# Microscopic models of quasicrystals

J. Jędrzejewski and J. Miękisz, Devil's staircase for nonconvex interactions,
Europhysics Letters 50: 307-311 (2000).

We proved that for certain non-convex one-dimensional infinite-range lattice-gas interactions, molecules consisting of two particles form a molecular devil's staircase in the unique ground-state measure. The structure of the ground set is that of a Cantor set. This is a generalization of Aubry result concerning convex interactions.

J. Miękisz, Stable quasiperiodic ground states, J. Stat. Phys. 88: 691-711 (1997).

Construction of a classical-lattice gas model with finite-range interactions and with a unique quasicrystalline (almost periodic) ground-state measure which is stable in the space of finite-range interactions.

J. Miękisz A Microscopic model with quasicrystalline properties,
J. Stat. Phys. 58: 1137-1149 (1990).

Construction of a classical-lattice gas model with finite-range interactions, without periodic ground-state configurations, and with an infinite sequence of phase transitions of Gibbs states with period-doubling at critical temperatures.

A. van Enter and J. Miękisz, Breaking of periodicity at positive temperatures,
Commun. Math. Phys. 134: 647-651 (1990).

We constructed a non-periodic low-temperature Gibbs state measure in a certain lattice-gas model without periodic ground-state configurations. This is a first known in literature example of such measure and the first microscopic model of a quasicrystal. The construction is based on tangent functionals.

J. Miękisz, Classical lattice gas model with a unique nondegenerate
but unstable periodic ground state configuration,
Commun. Math. Phys. 111: 533-538 (1987).

We constructed a classical lattice-gas model with finite-range interactions and with a unique periodic ground-state configuration which is unstable in arbitrarily low temperatures,
that is it is not in the support of any low-temperature Gibbs state. Our construction provided a counterexample to a Dobrushin-Shlosman hypothesis.

J. Miękisz and C. Radin, The unstable chemical structure of quasicrystalline alloys,
Phys. Lett. 119A: 133-134 (1986).

We proved that the ground-state configurations of a lattice gas-model based on Robinson's non-periodic tilings is unstable with respect to a chemical potential.

# Ergodic theory

J. Miękisz, Ultimate frustration in classical lattice-gas models,
J. Stat. Phys. 90: 285-300 (1998).

We showed that the class of uniquely ergodic ground-state measures of finite-range interactions in lattice-gas models is larger than the class of uniquely ergodic measures of subshifts of finite type. In other words, there are ground-state measures which minimize the energy functional of finite-range interactions which cannot be uniquely characterized by the absence of any finite set of patterns.

C. Gardner, J. Miękisz, C. Radin, and A. van Enter,
Fractal symmetry in an Ising model,
J. Phys. A.: Math. Gen. 22: L1019-L1023 (1989).

W. H. Gottschalk and G. A. Hedlund proved in 1964 that Thue-Morse sequences of two symbols are uniquely characterized by the absence of blocks BBb, where B is any block of symbols and b is the first symbol of the block B. We proved that to characterize Thue-Morse sequences it is enough to forbid blocks consisting of four symbols only. Then we showed that the uniquely ergodic Thue-Morse measure is the unique ground-state measure of the one-dimensional Ising model with exponentially-decaying spin interactions. The result is one of the counterexamples to the crystal problem.

J. Miękisz and C. Radin, Why solids are not really crystalline?
Phys. Rev. B. 39: 1950-1952 (1989).

We showed that in the space of summable interactions (l1 space), interactions with unique ground-state measures which are not supported by periodic ground-state configurations are generic that is they form the set of Baire second cathegory.

J. Miękisz, How low temperature causes long range order,
J. Phys. A.: Math. Gen. 21: L529-L531 (1988).

We showed that in the space of summable interactions (l1 space), interactions with unique ergodic ground-state measures which are not mixing (and therefore having long-range order) are generic that is they form the set of Baire second cathegory.

# Evolutionary game theory

J. Miękisz, Equilibrium selection in evolutionary games with random matching of players, J. Theor. Biol. 232: 47-53 (2005).

Long-run evolution of stochastic dynamics of evolutionary games depends on a method of matching players. We compare well-mixed models where individuals play against an average strategy of a population and random-matching ones where individuals play with few randomly selected opponents, payoffs are then random variables – they depend on particular matching realizations. We study the effects of the size of the population and noise level on the stationary distribution of evolutionary games. We proved the existence of multiple equilibrium transitions in systems of many interacting players.

J. Miękisz, Statistical mechanics of spatial evolutionary games, J. Phys. A: Math. Gen. 37: 9891-9906 (2004).

J. Miękisz, Stochastic stability in spatial games, J. Stat. Phys. 117: 99-110 (2004).

The notion of low-noise ensemble stability is introduced in spatial games with local interactions. It describes the equilibrium behavior of stochastic evolutionary games in the infinite population limit. We show that the infinite population limit and the zero-noise limit (describing the classic notion of stochastic stability) do not commute.

M. Bukowski and J. Miękisz, Evolutionary and asymptotic stability in symmetric multi-player games, Int. J. Game Theory 33: 41-54 (2004).

Classification of three-player symmetric games with respect to the number of Nash equilibria and their evolutionary and asymptotic stability in replicator dynamics.

# Time delays

J. Miękisz, J. Poleszczuk, M. Bodnar, and U. Foryś,
Stochastic model of gene expression with delayed degradation,
Bulletin of Mathematical Biology 73: 2231-2247 (2011).

Time delays usually cause oscillations in dynamical systems. In a recently published paper (Bratsun et al, PNAS, 2005), the authors claimed that delayed degradation causes oscillations in a simple model of gene expression. We pointed out a methodological error present in their paper, proposed our own model and showed that oscillations are not present.

J. Alboszta and J. Miękisz,
Stability of evolutionarily stable strategies in discrete replicator dynamics with time delay,
J. Theor. Biol. 231: 175-179 (2004).

We proposed a biological-type model of replicator dynamics with a time delay where new players are born from parents who played in the past. We proved that unlike in social-type models studied before, where in the adaptation process players take into account information about strategies and payoff of their opponents at some earlier time, interior evolutionarily stable strategy is also globally asymptotically stable. We also provided an elementary proof (without the reference to the theory of ordinary differential equations with time delays) that in the usual social-type replicator dynamics, the interior stationary state is asymptotically stable for small time delays and becomes unstable for large ones.

# Gene expression and regulation

M. Komorowski, J. Miękisz, and M. P. H. Stumpf,
Decomposing noise in biochemical signalling systems highlights the role of protein degradation,
Biophysical Journal, in press (2013).

We show that in a fairly general class of biochemical processes, the stationary variance attributed to the degradation of the final product is equal to its mean.

M. Komorowski, J. Miękisz, and A. Kierzek,
Translational repression contributes greater noise to gene expression than transcriptional repression, Biophysical Journal 96: 372--384 (2009).

We analyzed four simple modules of gene regulation: transcriptional and translational repression by means of etiher mRNA or proteins. We linearize Master equations and derived analytical formulas for the variance and the Fano factor of the number of proteins in the stationary state. We showed that the Fano factor in the transcription repression is bigger than in the translation one.