Final Exam Models of Applied Mathematics 6 February 2015

All problems are worth 10 points.

1. DNA can be in two different states with the following transition intensities:

$$1 \to^{\alpha} 2, \;\; 2 \to^{\beta} 1$$

At each state, mRNA is produced with the intensity $k_i, \;\; i = 1, 2$. mRNA molecules degrade with the intensity γ. Find the expected value and the variance of the number of mRNA molecules in the stationary state.

2. Construct a system of differential equations for the probability mass function of the number of molecules of type A and type B for the following birth and death processes:

$$\emptyset \to^{k_A} 2A, \;\; \emptyset \to^{k_B} B, \;\; A + 2B \to^{\gamma} \emptyset$$

3. Find all ground-state configurations in the following one-dimensional Ising model of interacting spins, $s_i = \pm 1, \pm 2$:

$$H = \sum_i (s_i - s_{i+1})^2.$$

4. Find the expected value of the magnetization in the Gibbs measure in the following spin model: $s_i = \pm 1, 0$,

$$H = -\sum_{i=1}^{N} (s_i^2 - s_i).$$

What is the magnetization in the ground state?

5. Let the price of the unit of a good be equal to $P(x, y) = 4 - (x + y)$ where x and y are production levels of two competing companies, production costs per unit are equal to 2 for the first company and 1 for the second one. Find the Nash equilibrium in the Stackelberg Duopoly.

6. Find all Nash equilibria in the following game:

	C	D
A	3, 3	0, 1
B	1, 2	1, 4

BONUS (10 points)

Find ground-state configurations of the antiferromagnetic Ising model on the triangular lattice, Hamiltonian of nearest-neighbor interactions is given by

$$H = \sum_{i,j} s_i s_j$$

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