Monadically Stable Graph Classes

LoGAlg 2023, Warsaw, November 15

Jan Dreier, TU Wien

Main Question

First-Order Model Checking

Given a graph G and a first-order sentence φ, decide whether $G \models \varphi$.

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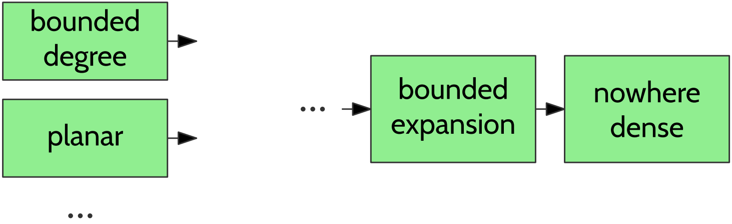
- Example: G has dominating set of size k if

$$G \models \exists x_1 \ldots \exists x_k \forall y \bigvee_i E(y, x_i) \vee y = x_i.$$

- Can be decided in $O(|G|^{|\varphi|})$.
- Question: On what graph classes is the problem fpt, i.e., solvable in time $f(|\varphi|) \cdot \text{poly}(|G|)$?

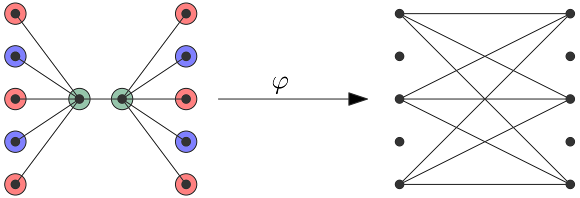
Tractable Classes



Bounded Degree Model Checking: Seese, 1996
Planar Model Checking: Flum, Grohe 2001
Bounded Expansion Model Checking: Dvořák, Král, Thomas, 2010
Nowhere Dense Model Checking: Grohe, Kreutzer, Siebertz, 2017

Transductions

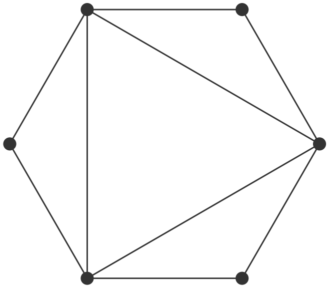
φ-transduction: color vertices + apply φ + take induced subgraph



$\varphi(x, y) :=$ Red$(x) \wedge$ Red$(y) \wedge$ dist$(x, y) = 3$

A class $\mathcal{D}$ is a *transduction* of a class $\mathcal{C}$ if there exists φ such that every graph in $\mathcal{D}$ is a φ-transduction of some graph in $\mathcal{C}$.

The class of subdivided cliques transduces the class of all graphs.



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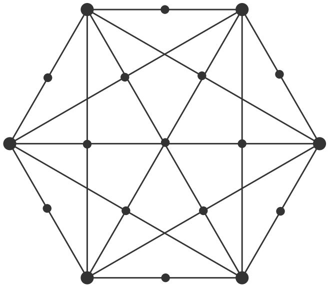

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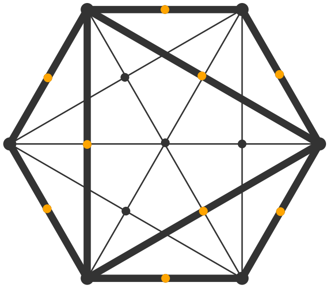

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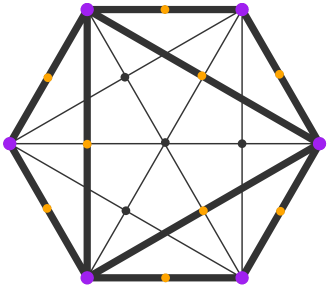

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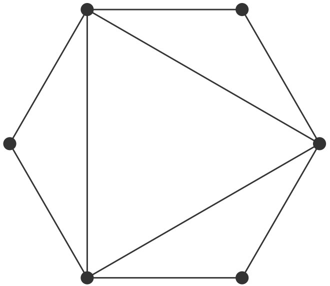
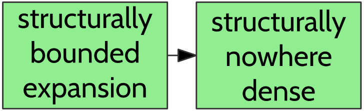


Tractable Classes

Gajarský, Kreutzer, Něsetřil, Ossona de Mendez, Pilipczuk, Siebertz, Toruńczyk, 2018.
Něsetřil, Ossona de Mendez, 2016

A class is *structurally nowhere dense*, if it is a transduction of a nowhere dense graph class.

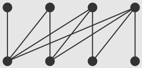


Structurally Nowhere Dense Model Checking: Dreier, Mählmann, Siebertz, 2022

Monadic Stability

Baldwin, Shelah, 1985

A class is *monadically stable*, if it does not transduce the class of all half-graphs.



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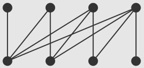

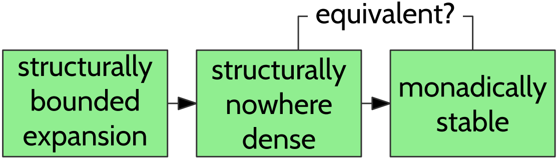

Monadically Stable Model Checking: Dreier, Eleftheriadis, Mählmann, McCarty, Pilipczuk, Toruńczyk, 2023

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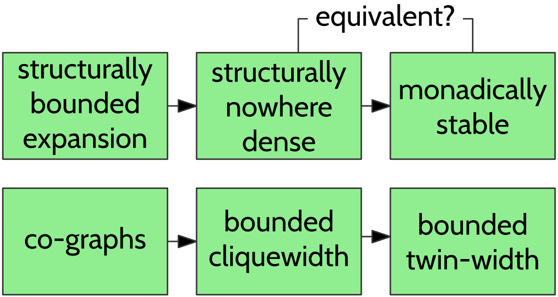
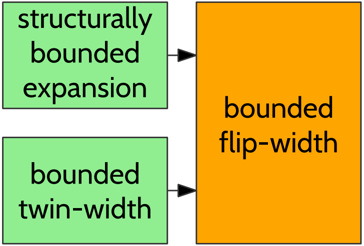


Cliquewidth Model Checking: Courcelle, Makowsky, Rotics, 2000
Twin-Width Model Checking: Bonnet, Kim, Thomassé, Watrigant,2021,

Tractable Classes

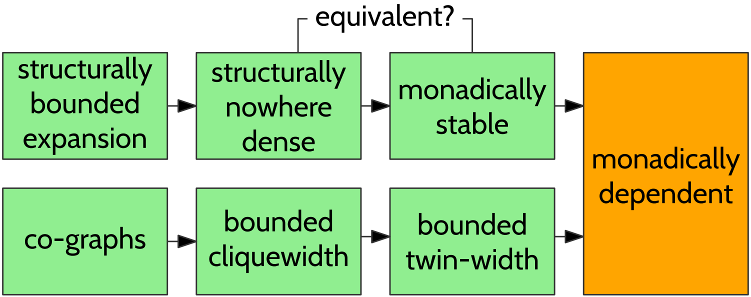


Flip-Width: Toruńczyk, 2023

Tractable Classes

Baldwin, Shelah, 1985

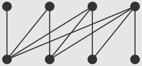
A class is *monadically dependent*, if it does not transduce the class of all graphs.



Tractable Classes

Dreier, Eleftheriadis, Mählmann, McCarty, Pilipczuk, Toruńczyk, 2023

Let $\mathcal{C}$ be a hereditary graph class that does not contain arbitrariliy large *semi-induced* half-graphs.



