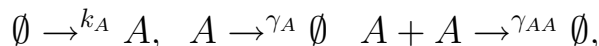


All problems are worth 8 points.

1. Find the stationary state for the following birth and death process:



where  $A = 0, 1, 2$ .

2. Find ground-state configurations (as a function of  $d$  and  $h$ ) of the following Hamiltonian of two interacting spins  $s_i = 0, \pm 1$ ,

$$H(s_1, s_2) = -s_1 s_2 - d(s_1^2 + s_2^2) - h(s_1 + s_2)$$

3. Let us suppose that the price of a product is given by the following function of production levels of two firms:  $P(x, y) = 4 - (x + y)$ , and the cost of producing the unit of the product is equal to 2 for the first company and 1 for the second one. Find the Nash equilibrium of the Stackelberg Duopoly.

4. Find all Nash equilibria for the symmetric game with the following payoff matrix:

|     |   |   |   |   |
|-----|---|---|---|---|
|     |   | A | B | C |
|     | A | 6 | 2 | 0 |
| U = | B | 2 | 4 | 0 |
|     | C | 2 | 2 | 1 |

5. Find the price of the financial instrument at  $t = 0$  paying  $\frac{1}{2}S_1 - 10$  at  $t = 1$ . Assume that  $S_0 = 50$ ,  $r = 0.2$ ,  $u = 0.6$ , and  $d = -0.2$ . Construct the replicating portfolio at  $t = 0$ .

6. Consider the put option with the expiration date at  $t = 2$  and  $S_0 = 100$ ,  $X = 91$ ,  $r = 0.2$ ,  $u = 0.3$ , and  $d = -0.1$ . Find the value of the option at  $t = 0$ .

**BONUS** (10 points)

Consumers are uniformly distributed in the linear town of length 1. There are two stores on boundaries of the town which sell the same product. The unit production cost is  $c$  and the buyer pays the transport cost  $t$  per unit length. Consumers buy the the unit of the product in a store for which the sum of the price and the transportation cost is smaller. Find the the price of the product in both stores in the Nash equilibrium