A 1.4 "Real" parts of $\mathbb{C}$-add. measure and "positive" parts of $\mathbb{R}$-add. one

Obviously, if $\mathbb{K}=\mathbb{C}$ and $\mu \in l_{add}(\mathfrak{M})$, then $\operatorname{Re}\mu$ and $\operatorname{Im}\mu$ are also in $l_{add}(\mathfrak{M})$, and $\mu \in l^{\infty}_{add}(\mathfrak{M})$ iff $\operatorname{Re}\mu, \operatorname{Im}\mu \in l^{\infty}_{add}(\mathfrak{M})$ *). — The resolution $\mu = \operatorname{Re}\mu + i \operatorname{Im}\mu$ was just used e.g. in the proof of 8. of fact "On variation" (see p. APP-9)

Assume now that $\mathbb{K}=\mathbb{R}$ and $\mu \in l^{\infty}_{add}(\mathfrak{M})$. The following resolution onto μ_+ and μ_- is called The Jordan resolution of a real (additive) measure:

$$\mu_+ := \tfrac{1}{2}(\operatorname{var}\mu + \mu), \quad \mu_- := \tfrac{1}{2}(\operatorname{var}\mu - \mu)$$

(the names "positive/negative variation" are also used).

Obviously we have:

Fact

Both $\mu_+, \mu_- \in l^{\infty}_{add}(\mathfrak{M})$, they are ≥ 0 and $\begin{cases}\operatorname{var}\mu = \mu_+ + \mu_- \\ \mu = \mu_+ - \mu_- .\end{cases}$

Moreover, (if $\mathfrak{M}$ is a σ-algebra, then) $\mu \in l_{\sigma\text{-add}}(\mathfrak{M})$ iff $\mu_+, \mu_- \in l_{\sigma\text{-add}}(\mathfrak{M})$.

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*) And the analogic equivalence holds for $l_{\sigma\text{-add}}(\mathfrak{M})$, if $\mathfrak{M}$ is a σ-algebra.