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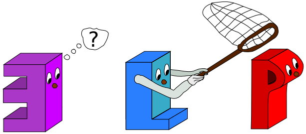

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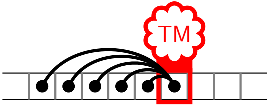

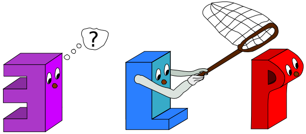

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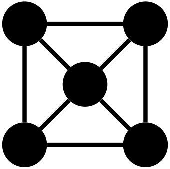

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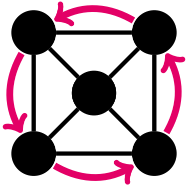

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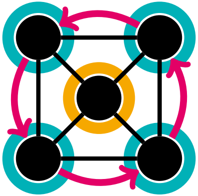

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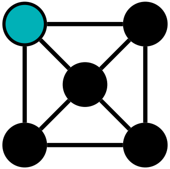

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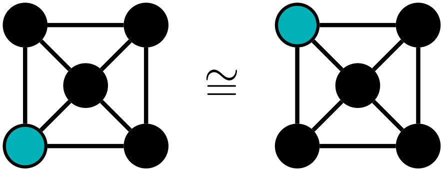

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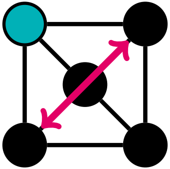

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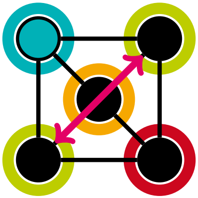

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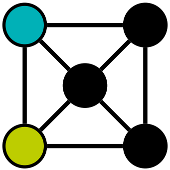

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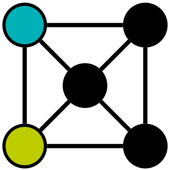

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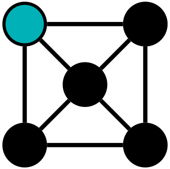

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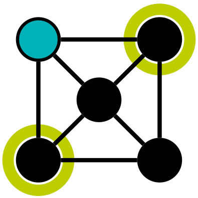

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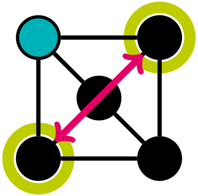

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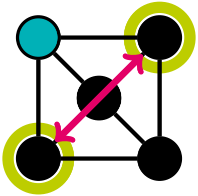

fixed-point operators with (W)SC

$$\textbf{fp-wsc}\Big(\Phi_{\textbf{step}}(x, y), \Phi_{\textbf{choice}}(x), \Phi_{\textbf{wit}}(x, z), \Phi_{\textbf{out}}(x)\Big)$$

Capturing PTIME and Canonization in Choiceless Polynomial Time with Witnessed Symmetric Choice

The Role of Canonization

Capturing PTIME via definable canonization

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graph class G

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Capturing PTIME via definable canonization

graph class G

$\downarrow$ canonization

ordered graph class G

canonization:

for all $A, B \in \mathbf{G}$

- $\text{can}(A) \cong A$
- $\text{can}(A) = \text{can}(B)$ $\Leftrightarrow$ $A \cong B$

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Capturing PTIME via definable canonization

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capturing PTIME on G

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- Does isomorphism testing imply canonization?

CPT and Isomorphism Testing

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Deep Weisfeiler Leman (Grohe, Schweitzer, Wiebking, '19)

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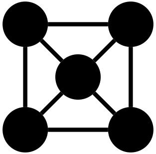

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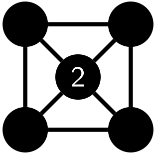
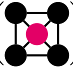


inv () = 2

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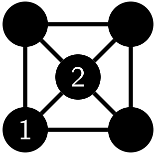
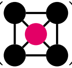
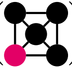


inv () = 2, inv () = 1

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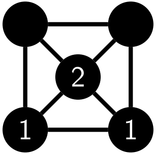
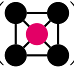
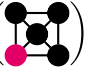
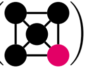


inv () = 2, inv () = 1, inv () = 1

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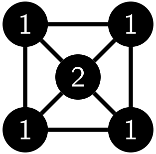
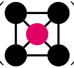
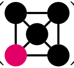
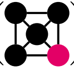
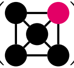
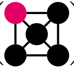


inv () = 2, inv () = 1, inv () = 1, inv () = 1, inv () = 1

CPT and Isomorphism Testing

Deep Weisfeiler Leman (Grohe, Schweitzer, Wiebking, '19)

