Initial value problem of nonlinear Bogolyubov hierarchy

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We investigate the initial value problem for the sequence of non-linear Lioville equations and non-linear Bogolyubov hierarchy. The nonlinear Liouville hierarchy that describes the evolution of correlation functions, arises in many problems of statistical mechanics concerning many-particle systems. It is well known, that all possible states of a classical system of a finite number of particles are described by the functions interpreted as probability density functions. These functions are solutions of the initial-value problem of the Liouville hierarchy If the state of a system is presented in terms of a cluster expansion in new (correlation) functions one evidently obtains an equivalent description of this state. Now evolution of the correlation functions is determined by the nonlinear Liouville hierarchy. An explicit solution of such nonlinear equations is constructed and presented as an expansion in terms of particle clusters whose evolution is described by a cumulant (semi-invariant) of the evolution operators. The existence of a strong solution of the Cauchy problem with initial data from the space of integrable functions is proved. The nonlinear Liouville hierarchy is basic in the substantiation of the derivation of the nonlinear Bogolyubov hierarchy whose solutions describe the correlation dynamics of infinite systems of particles. A solution for the initial value problem of non-linear Bogolyubov hierarchy we construct by way of solutions of the nonlinear Liouville equations and in the form of the expansion over particle clusters whose evolution is governed by the certain combination of cumulants of the evolution operator of the corresponding particle cluster. This concerns the mathematical substantiation of the correlation-weakening principle. The correlation functions may be employed to directly calculate the specific characteristics of the system, i.e., fluctuations, defined as the average values of the square deviations of an observable from its average value, as well as macroscopic values which are not averages of observables.