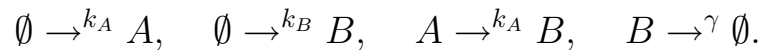


All problems are worth 10 points.

1. Construct the system of differential equations for the probability mass function (master equations) for the number of particles of A and B types for the following reactions:



2. Find the stationary state for the self-repressed gene with the maximal number of protein molecules (present in the system) equal to 2. Assume that proteins are produced directly from DNA.

3. Find all periodic ground-state configurations for the following one-dimensional Ising model of interacting spins, $s_i = \pm 1$:

$$H = - \sum_{i \in \mathbb{Z}} (s_i s_{i+1} - s_i s_{i+2} - 4s_i s_{i+3}).$$

4. Find the expected value of the magnetization in the Gibbs measure for the following spin model: $s_i = \pm 1, 0$,

$$H = - \sum_{i=1}^N (s_i^2 - s_i).$$

What is the magnetization in the ground state that is at zero temperature?

BONUS

Prove that every nearest-neighbor and next-nearest-neighbor interaction on the square lattice \mathbf{Z}^2 which is symmetric (it depends only on types of particles and distances between them) has at least one periodic ground-state configuration.