CPT-definable isomorphism test for **G** implies

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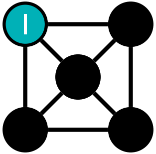

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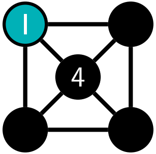
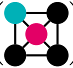


inv () = 4

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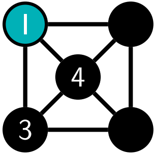
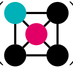
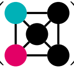


inv () = 4, inv () = 3

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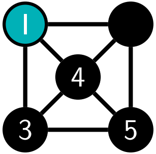
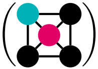
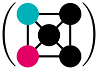
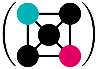


inv  = 4, inv  = 3, inv  = 5

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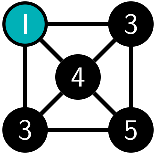
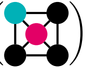
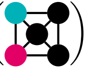
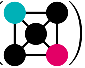
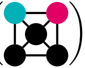


inv () = 4, inv () = 3, inv () = 5, inv () = 3

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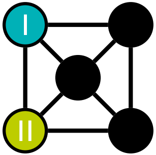

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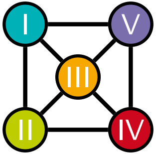

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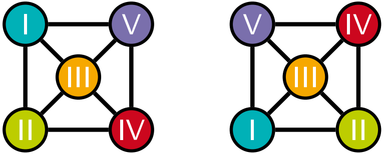

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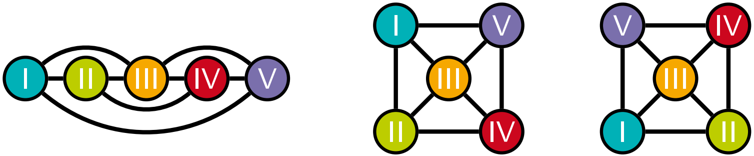

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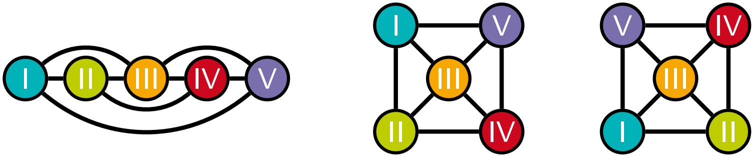

Not definable in CPT but definable in CPT+WSC!

Defining Isomorphisms and Canonization in CPT+WSC

Theorem. (L., Schweitzer, '21)

For every individualization-closed graph class **G**, the following are equivalent:

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Corollary.
For every individualization-closed graph class **G** with a PTIME isomorphism test
CPT+WSC defines **isomorphism** of **G** $\Longleftrightarrow$ CPT+WSC **captures PTIME** on **G**.

Expressiveness of Symmetric Choice in Fixed-Point Logic with Counting

Symmetric Choice, Asymmetric Structures, and Interpretations

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symmetric choice on **asymmetric structures** is useless

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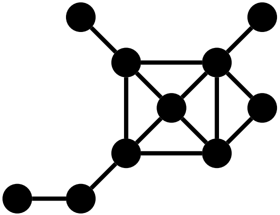

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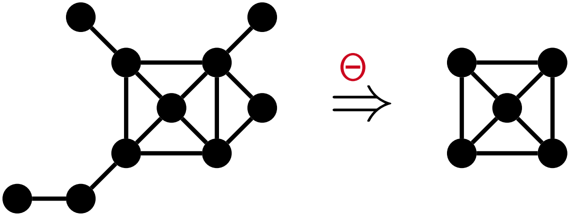