Model checking is fpt on $\mathcal{C}$

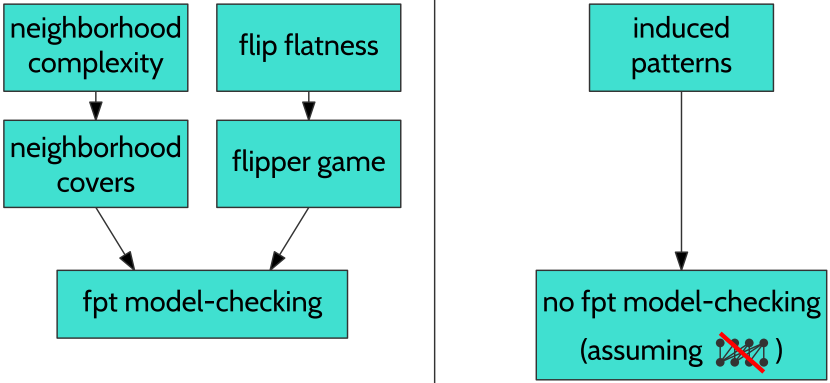
$\Leftrightarrow$

$\mathcal{C}$ is monadically stable

Outline

$\mathcal{C}$ monadically stable

$\mathcal{C}$ not monadically stable



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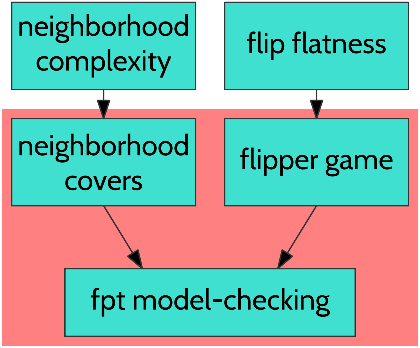

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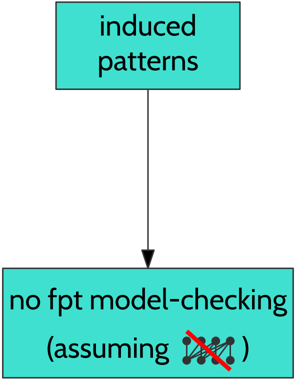

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Denote by $G \oplus F$ the graph obtained from G by complementing edges between pairs of vertices from F.

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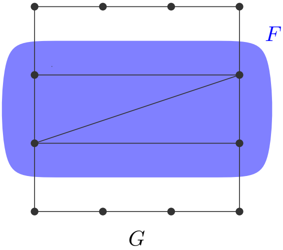

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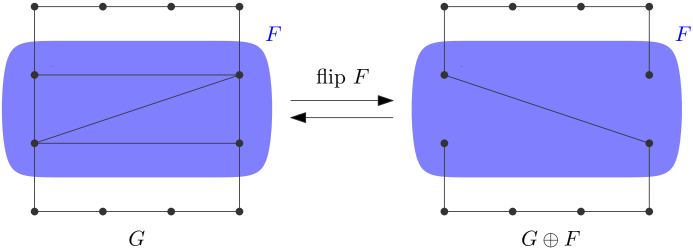

Flipper Game

The radius-r Flipper game is played on a graph G_1. In round i

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Example play of the radius-2 Flipper game:

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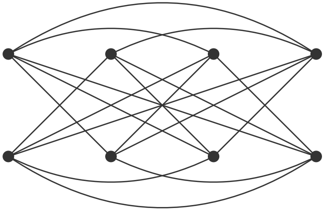

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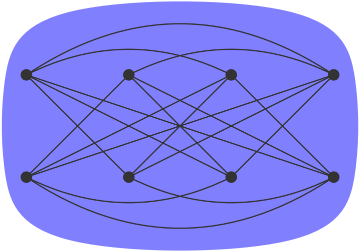

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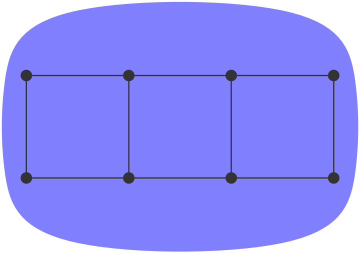

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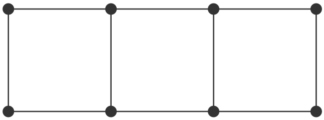

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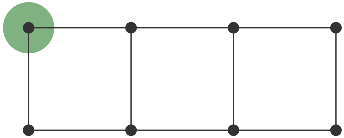

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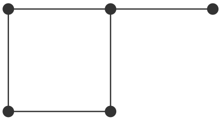

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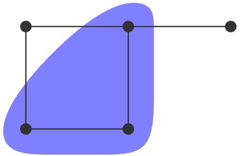

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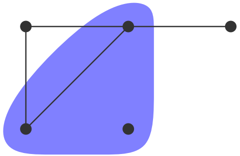

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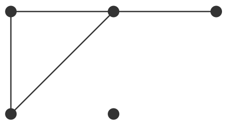

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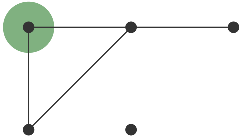

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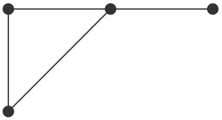

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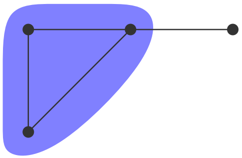

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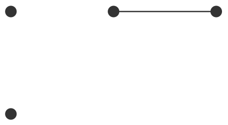

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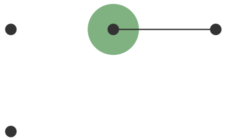

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Flipper Game and Monadic Stability

Gajarský, Mählmann, McCarty, Ohlmann, Pilipczuk, Przybyszewski, Siebertz, Sokołowski, Toruńczyk, 2023

A class of graphs $\mathcal{C}$ is monadically stable $\Leftrightarrow$

$\forall r \exists \ell$ such that Flipper wins the radius-r game on all graphs from $\mathcal{C}$ in ℓ rounds.

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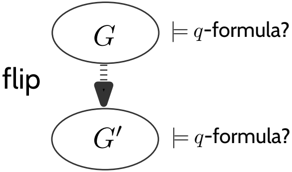
$\forall r \exists \ell$ such that Flipper wins the radius-r game on all graphs from $\mathcal{C}$ in ℓ rounds.

Moreover, Flipper's moves can be computed in time $O(n^2)$.

Guiding the Recursion with Flipper Games



round 1

Algorithm from: Dreier, Mählmann, Siebertz, 2022

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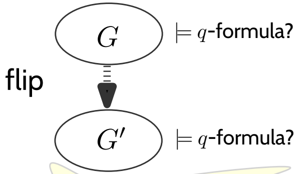

round 1

Update q-formula by replacing each edge relation:

$$G \models E(x,y) \iff G' \models E(x,y) \oplus (x \in F \wedge y \in F)$$

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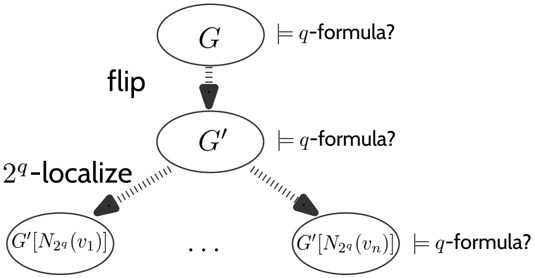

round 1

round 2

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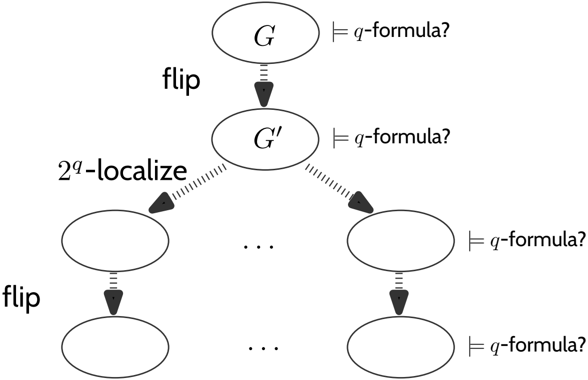

