International Banach Prize PTM Conference

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Banach Center Warsaw

Invited lecture

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Title:

Dynamics of $\mathscr{B}$-free systems generated by Behrend sets

Abstract:

This is a joint work with Mariusz Lemańczyk and Sebastian Zuniga Alterman. Given a set $\mathscr{B}$ of natural numbers, not containing 1, we denote by $\mathcal{F}_{\mathscr{B}}$ the set of $\mathscr{B}$-free numbers, that is, $\mathcal{F}_{\mathscr{B}} = \mathbb{Z} \setminus \bigcup_{b\in\mathscr{B}} b\mathbb{Z}$. Let X_η be the $\mathscr{B}$-free subshifts, that is the subshift induced by η, where η denotes the characteristic function of $\mathcal{F}_{\mathscr{B}}$. That means, X_η is the closure of the set of all shifts of η in the space $\{0,1\}^{\mathbb{Z}}$ equipped with the product topology. We are interested in the case when $\mathscr{B}$ is a Behrend set, that is, when the set of $\mathscr{B}$-free numbers has zero density. It turns out that this happens precisely when X_η is proximal and has zero entropy. We prove that the complexity of X_η, with $\mathscr{B}$ being a Behrend set, can achieve any subexponential growth. We also estimate the complexity for some classical subshifts (prime and semiprime subshifts). The lower estimates are obtained conditionally on Hardy-Littlewood Conjecture or Dickson Conjecture. Toghether with the $\mathscr{B}$-free shift we investigate the $\mathscr{B}$-admissible shift $X_{\mathscr{B}}$ and we provide a dynamical characterization of the subshifts generated by the Erdös sets $\mathscr{B}$ (i.e. infinite, coprime and not Behrend).