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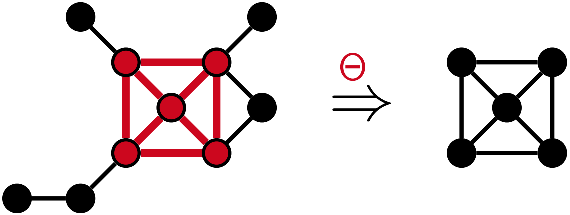

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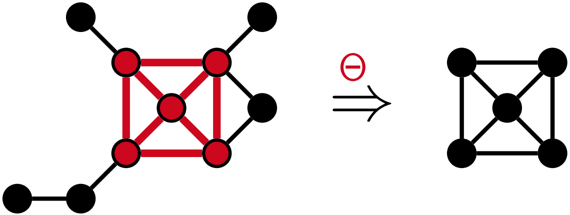

Interpretation operator $\mathrm{I}(\Theta, \Phi)$ (Gire and Hoang, '98)

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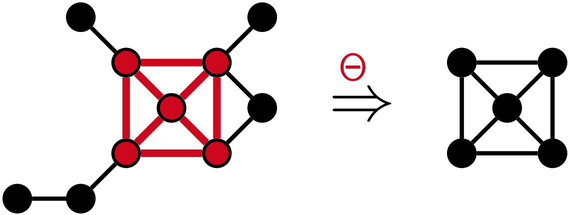

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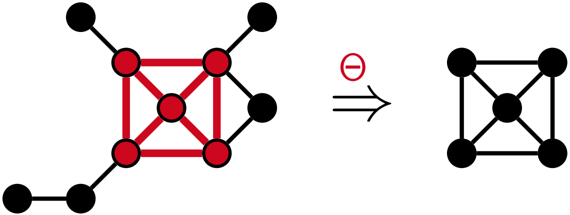
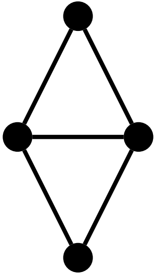


Interpretation operator $I(\Theta, \Phi)$ (Gire and Hoang, '98)

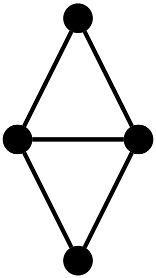
- Is IFPC+WSC+I more expressive than IFPC+WSC?
- Is IFP+SC+I more expressive than IFP+SC? (Dawar, Richerby, '03)

The CFI Query



base graph

The CFI Query



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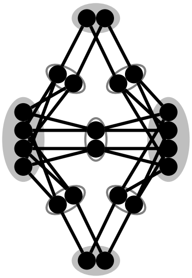

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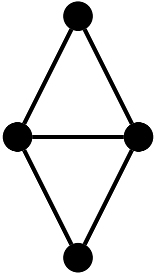

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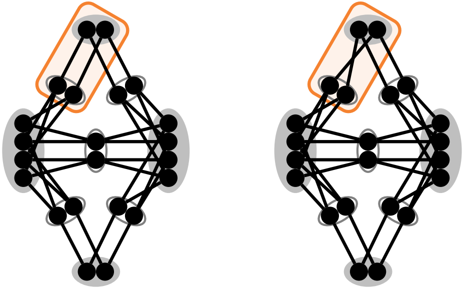

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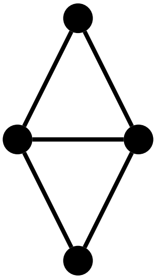

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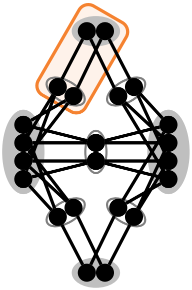
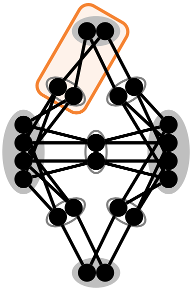


