Final Exam Models of Applied Mathematics 25 January 2014

All problems are worth 8 points.

1. Find the stationary state for the following birth and death process:

$$\emptyset \to^{k_A} A, \quad A \to^{\gamma_A} \emptyset \quad A + A \to^{\gamma_{AA}} \emptyset,$$

where $A = 0, 1, 2$.

2. Find ground-state configurations (as a function of d and h) of the following Hamiltonian of two interacting spins $s_i = 0, \pm 1$,

$$H(s_1, s_2) = -s_1 s_2 - d(s_1^2 + s_2^2) - h(s_1 + s_2)$$

3. Let us suppose that the price of a product is given by the following function of production levels of two firms: $P(x, y) = 4-(x+y)$, and the cost of producing the unit of the product is equal to 2 for the first company and 1 for the second one. Find the Nash equilibrium of the Stackelberg Duopoly.

4. Find all Nash equilibria for the symmetric game with the following payoff matrix:

U =		A	B	C
	A	6	2	0
	B	2	4	0
	C	2	2	1

5. Find the price of the financial instrument at $t = 0$ paying $\frac{1}{2}S_1 - 10$ at $t = 1$. Assume that $S_0 = 50$, $r = 0.2$, $u = 0.6$, and $d = -0.2$. Construct the replicating portfolio at $t = 0$.

6. Consider the put option with the expiration date at $t = 2$ and $S_0 = 100$, $X = 91$, $r = 0.2$, $u = 0.3$, and $d = -0.1$. Find the value of the option at $t = 0$.

BONUS (10 points)

Consumers are uniformly distributed in the linear town of length 1. There are two stores on boundaries of the town which sell the same product. The unit production cost is c and the buyer pays the transport cost t per unit length. Consumers buy the the unit of the product in a store for which the sum of the price and the transportation cost is smaller. Find the the price of the product in both stores in the Nash equilibrium