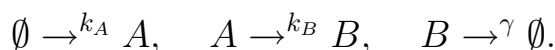


All problems are worth 10 points.

1. Construct the system of differential equations for the probability mass function (master equations) for the number of particles of  $A$ ,  $B$ , and  $C$  types for the following reactions:



2. Find the stationary state for the following family of birth and death processes:



with the maximal number of  $A$  and  $B$  molecules equal to 1.

3. Find all periodic ground-state configurations for the following one-dimensional Ising model of interacting spins,  $s_i = \pm 1$ :

$$H = \sum_{i \in Z} (s_i s_{i+2} - \delta s_i s_{i+3}).$$

4. Express the expected value of the energy in the ferromagnetic Ising model through thermodynamic potential (free energy).

## BONUS

Find all periodic ground-state configurations in the following Ising model on the square lattice:

$$H = -\sum_{i,j \in Z} (s_{i,j} s_{i+1,j} + s_{i,j} s_{i,j+1}) + 8 \sum_{i,j \in Z} (s_{i,j} s_{i+2,j} + s_{i,j} s_{i,j+2}).$$

**Pogodnych Świąt Bożego Narodzenia  
Wszystkiego Najlepszego w 2016  
Joyeux Noël et Bonne année**