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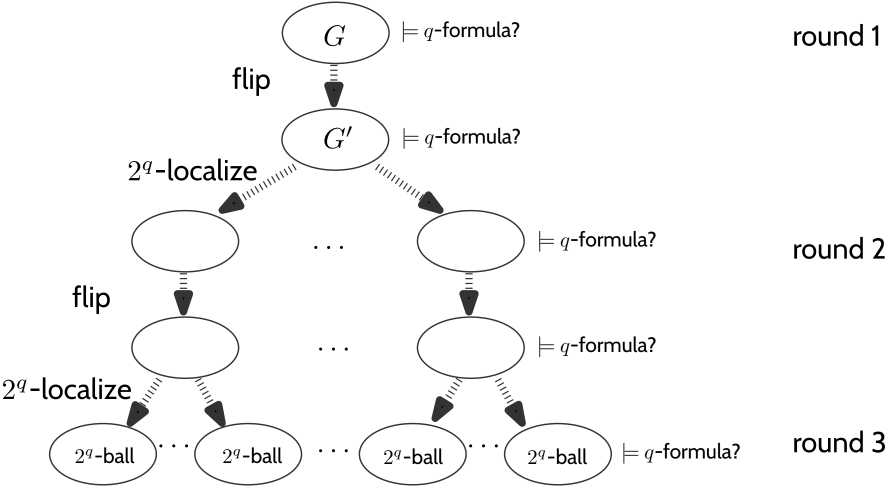

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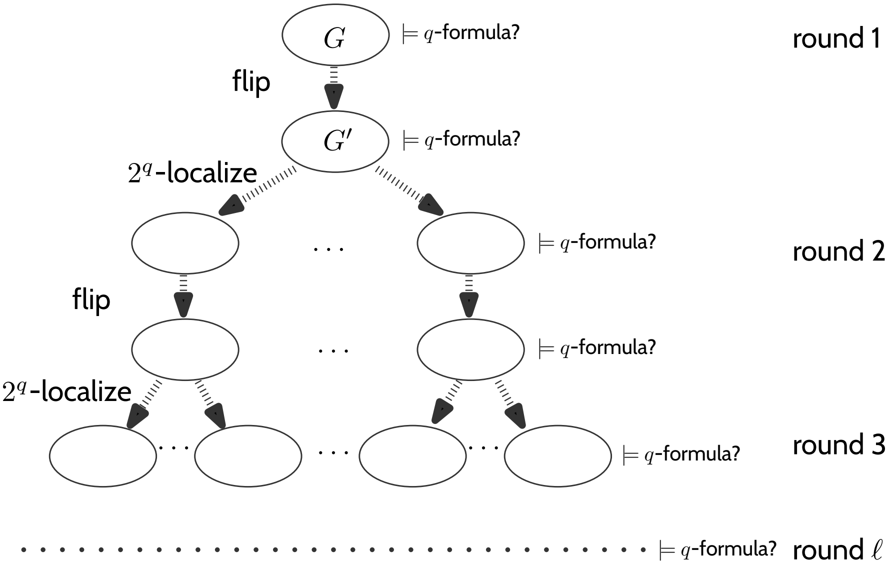

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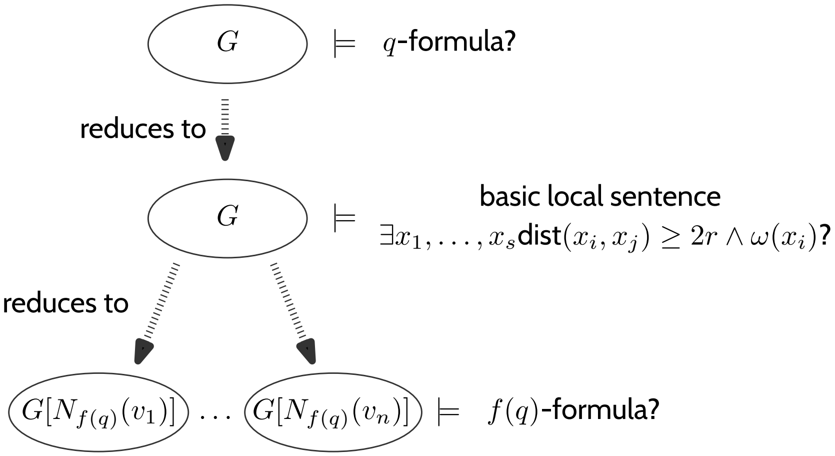
Gaifman–Approach

$G \models$ q-formula?

reduces to ↓

$G \models$ basic local sentence $\exists x_1, \ldots, x_s \text{dist}(x_i, x_j) \geq 2r \wedge \omega(x_i)$?

Gaifman–Approach



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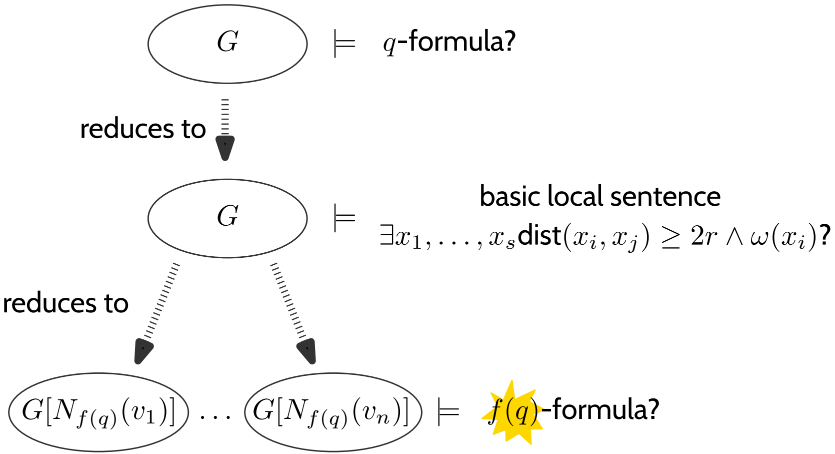

Local Types

Reduce model checking to the problem of deciding whether two vertices have the same q-type.

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Gajarský, Gorsky, Kreutzer, 2020 Toruńczyk, 2022 Dreier, Mählmann, Siebertz, 2022

Let G be a graph and a, b be two vertices with distance more than 2^q and

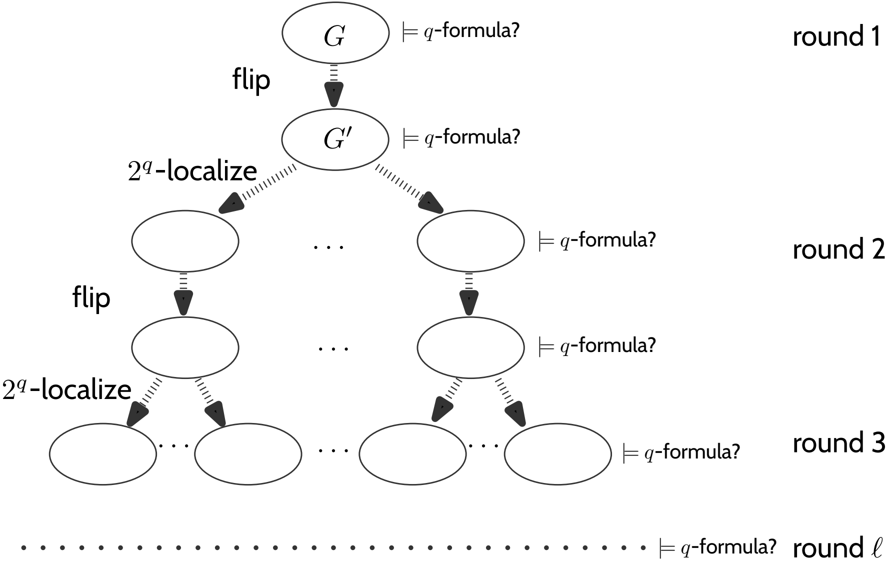
$$q\text{-type}(G[N_{2^q}(a)], a) = q\text{-type}(G[N_{2^q}(b)], b).$$

Then

$$q\text{-type}(G, a) = q\text{-type}(G, b).$$

There is just one more problem...

