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Title:

Singular integrals - classical and Dunkl setting

Abstract:

The singular integral operators are the central objects of classical harmonic analysis having connections with many other objects, like e.g.: the Fourier transform, Fourier multipliers, the Riesz and Bessel potentials, harmonic functions, Sobolev spaces, Hardy and BMO spaces. They are broadly applicable in the other branch of mathematics, especially in partial differential equations. The simplest example of a singular integral is the Hilbert transform

$$Hu(x) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \int_{|x-y|>\varepsilon} \frac{u(y)}{x-y}\, dy$$

which relates the boundary values of the real and imaginary parts a holomorphic function in the upper half space. The operator H is bounded on $L^p(\mathbb{R})$ for $1 < p < \infty$, but it is not the case for $p = 1$, which leads us to a definition of the Hardy space $H^1 = \{u \in L^1(\mathbb{R}) : Hu \in L^1(\mathbb{R})\}$. The theory of H was generalized to higher dimensions by considering systems of functions satisfying the generalized Cauchy-Riemann equations. Therefore, a very natural question is to study the other operators, which generalize Hilbert and Riesz transforms and still posses their crucial properties. The motivation comes from the classic works of A. Calderón and A. Zygmund [2], where the authors consider singular integrals of convolution type

$$Kf(\mathbf{x}) = \text{p.v.} \int_{\mathbb{R}^N} K(\mathbf{x}-\mathbf{y}) f(\mathbf{y})\, d\mathbf{y}$$

and provide conditions on the kernel $K(\mathbf{x})$ which guarantee the boundedness of K on L^p-spaces. Later on, this concept was intensively studied and generalized by many mathematicians. During my talk, I am going to present the main ideas of classical theory and some recent results concerning singular integrals in the Dunkl setting. The Riesz transforms in the Dunkl setting were introduced in [3, Theorem 5.3]. Then, in [1, Theorem 3.3], Amri and Sifi proved that they extend to a bounded operators $L^p(dw) \longmapsto L^p(dw)$ for $p > 1$. The results were inspiration for further studies on more generalized Dunkl singular integrals, which can be helpful in better understanding of the classical theory as well.

References

[1] B. Amri and M. Sifi, *Riesz transforms for Dunkl transform*, Ann. Math. Blaise Pascal 19 (2012), no. 1, 247–262.

[2] A. Calderón, A. Zygmund, *On the existence of certain singular integrals*, Acta Math. 88 (1952), 85–139.

[3] S. Thangavelu and Y. Xu, *Convolution operator and maximal function for the Dunkl transform*, J. Anal. Math. 97 (2005), 25–55.