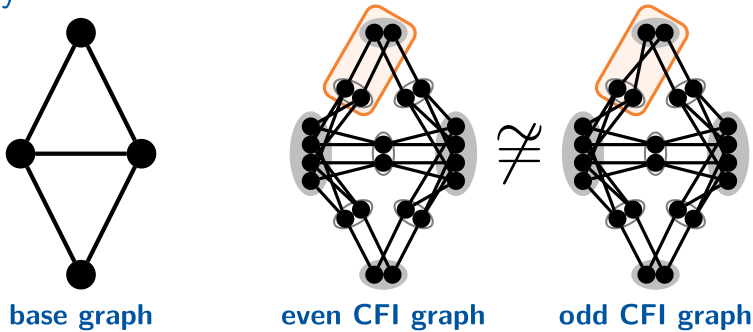
even CFI graph

$\ncong$



odd CFI graph

The CFI Query



CFI query: define whether a CFI graph is even

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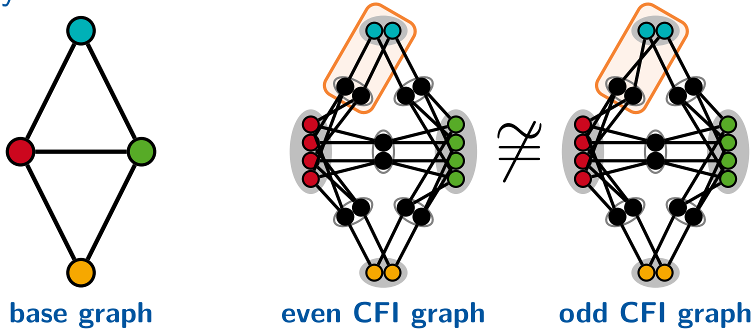

CFI query: define whether a CFI graph is even
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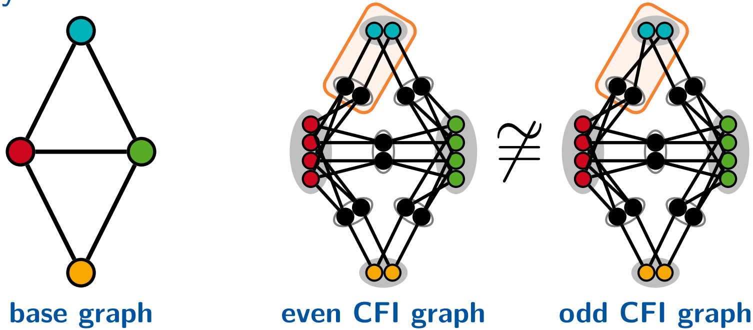


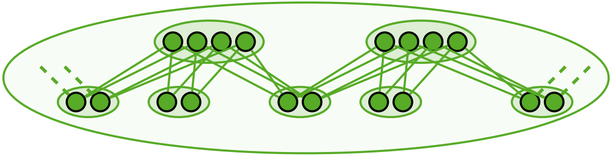
CFI query: define whether a CFI graph is even

ordered CFI query: ordered base graphs

Theorem (Cai, Fürer, Immerman, '92). The (ordered) CFI query is not IFPC-definable.

Theorem (Gire, Hoang, '98). The ordered CFI query is IFP+WSC-definable.

Separating IFPC+WSC from IFPC+WSC+I



ordered CFI graph

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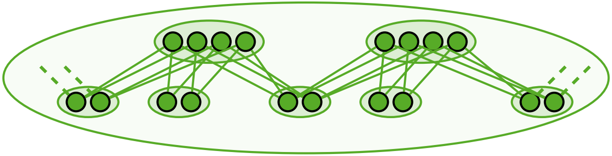

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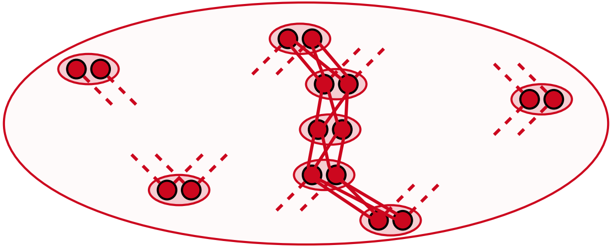

multipede

(Gurevich, Shelah, '96)

asymmetric structures

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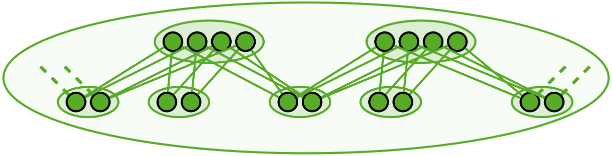

