

Complex interpolation theory and Sobolev spaces

Agnieszka Kałamańska

University of Warsaw

Abstract

Let A and B be the given Banach spaces. We are interested in complex interpolation theory, roughly speaking in establishing the “intermediate spaces” between $A \cap B$ and $A + B$:

$$A \cap B \subseteq [A, B]_{\Theta} \subseteq A + B, \quad \Theta \in (0, 1),$$

which are constructed in a certain way with the help of complex analysis.

By the Stein-Weiss Theorem (1958), when considering weighted L^p spaces as A, B , one has the complete description of the interpolation spaces. We will focus on the case when A and B are first order weighted Sobolev spaces. They consist of functions such that the functions and their first order derivatives belong to the weighted L^p spaces.

However some results in this direction are known, for example those by Jörgen and Löffström (1982), or Cwikel and Einav (2019), in general there are many open questions in that theory.