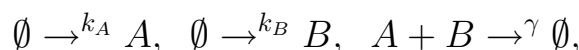


All problems are worth 10 points.

1. Construct a system of differential equations for the probability mass function of the number of protein molecules for the self-repressing gene with two binding sites of the promoter.

2. Find the stationary state for the following birth and death process:



where the number of molecules of type  $A$  and  $B$  is at most one.

3. Find all ground-state configurations for the following one-dimensional Ising model of interacting spins,  $s_i = \pm 1, 0$ :

$$H(s_1, s_2) = -(s_1 - s_2)^2.$$

4. Find the expected value of the magnetization in the Gibbs measure for the following spin model:  $s_i = \pm 1$ ,

$$H = - \sum_{i=1}^N (s_i^2 - s_i).$$

What is the magnetization in the ground state that is at zero temperature?

**BONUS** (10 points)

Prove that every nearest-neighbor and next-nearest-neighbor interaction on the square lattice  $\mathbf{Z}^2$  which is symmetric (it depends only on types of particles and distances between them) has at least one periodic ground-state configuration.

All the best in 2015