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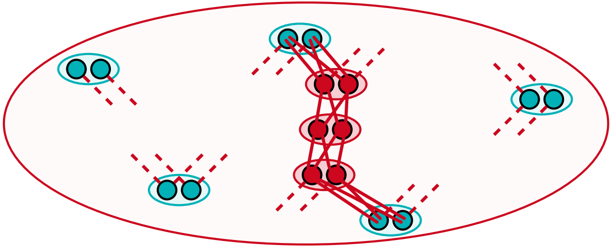

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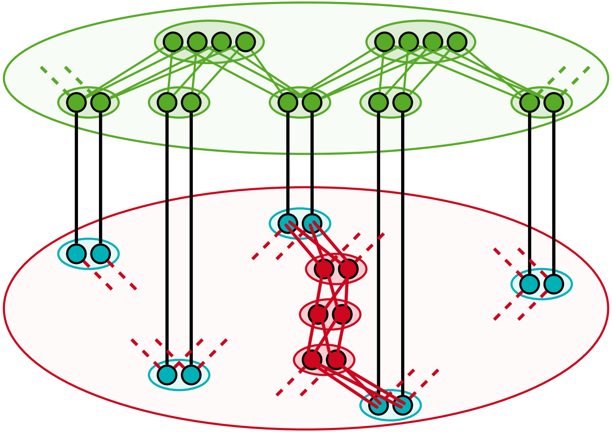

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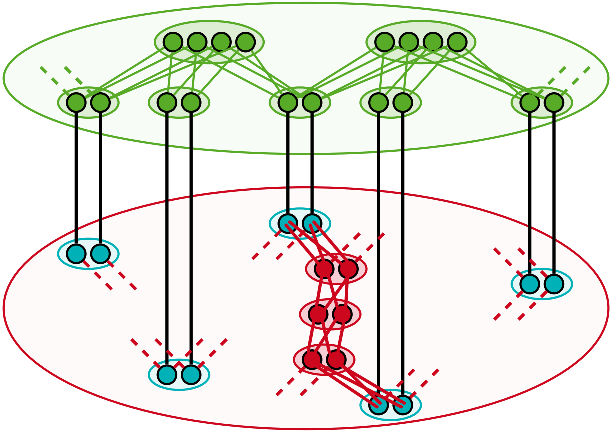

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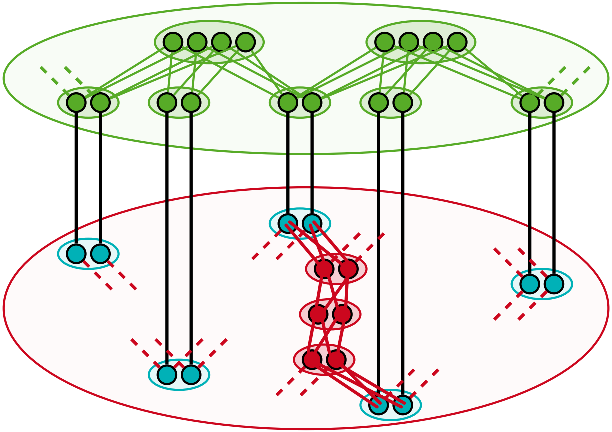

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asymmetric structures

orbits not IFPC-definable

Theorem (L, '23). IFPC+WSC $<$ IFPC+WSC+I $\leq$ PTIME

IFP(C)+(W)SC does not define the ordered CFI+multipede query.
IFP(C)+(W)SC+I defines the ordered CFI+multipede query.