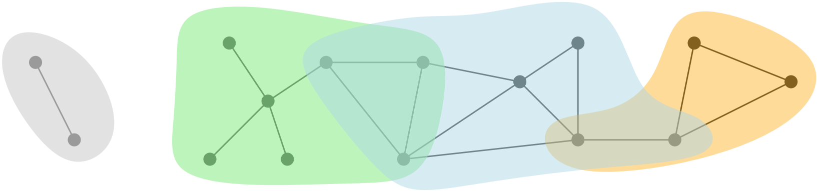
Guiding the Recursion with Flipper Games



Neighborhood Covers

Technique introduced by: Grohe, Kreutzer, Siebertz, 2017

We say an r-ball is a subgraph with radius r. An r-*neighborhood cover* with degree Δ in a graph G is a collection of sets $C_1, \ldots, C_l \subseteq V(G)$ such that



1-neigbhorhood cover with degree 2

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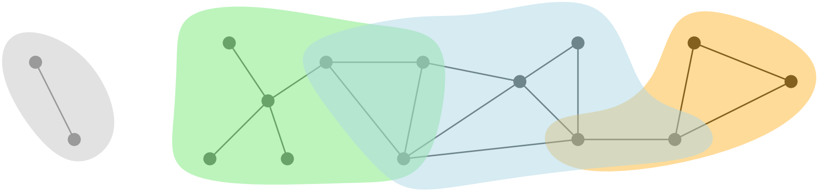

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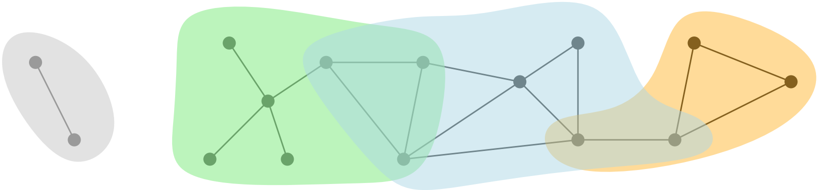

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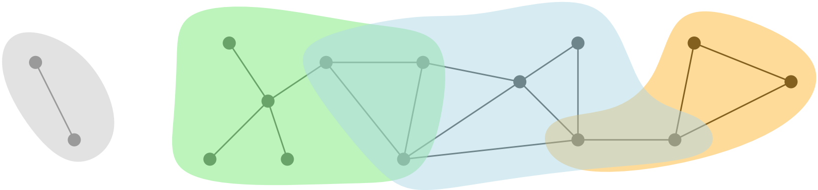

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Let $\mathcal{C}$ be a monadically stable graph class. Every $G \in \mathcal{C}$ has an r-neighborhood cover with degree $O_{\mathcal{C},\varepsilon,r}(|G|^{\varepsilon})$ for all $\varepsilon > 0$, $r \in \mathbb{N}$.

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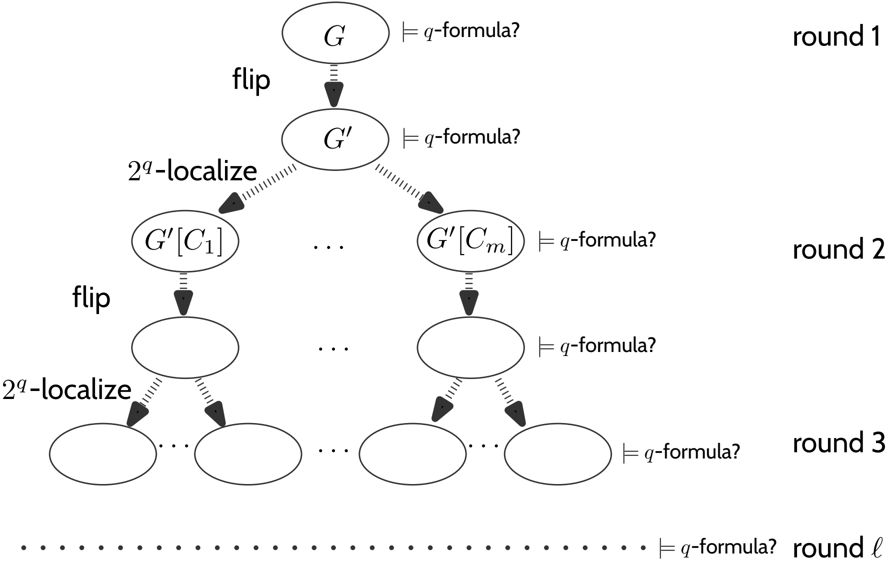
Then in particular, $\sum_{i=1}^{l} |C_i| \leq n \cdot O_{\mathcal{C},\varepsilon,r}(|G|^{\varepsilon})$.

Bounding the Size using Neighborhood Covers



Model Checking Summary

In summary, we used three ingredients.

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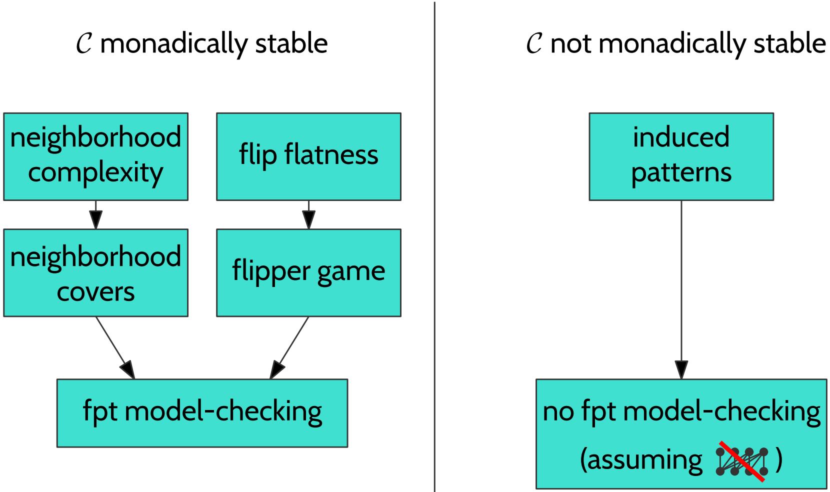
- The neighborhood covers bound the size of the recursion tree.

Outline



𝒞 monadically stable
neighborhood complexity
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Outline

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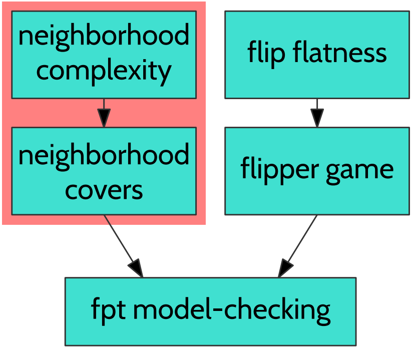

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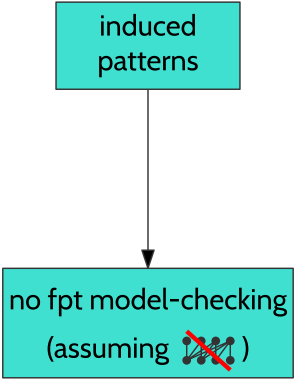
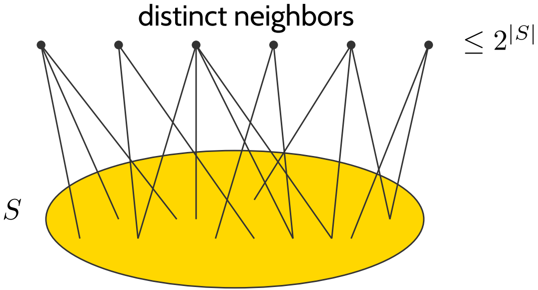