Towards Separating IFPC+WSC+I from PTIME

Towards Separating IFPC+WSC+I from PTIME

Goal: Prove an operator nesting hierarchy for IFPC+WSC+I on CFI graphs

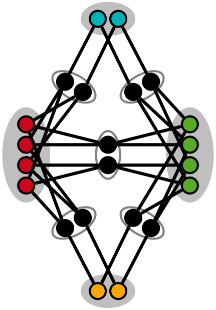
$$\mathsf{IFPC} \leq \mathrm{WSCI}(\mathsf{IFPC}) \leq \mathrm{WSCI}(\mathrm{WSCI}(\mathsf{IFPC})) \leq \cdots \leq \textsc{Ptime}$$

Towards Separating IFPC+WSC+I from PTIME

Goal: Prove an operator nesting hierarchy for IFPC+WSC+I on CFI graphs

$$\text{IFPC} < \text{WSCI(IFPC)} \leq \text{WSCI(WSCI(IFPC))} \leq \cdots \leq \text{PTIME}$$

(Gire, Hoang, '98)



Towards Separating IFPC+WSC+I from PTIME

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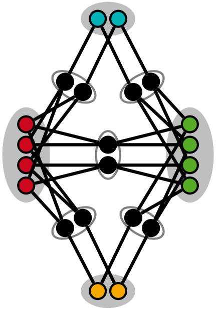
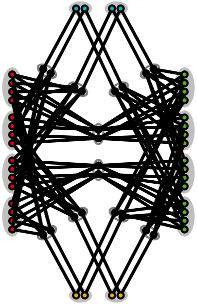


(L., '23)

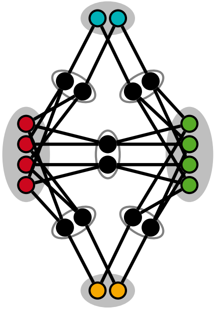


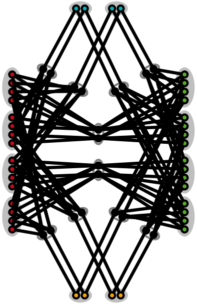
Towards Separating IFPC+WSC+I from PTIME

Goal: Prove an operator nesting hierarchy for IFPC+WSC+I on CFI graphs

$$\text{IFPC} < \text{WSCI(IFPC)} < \text{WSCI(WSCI(IFPC))} \leq \cdots \leq \text{PTIME}$$

(Gire, Hoang, '98) (L., '23) (ongoing work ...)





Summary: Witnessed Symmetric Choice

$$\textbf{fp-wsc}\Big(\Phi_{\textbf{step}}(x, y), \Phi_{\textbf{choice}}(x), \Phi_{\textbf{wit}}(x, z), \Phi_{\textbf{out}}(x)\Big)$$

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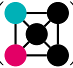
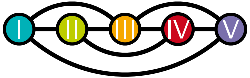
isomorphism
⇔ canonization
in CPT+WSC

inv () = 3

⇓



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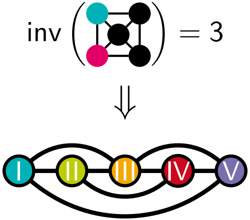
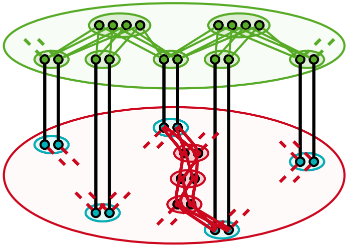


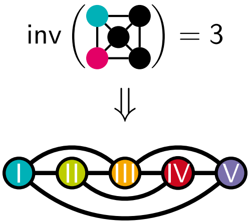
IFPC+WSC
does not capture PTIME



Summary: Witnessed Symmetric Choice

$$\textbf{fp-wsc}\Big(\Phi_{\textbf{step}}(x, y), \Phi_{\textbf{choice}}(x), \Phi_{\textbf{wit}}(x, z), \Phi_{\textbf{out}}(x)\Big)$$

isomorphism
$\Leftrightarrow$ canonization
in CPT+WSC



IFPC+WSC
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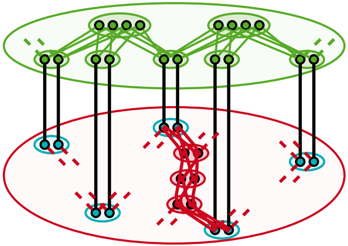

IFPC+WSC and
interpretations