Neighborhood Complexity

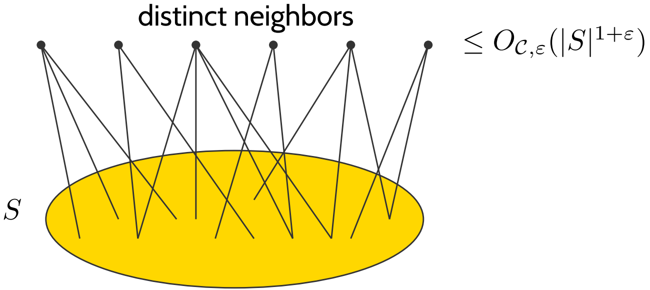


Neighborhood Complexity

Eickmeyer, Giannopoulou, Kreutzer, Kwon, Pilipczuk, Rabinovich, Siebertz, 2016

Let $\mathcal{C}$ be a nowhere dense graph class. For all $\varepsilon > 0$, $G \in \mathcal{C}$, $S \subseteq V(G)$,

$$|\{N(v) \cap S \mid v \in V(G)\}| \leq O_{\mathcal{C},\varepsilon}(|S|^{1+\varepsilon}).$$



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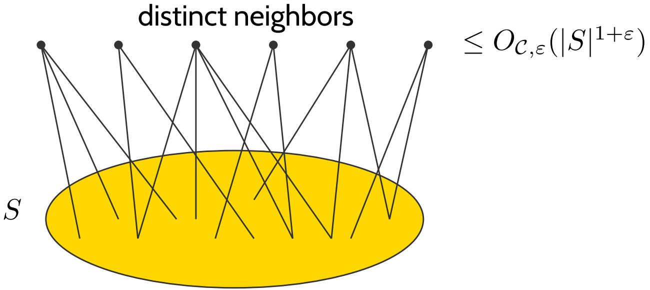
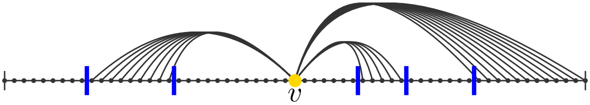
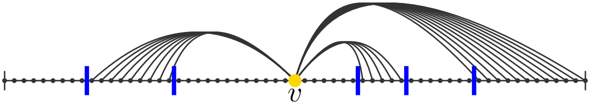


Welzl Orders



Welzl-order where v has 5 alternations

Welzl Orders



Welzl-order where v has 5 alternations

Corollary, Welzl, 1988

If a graph class $\mathcal{C}$ is monadically stable, then for all $\varepsilon > 0$ exists $c \in \mathbb{N}$ such that all $G \in \mathcal{C}$ admit Welzl orders where each vertex has $c \cdot |G|^{\varepsilon}$ alternations.

Building 1-Neighborhood Covers

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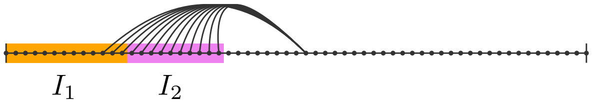

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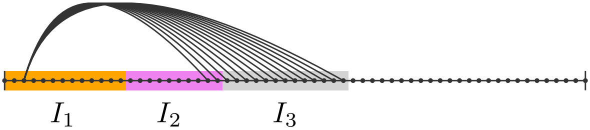

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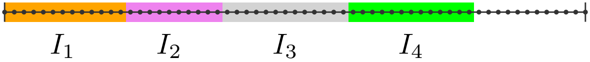

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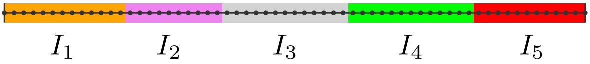

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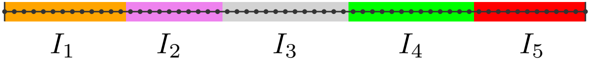

Let $N(I_1), N(I_2), \ldots$ be the clusters of the 1-neighborhood cover.

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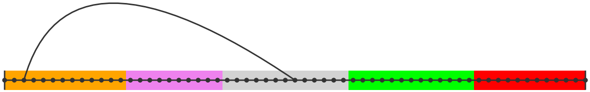

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Every vertex in $N(I_i)$ has alternation point in I_i.

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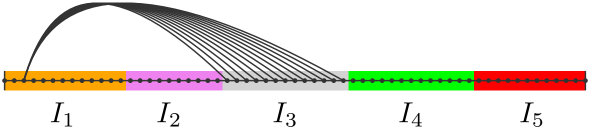

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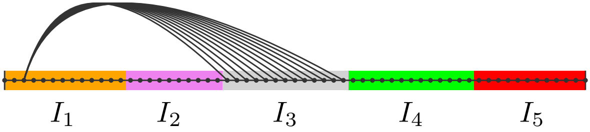

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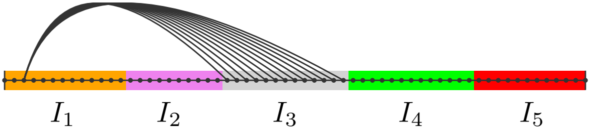

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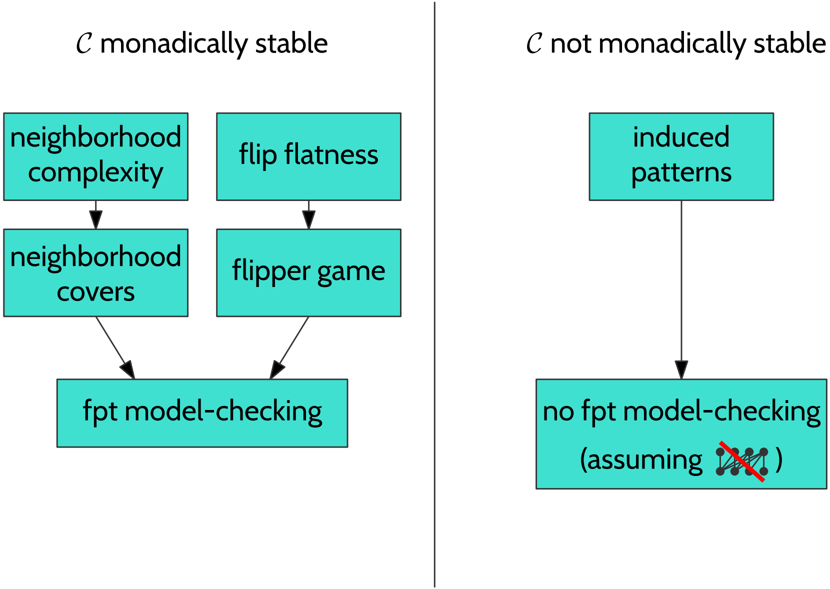
Outline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neighborhood complexity
flip flatness
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flipper game
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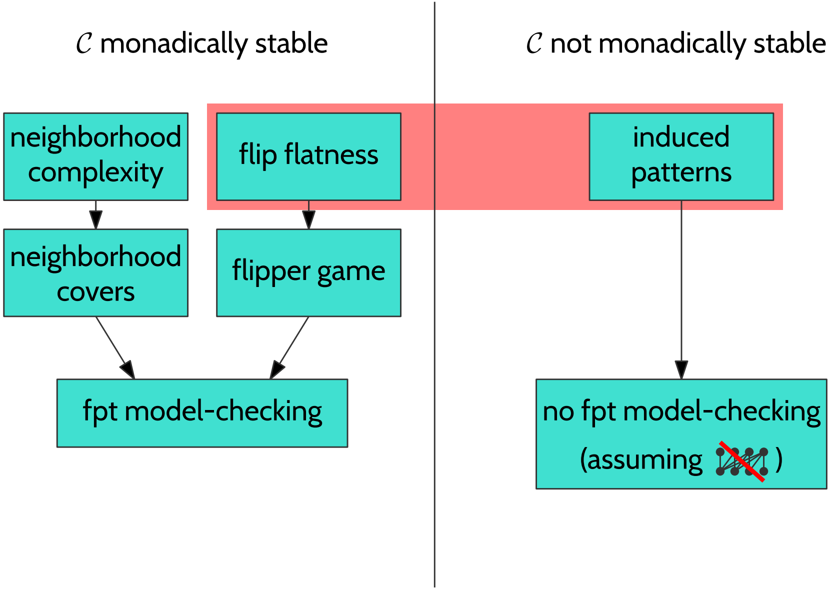
Outline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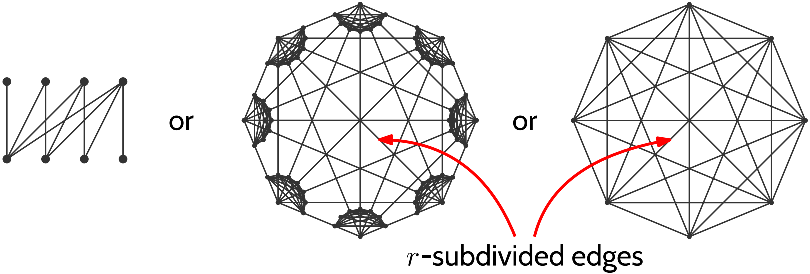
A class $\mathcal{C}$ is monadically stable if and only if for every r, these three types of induced subgraphs appear only up to a certain size.



Forbidden Patterns

Dreier, Eleftheriadis, Mählmann, McCarty, Pilipczuk, Toruńczyk, 2023

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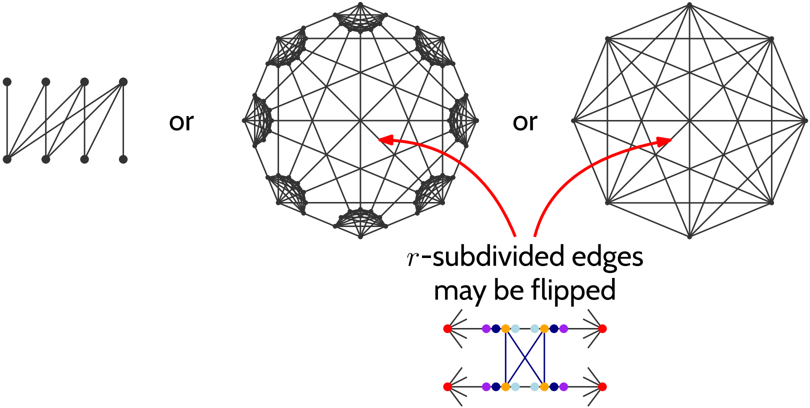
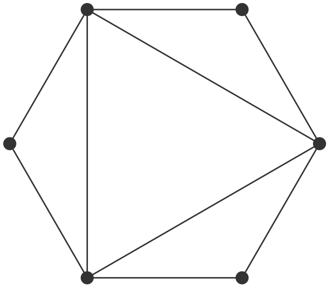


Transductions

The right two patterns can be used to show hardness of model checking.



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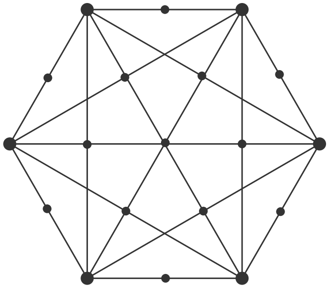

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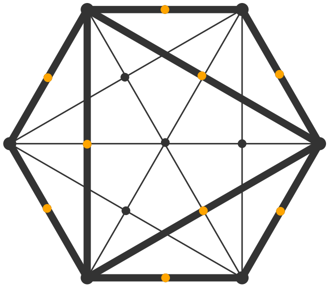

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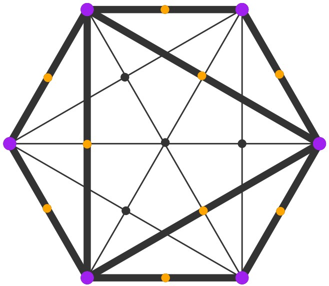

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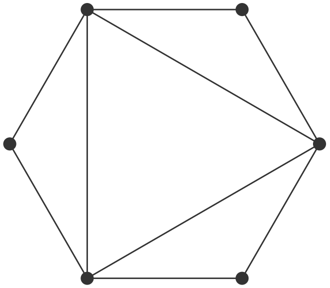

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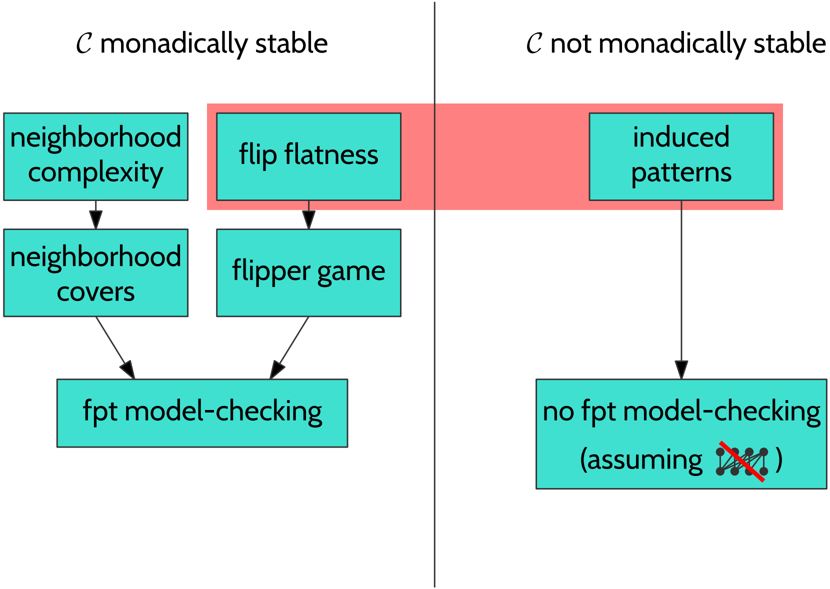

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Ramsey

Ramsey's Theorem

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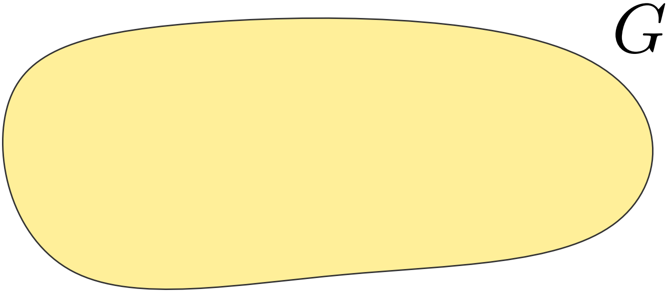

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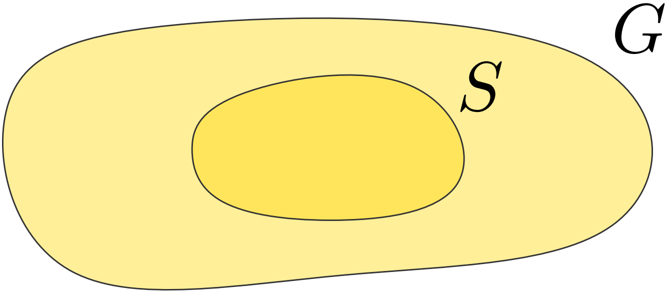
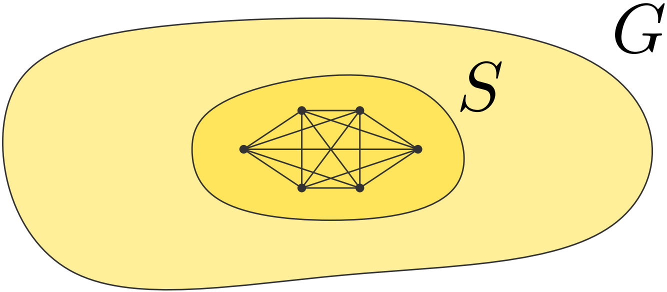


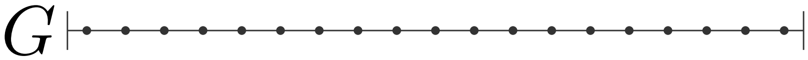
Ramsey's Theorem

In every graph G we can find $S \subseteq V(G)$, $|S| = U(|G|)$ such that S forms an independent set or clique.



Can we force even more structure onto S?

Fix $q \in \mathbb{N}$ and order the vertices of G.



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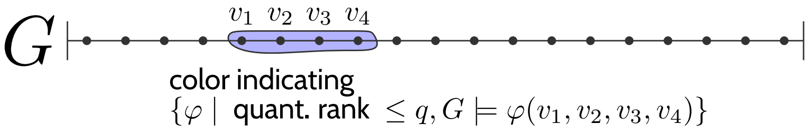

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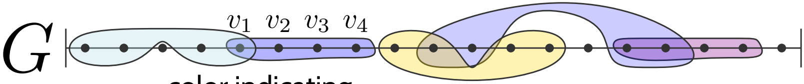

color indicating
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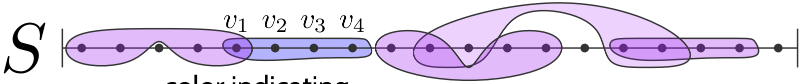

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all tuples have same color

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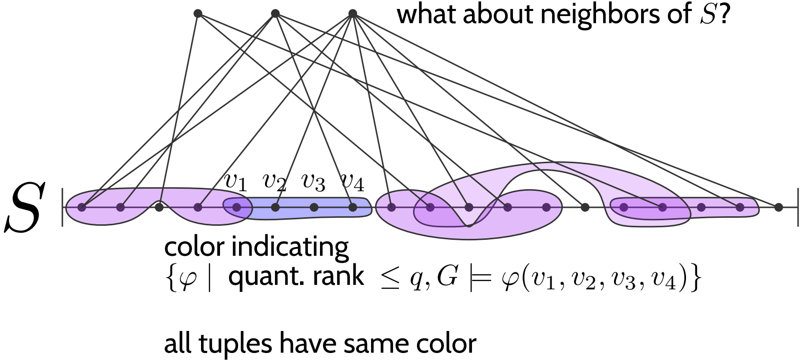

Ramsey

Can we force even more structure onto S?

Fix $q \in \mathbb{N}$ and order the vertices of G.

what about neighbors of S?

Question

Can we use monadic stability to force structure not only onto S, but also its neighbors?

Motivation: Uniform Quasi-Wideness

all tuples have same color

Flip Flatness

Dreier, Mählmann, Toruńczyk, Siebertz 2023

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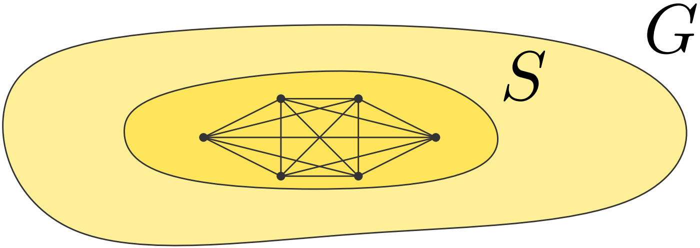

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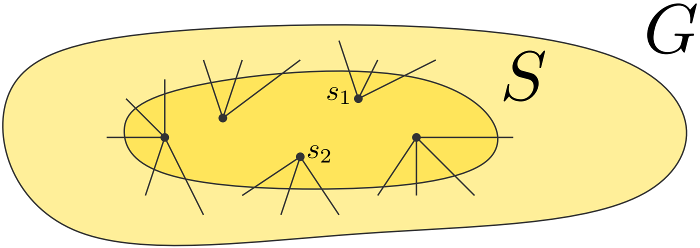
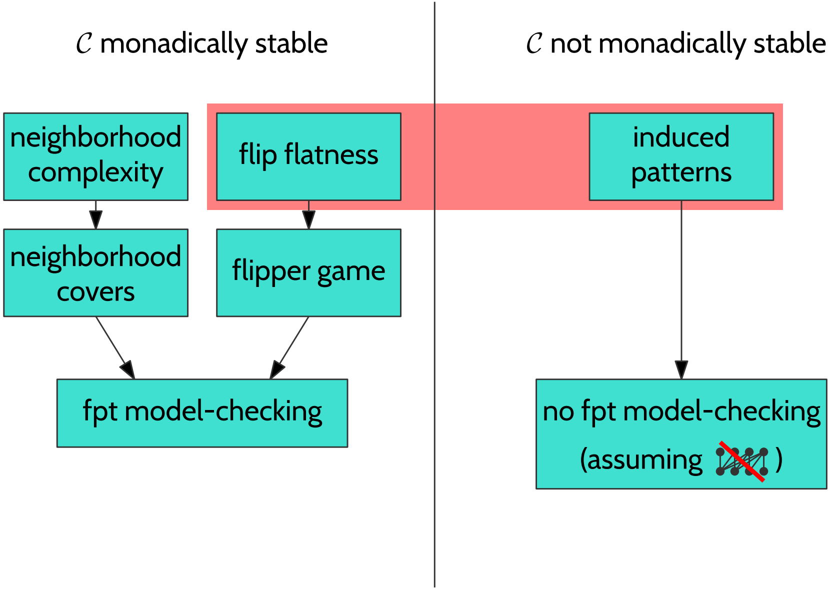
In every $G \in \mathcal{C}$ we find $S \subseteq V(G)$, $|S| = U_r(|G|)$, such that ~~S forms an independent set of clique~~

after performing c flips, $\forall s_1, s_2 \in S \;\; N_r(s_1) \cap N_r(s_2) = \emptyset$.



Outline


$\mathcal{C}$ monadically stable
$\mathcal{C}$ not monadically stable
neighborhood complexity
flip flatness
induced patterns
neighborhood covers
flipper game
fpt model-checking
no fpt model-checking
(assuming)

Goal

Let $\mathcal{C}$ be *any* graph class.

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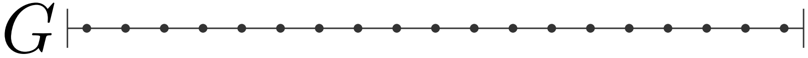
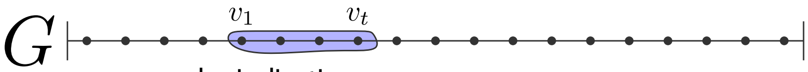
Use this to show radius-1 flip-flatness of $\mathcal{C}$.

Ramsey

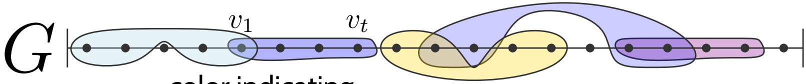


Ramsey



color indicating
$\{\varphi \mid$ quant. rank $\leq q, G \models \varphi(v_1, \ldots v_t)\}$

Ramsey



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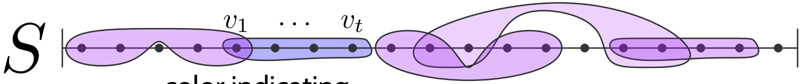
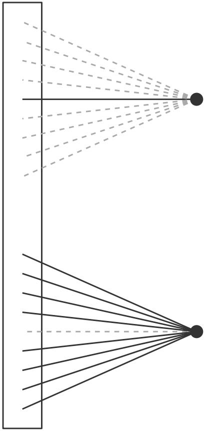


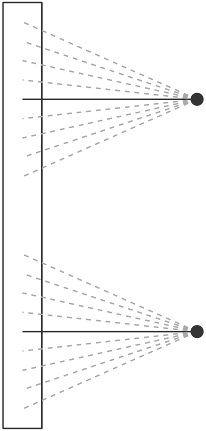
color indicating
$\{\varphi \mid$ quant. rank $\leq q, G \models \varphi(v_1, \ldots v_t)\}$

all tuples have same color

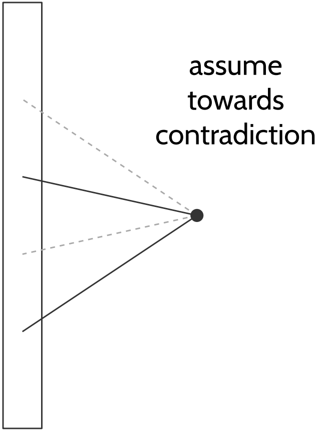
Construction



Construction



Construction



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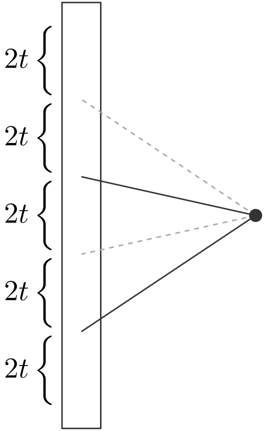

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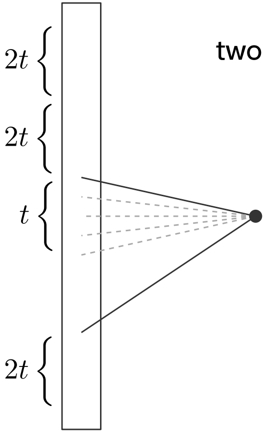
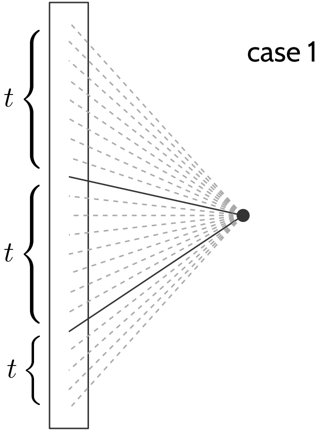


two cases

Construction



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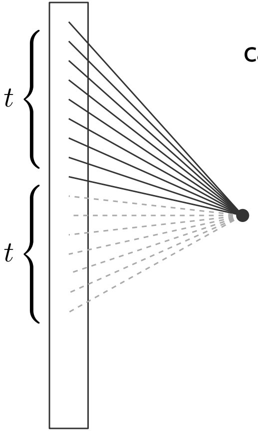
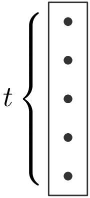
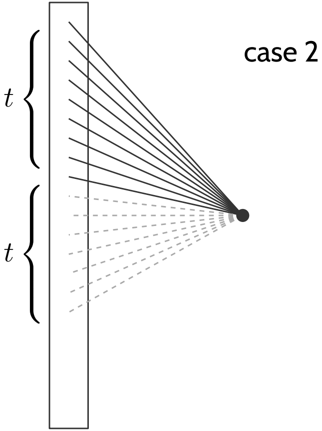


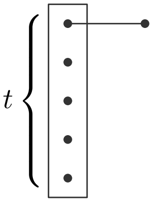
case 2

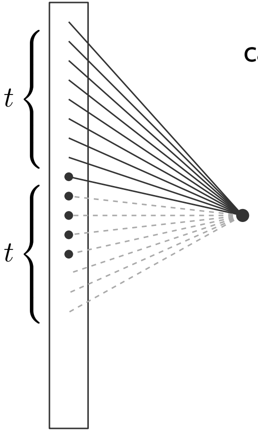
Construction





Construction





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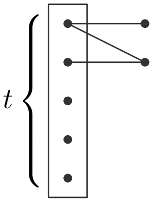
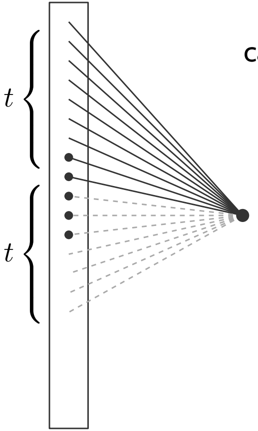
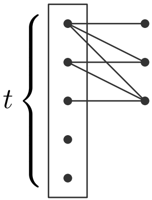
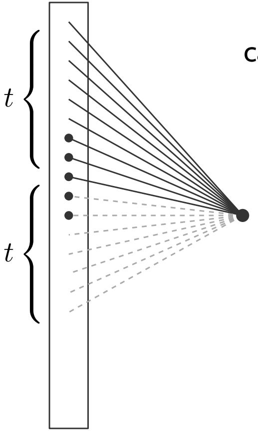




case 2

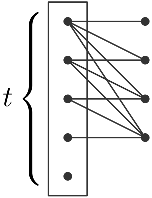
Construction

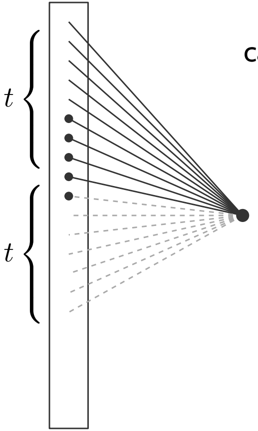




case 2

Construction





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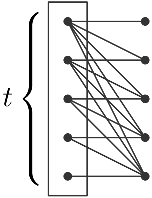

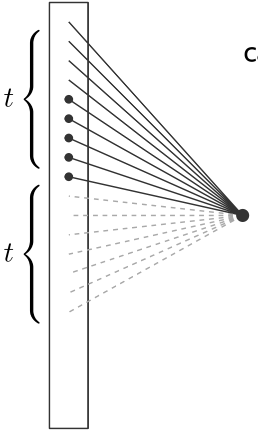

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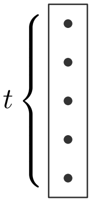
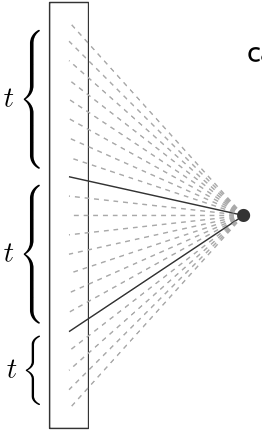
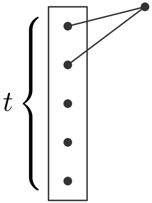
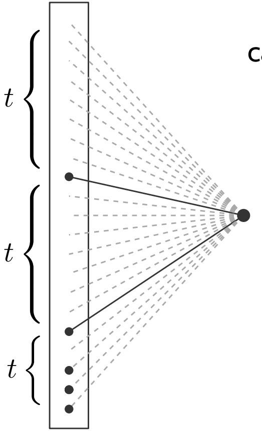




Construction





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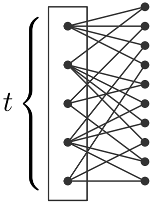
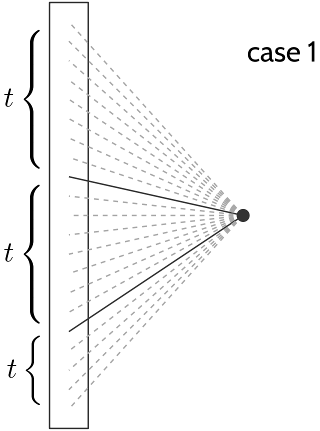
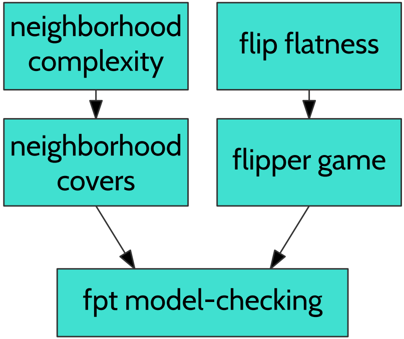


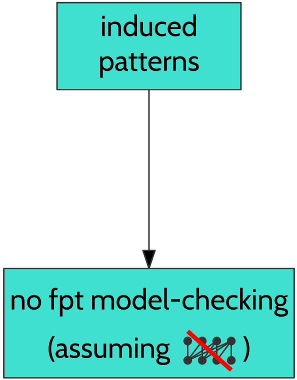


Outline

$\mathcal{C}$ monadically stable



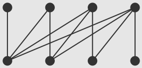
$\mathcal{C}$ not monadically stable



Summary

Dreier, Eleftheriadis, Mählmann, McCarty, Pilipczuk, Toruńczyk, 2023

Let $\mathcal{C}$ be a hereditary graph class that does not contain arbitrariliy large *semi-induced* half-graphs.



Model checking is fpt on $\mathcal{C}$

$\Leftrightarrow$

$\mathcal{C}$ is monadically stable

The End